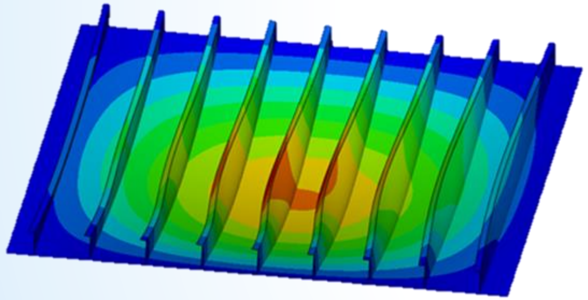

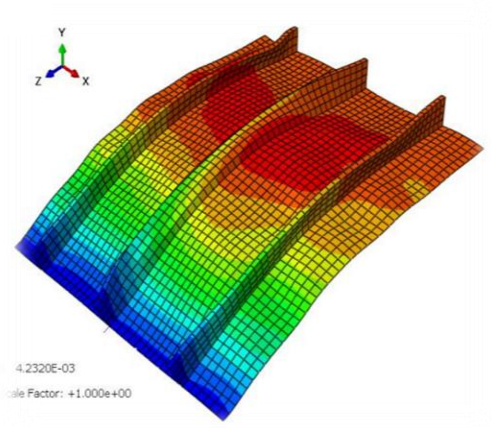

* Buckling of Stiffened Panels (Topic 9)

(Tripping)

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Adv. Local Structural Design & Analysis of Marine Structures (Overview)

[Theory of Plates and Grillages]

[Part I] Plastic Design of Structures

- Plastic theory of bending (Topic 1)
- Ultimate loads on beams (Topic 2)
- Collapse of frames and grillage structures (Topic 3)

[Part II] Elastic Plate Theory under Pressure

- Basic (Topic 4)
- Simply supported plates under Sinusoidal Loading (Topic 5)
- Long clamped plates (Topic 6)
- Short clamped plates (Topic 7)
- Low aspect ratio plates, strength & permanent set (Topic 7A)

[Part III] Buckling of Plates & Stiffened Panels

- Failure modes (Topic 8)
- Tripping (Topic 9) + Post-buckling strength of plate (Topic 9A)
- Post-buckling behaviour (Topic 10)

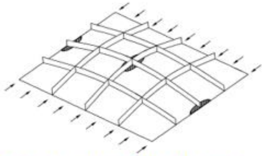
Reminder

Overall Picture

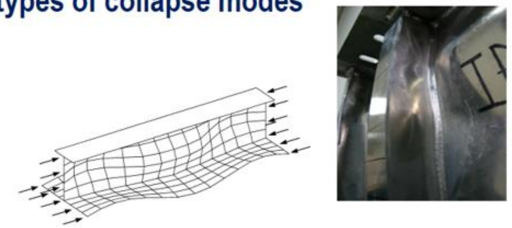
ALPS/ULSAP

- Ultimate strength of stiffened panels: 6 types of collapse modes

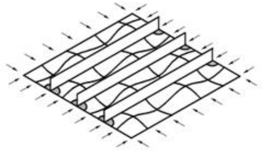
$\sigma_u = \min.(\sigma_u^{I}, \sigma_u^{II}, \sigma_u^{III}, \sigma_u^{IV}, \sigma_u^{V}, \sigma_u^{VI})$



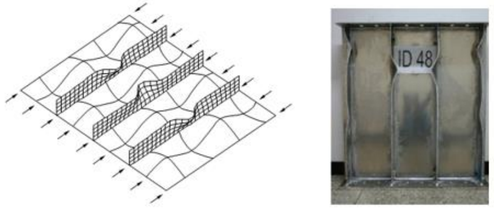
Mode I – overall collapse



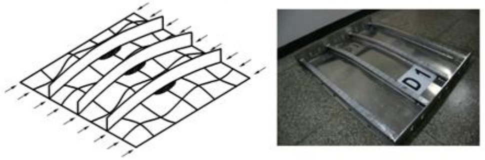
Mode IV – stiffener-induced collapse by web buckling



Mode II – plate-induced collapse

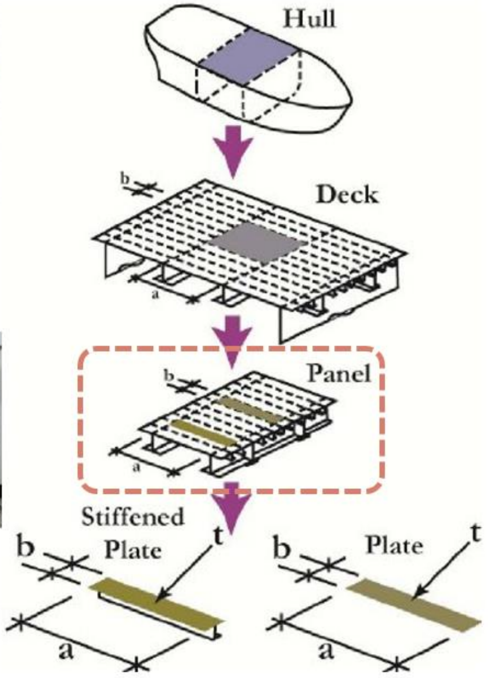


Mode V – stiffener-induced collapse by tripping



Mode III – stiffener-induced collapse by beam-column type collapse

Mode VI: Gross yielding



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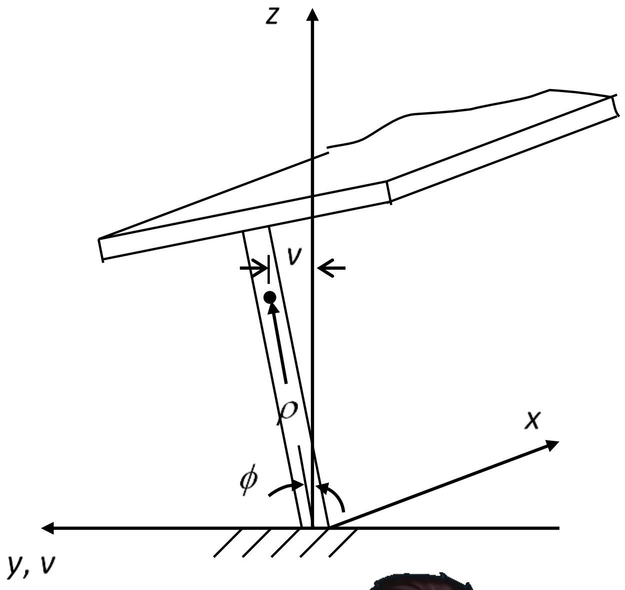
Objectives

The aim of this lecture is:

- To equip you with the knowledge & understanding of tripping of stiffeners.



At the end of this lecture, we should be able to:

- Derive the formula for the critical stress of tripping a stiffener.
- Calculate the critical stress of tripping a stiffener of rolled or built-up sections.
- Be aware of warping effects on tripping.

#Tripping #Stiffener #StiffenedPanel

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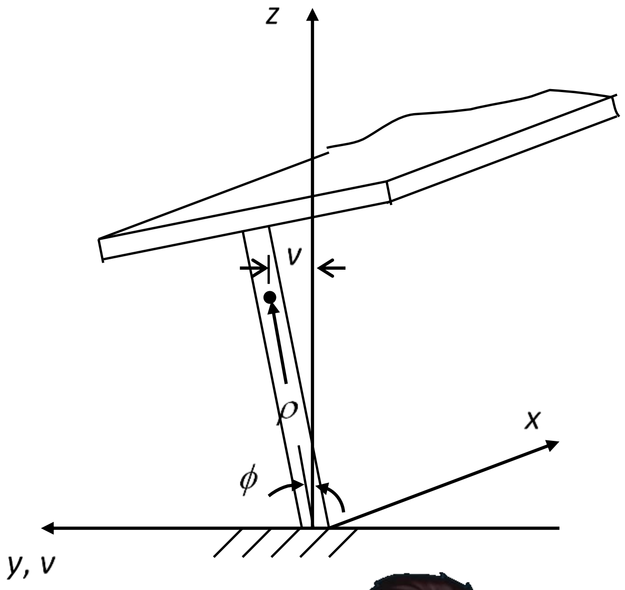
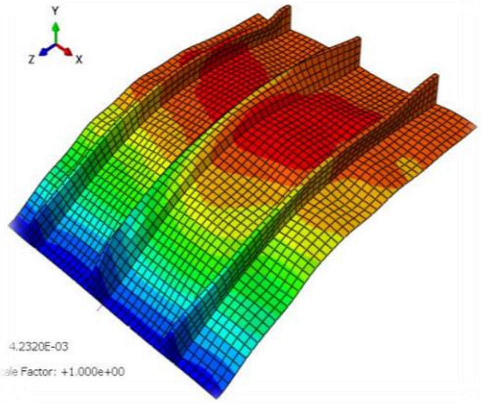
Introduction

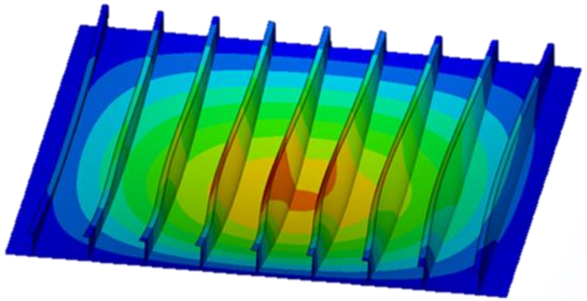
- When a grillage experiences in-plane compression, tripping of stiffeners will occur if
  - stiffeners have weak torsional strength
  - long slender web (depth (d_w)/thickness (t_w) > 15)



- To avoid tripping
  - $d_w/t_w < 12$ is required.
  - tripping brackets are fitted

However, it is necessary to estimate the strength of stiffener against tripping.





Lecture 9.1:

Tripping of stiffeners

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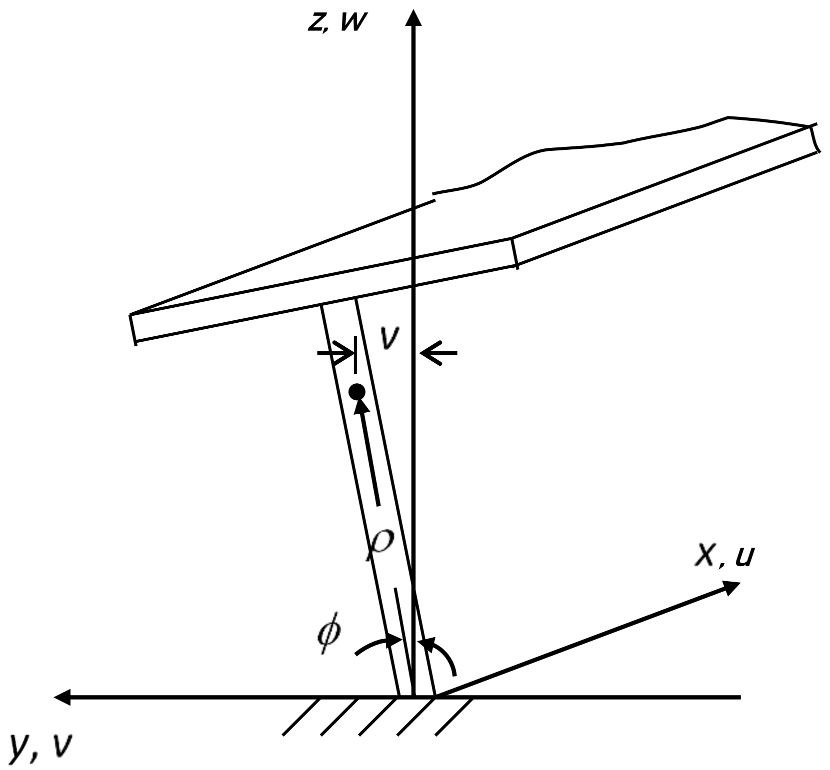
Tripping of stiffeners (1/6)

The word 'tripping' is used because by virtual of the plate to which the stiffener is attached. The toe cannot move sideways and therefore any lateral buckling of the stiffener entails some torsion (about the centroid of stiffener) and some lateral bending of the stiffener.



Tripping of stiffeners (2/6)

There are 3 major components of strain energy in the buckled form:

1. Sideways (lateral) bending strain energy

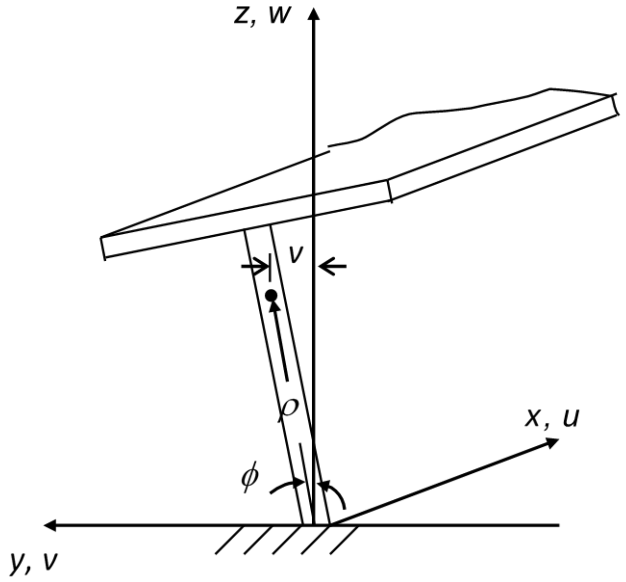
$$U_B = \tfrac{1}{2} EI_z \int_0^L \left(\frac{\partial^2 v}{\partial x^2} \right)^2 dx$$

where,

v is lateral deflection due to stiffener buckling.

I_z is second moment area of the stiffener cross-section about z-axis.

L is the span of the stiffener.



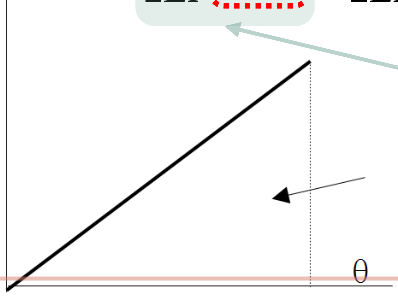
$\boxed{L = R\theta}$

$$\boxed{M = EI \frac{d^2 v}{dx^2}}$$

Simple beam theory

$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}} \rightarrow \boxed{\frac{1}{R} = \frac{M}{EI}} \rightarrow \boxed{\frac{1}{R} = \frac{d^2 v}{dx^2} \quad or \quad \frac{d^2 w}{dx^2}}$$

$$\boxed{\theta} = \frac{L}{R} = \left(\frac{1}{R}\right) L = \left(\frac{M}{EI}\right) L = \boxed{\frac{ML}{EI}}$$



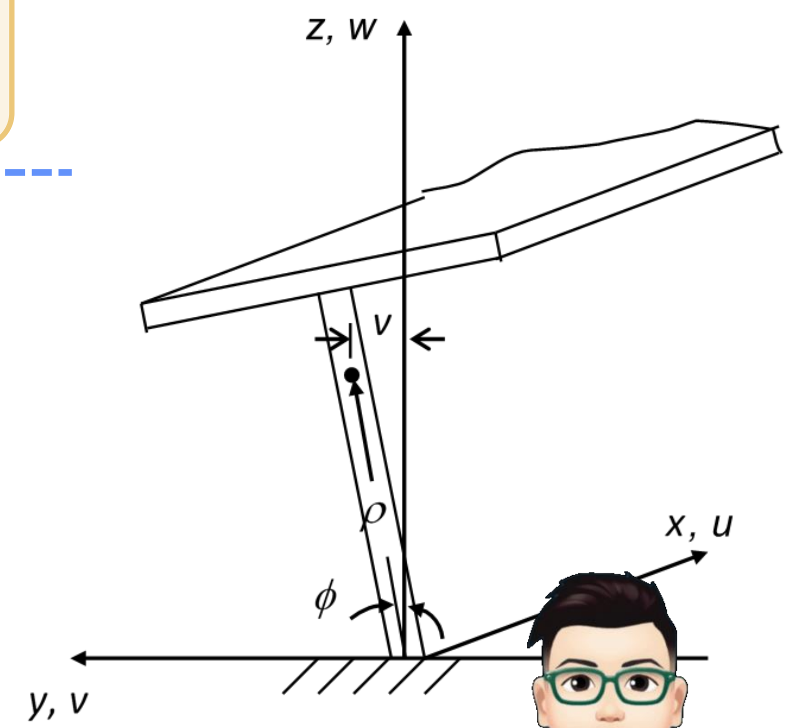
$$U_B = \frac{L}{2EI}\left(M^2\right) = \frac{L}{2EI}\left(EI\frac{d^2 v}{dx^2}\right)^2 = \tfrac{1}{2} EI_z \int_0^L \left(\frac{\partial^2 v}{\partial x^2} \right)^2 dx$$

$$U = W = \frac{M\theta}{2} = \frac{M^2 L}{2EI}$$

Tripping of stiffeners (3/6)

1. Sideways (lateral) bending strain energy

$$U_B = \tfrac{1}{2} EI_z \int_0^L \left(\frac{\partial^2 v}{\partial x^2} \right)^2 dx$$



2. Torsional strain energy

$$U_T = \tfrac{1}{2} GJ \int_0^L \left(\frac{\partial \phi}{\partial x} \right)^2 dx$$

where

ϕ is angular displacement of the stiffener.

J is torsional constant.

3. Strain energy due to stiffness k (= M(x)/ϕ)

$$U_S = \tfrac{1}{2} k \int_0^L \phi^2 dx$$

where

k is the stiffness of rotational constraint provided by attached plating

Tripping of stiffeners (4/6)

$$U_B = \tfrac{1}{2} EI_z \int_0^L \left(\frac{\partial^2 v}{\partial x^2} \right)^2 dx \qquad U_T = \tfrac{1}{2} GJ \int_0^L \left(\frac{\partial \phi}{\partial x} \right)^2 dx \qquad U_S = \tfrac{1}{2} k \int_0^L \phi^2 dx$$

- There are also some strain energy components due to longitudinal and transverse warping of the stiffener cross-section but for most purpose they can be ignored.
- The total strain energy is $U = U_B + U_T + U_S$

$$\therefore \quad U = \tfrac{1}{2} EI_z \int_0^L \left(\frac{\partial^2 v}{\partial x^2} \right)^2 dx + \tfrac{1}{2} GJ \int_0^L \left(\frac{\partial \phi}{\partial x} \right)^2 dx + \tfrac{1}{2} k \int_0^L \phi^2 dx$$

[Tip] How to remember?

The effect of

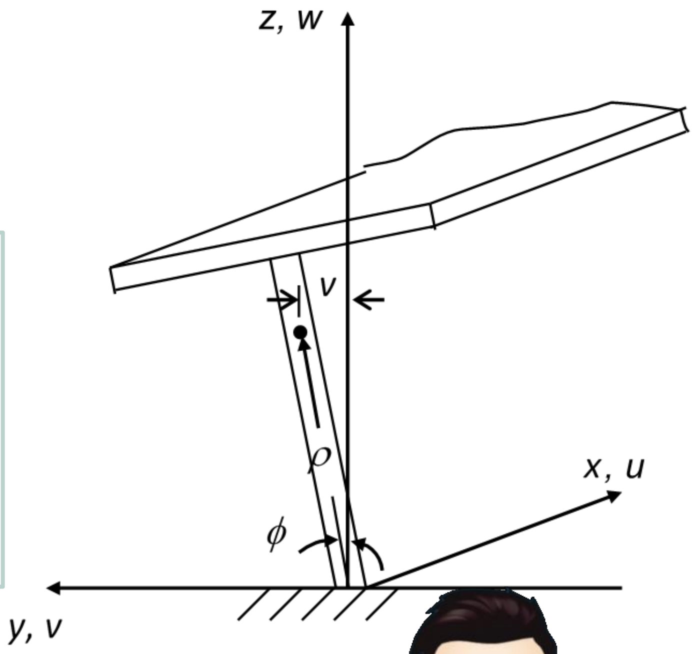
Bending + Torsion + Stiffness

Source: BTS ARMY

Tripping of stiffeners (5/6)

Work done in compression by critical end stress σ_{cr} is

$$W = F \times d = \oint \sigma_{cr}\, u\, dA$$



where integration is throughout the cross-section A and u is the approach of one end of a filament of area dA to the other end, i.e. the shortening of the stiffener over the span.

Since $u = \frac{1}{2}\int_0^L \left(\frac{\partial v}{\partial x}\right)^2 dx$ and $v = \rho\, \phi$ Then $u = \frac{1}{2}\rho^2 \int_0^L \left(\frac{\partial \phi}{\partial x}\right)^2 dx$

Hence, $W = \oint \left[\sigma_{cr}\, \frac{1}{2}\rho^2 \int_0^L \left(\frac{\partial \phi}{\partial x}\right)^2 dx\right] dA$ $\quad \therefore \quad W = \frac{1}{2}\sigma_{cr} \oint \rho^2 dA \int_0^L \left(\frac{\partial \phi}{\partial x}\right)^2 dx$

$$\therefore \quad W = \frac{1}{2}\sigma_{cr}\, I_o \int_0^L \left(\frac{\partial \phi}{\partial x}\right)^2 dx$$

where I_o is the polar moment of stiffener cross-section.

Tripping of stiffeners (6/6)

$$\prod = U + W \ \ (Total\ \ Potential\ Energy)$$

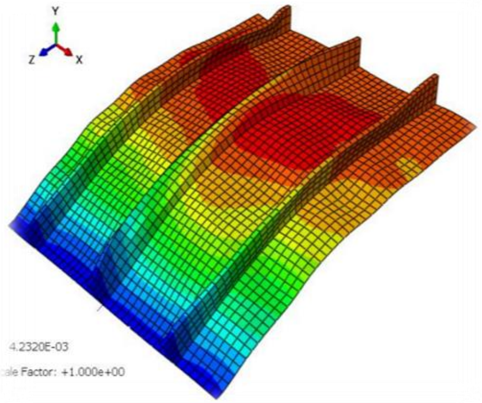
For stability, $W = U$ or $W \le U$

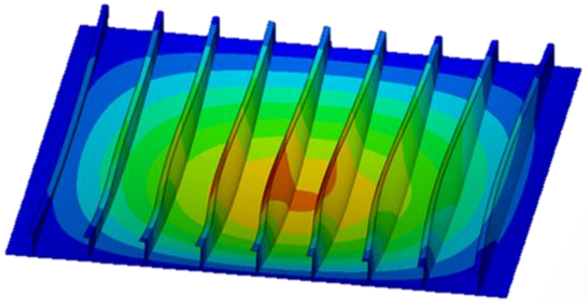
$$\boxed{\therefore \quad \tfrac{1}{2}\sigma_{cr} I_o \int_0^L \left(\frac{\partial \phi}{\partial x}\right)^2 dx = \tfrac{1}{2} EI_z \int_0^L \left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx + \tfrac{1}{2} GJ \int_0^L \left(\frac{\partial \phi}{\partial x}\right)^2 dx + \tfrac{1}{2} k \int_0^L \phi^2 dx}$$

As $\upsilon = \rho\phi$, we have $$\left(\frac{\partial^2 v}{\partial x^2}\right)^2 = \rho^2 \left(\frac{\partial^2 \phi}{\partial x^2}\right)^2 \approx \bar{z}^2 \left(\frac{\partial^2 \phi}{\partial x^2}\right)^2$$

$$\therefore \quad \sigma_{cr} = \frac{EI_z \bar{z}^2 \int_0^L \left(\frac{\partial^2 \phi}{\partial x^2}\right)^2 dx + GJ \int_0^L \left(\frac{\partial \phi}{\partial x}\right)^2 dx + k \int_0^L \phi^2 dx}{I_o \int_0^L \left(\frac{\partial \phi}{\partial x}\right)^2 dx}$$

This critical buckling stress due to tripping of stiffener can be determined when the buckling shape which satisfies the necessary boundary conditions of the stiffener is found.





Lecture 9.2:

Simply supported stiffener ends

w/o rotation about x-axis but free to warp

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[Reminder of the previous learning] Column collapse (1/3)

Outline of Lecture

- Effect of initial deflect
- Effect of eccentric loa
- Slenderness / Yield e

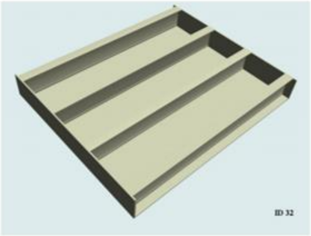

Figure 15(a) One-bay prototype structure

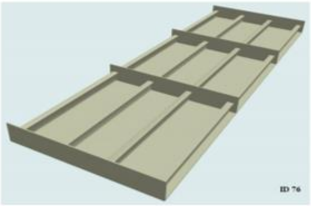

Figure 15(b) Three-bay prototype structure

http://www.shipstructure.org/pdf/2007symp01.pdf

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[Reminder of the previous learning] Column collapse (2/3)

Outline of Lecture

- Effect of initial deflection.
- Effect of eccentric load.
- Slenderness / Yield effects / Inelastic effects.

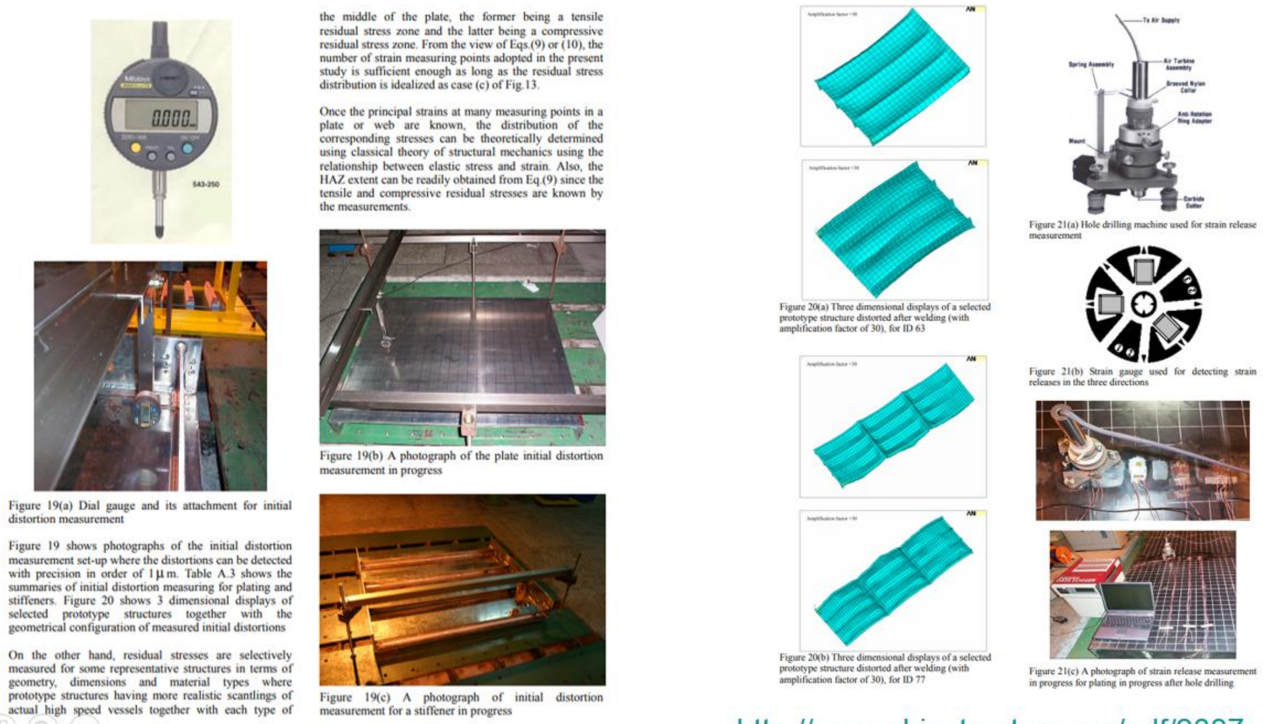


Figure 19(a) Dial gauge and its attachment for initial distortion measurement

Figure 19 shows photographs of the initial distortion measurement set-up where the distortions can be detected with precision in order of 1 µm. Table A.3 shows the summaries of initial distortion measuring for plating and stiffeners. Figure 20 shows 3 dimensional displays of selected prototype structures together with the geometrical configuration of measured initial distortions

On the other hand, residual stresses are selectively measured for some representative structures in terms of geometry, dimensions and material types where prototype structures having more realistic scantlings of actual high speed vessels together with each type of

the middle of the plate, the former being a tensile residual stress zone and the latter being a compressive residual stress zone. From the view of Eqs.(9) or (10), the number of strain measuring points adopted in the present study is sufficient enough as long as the residual stress distribution is idealized as case (c) of Fig.13.

Once the principal strains at many measuring points in a plate or web are known, the distribution of the corresponding stresses can be theoretically determined using classical theory of structural mechanics using the relationship between elastic stress and strain. Also, the HAZ extent can be readily obtained from Eq.(9) since the tensile and compressive residual stresses are known by the measurements.

Figure 19(b) A photograph of the plate initial distortion measurement in progress

Figure 19(c) A photograph of initial distortion measurement for a stiffener in progress

Figure 20(a) Three dimensional displays of a selected prototype structure distorted after welding (with amplification factor of 30), for ID 63

Figure 20(b) Three dimensional displays of a selected prototype structure distorted after welding (with amplification factor of 30), for ID 77

Figure 21(a) Hole drilling machine used for strain release measurement

Figure 21(b) Strain gauge used for detecting strain releases in the three directions

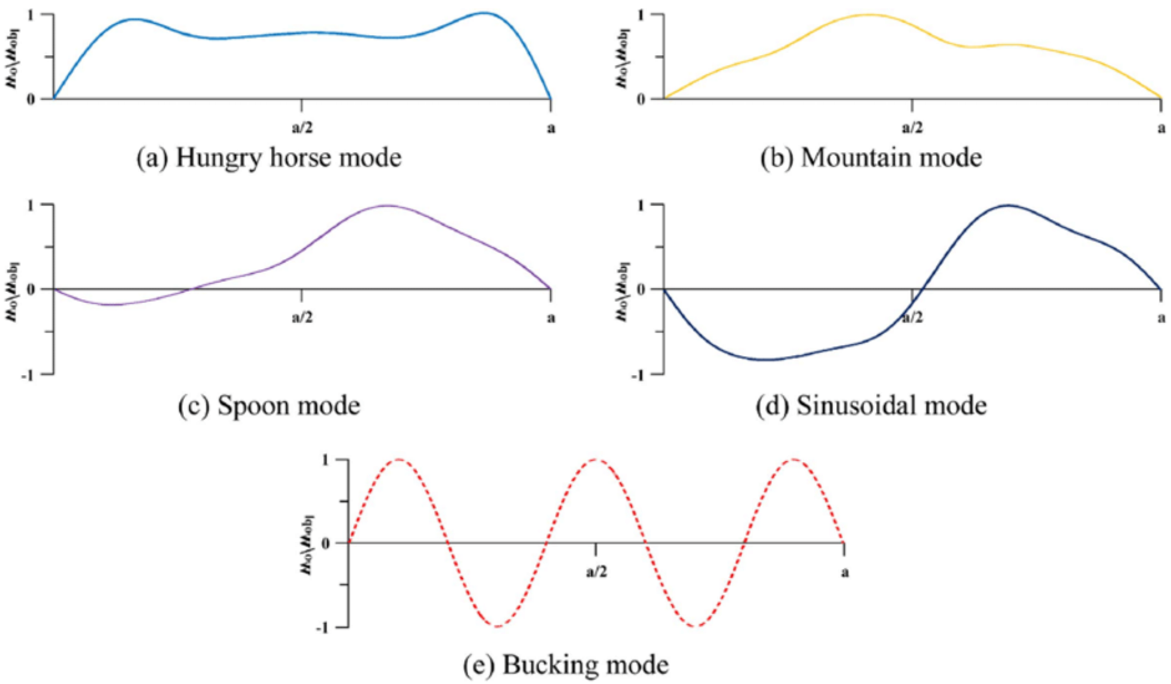
Figure 21(c) A photograph of strain release measurement in progress for plating in progress after hole drilling

http://www.shipstructure.org/pdf/2007symp01.pdf

[Reminder of the previous learning] Column collapse (3/3)

Outline of Lecture

- Effect of initial deflection.
- Effect of eccentric load.
- Slenderness / Yield effects / Inelastic effects.



Source: J.K. Paik et al. (2018) - Book or D.K. Kim et al. (2012) Structural Engineering and Mechanics – Journal paper

Simply supported stiffener ends – w/o rotation about x-axis but free to warp (1/4)

$$\therefore \quad \sigma_{cr} = \left[EI_z \bar{z}^2 \int_0^L \left(\frac{\partial^2 \varphi}{\partial x^2} \right)^2 dx + GJ \int_0^L \left(\frac{\partial \varphi}{\partial x} \right)^2 dx + k \int_0^L \varphi^2 dx \right] \Bigg/ \left[I_o \int_0^L \left(\frac{\partial \varphi}{\partial x} \right)^2 dx \right]$$

Simply supported stiffener ends without rotation about x-axis but free to warp

Assume buckling shape is $\boxed{\phi = a_m \sin \frac{m\pi x}{L}}$

which satisfies boundary conditions: $\phi = 0$ and $\partial^2 \phi / \partial x^2 = 0$ at x = 0 and x = L.

Then, $\frac{\partial \phi}{\partial x} = a_m \frac{m\pi}{L} \cos \frac{m\pi x}{L}$ and $\frac{\partial^2 \phi}{\partial x^2} = -a_m \frac{m^2 \pi^2}{L^2} \sin \frac{m\pi x}{L}$

since $\int_0^L \sin^2 \frac{m\pi x}{L} dx = \int_0^L \cos^2 \frac{m\pi x}{L} dx = \boxed{\frac{L}{2}}$

we have $\boxed{\sigma_{cr} = \left(EI_z \bar{z}^2 \frac{m^2 \pi^2}{L^2} + GJ + k \frac{L^2}{m^2 \pi^2} \right) / I_o}$

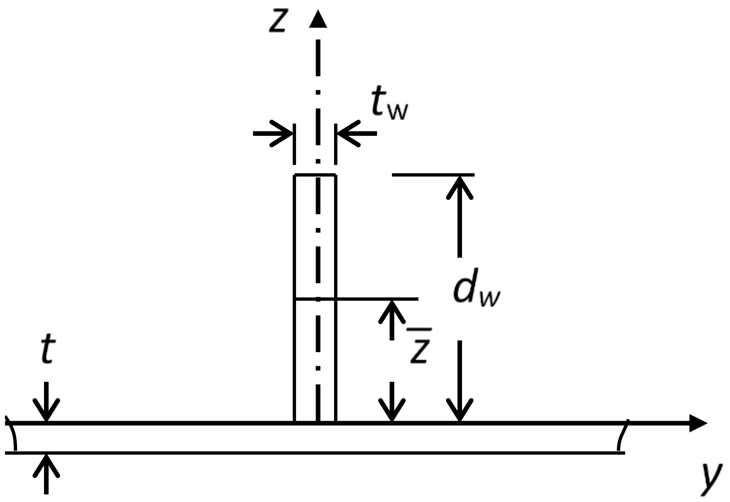
when $\partial \sigma_{cr} / \partial m = 0$ minimum σ_{cr} occurs. This leads to

$$\boxed{\text{Integer } m = \frac{L}{\pi} \left(\frac{k}{EI_z \bar{z}^2} \right)^{1/4} > 1}$$

Simply supported stiffener ends – w/o rotation about x-axis but free to warp (2/4)

Example

Find the expression for the tripping of flat bar stiffener due to buckling.



Simply supported stiffener ends – w/o rotation about x-axis but free to warp (3/4)

Example (Solution)

Section properties of a flat bar stiffener:

- The second moment of area about the base is

$$I_y = \frac{d_w^3 t_w}{3}$$

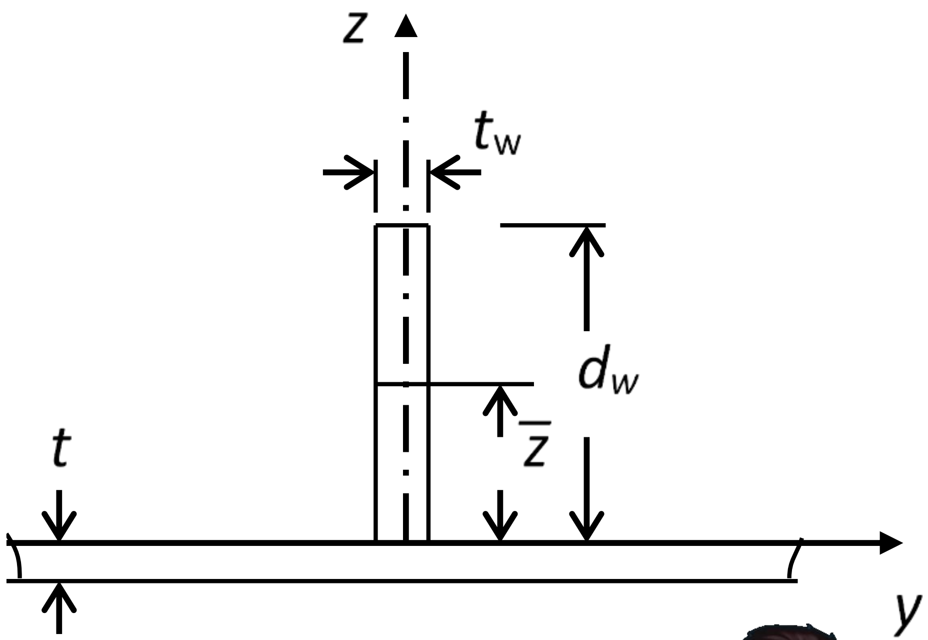
- The second moment of area about z-axis is

$$I_z = \frac{d_w t_w^3}{12}$$

- The torsional constant is $J = \dfrac{d_w t_w^3}{3}$

- The polar moment of area about the base is $I_o = I_y + I_z = \dfrac{d_w^3 t_w}{3} + \dfrac{d_w t_w^3}{12} \approx \dfrac{d_w^3 t_w}{3}$

- The centroid of cross-sectional area above the base is $\bar{z} = \frac{1}{2} d_w$



Simply supported stiffener ends - w/o rotation about x-axis but free to warp (4/4)

$$\sigma_{cr} = \left(EI_z \bar{z}^2 \frac{m^2\pi^2}{L^2} + GJ + k\frac{L^2}{m^2\pi^2} \right) / I_o$$

$$I_y = \frac{d_w^3 t_w}{3} \qquad I_z = \frac{d_w t_w^3}{12} \qquad J = \frac{d_w t_w^3}{3} \qquad I_o = \frac{d_w^3 t_w}{3} \qquad \bar{z} = \tfrac{1}{2} d_w$$

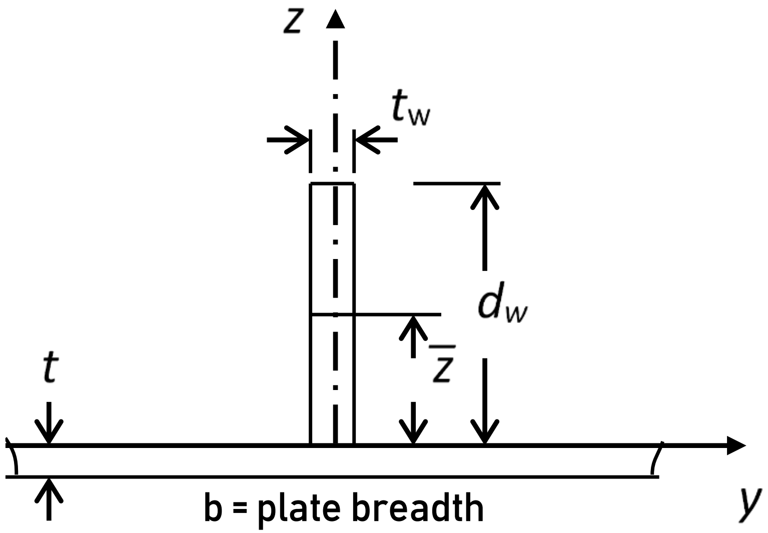
Example (Solution)

The span L = a

$$G = \frac{E}{2(1+\nu)}$$

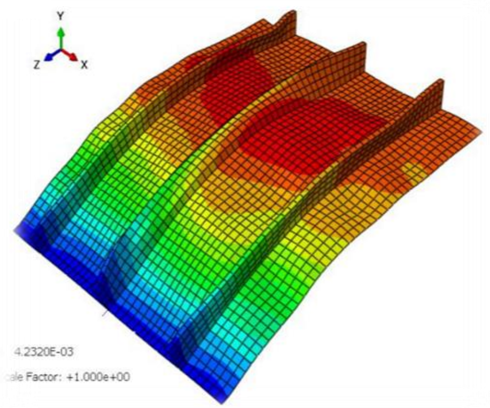
The stiffness (plate)

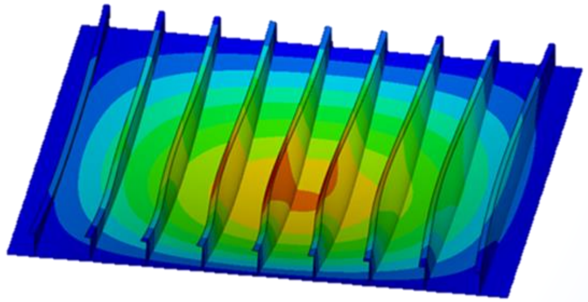
$$k = \frac{Et^3}{3b(1-\nu^2)}$$



Substituting the above information into the equation for critical tripping of stiffener gives

$$\sigma_{cr} = E\frac{t_w^2 m^2\pi^2}{16a^2} + G\left(\frac{t_w}{d_w}\right)^2 + \frac{Et^3a^2}{b(1-\nu^2)d_w^3 t_w m^2\pi^2} \quad \text{for flat bar}$$





Lecture 9.3:

Built-in stiffener ends w/o warping

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Built-in Stiffener Ends w/o Warping

- Assume buckling shape is $\phi = a_m\left(1-\cos\frac{2m\pi x}{L}\right)$ which satisfies boundary conditions: $\phi = 0$ and $\partial\phi/\partial x = 0$ at x = 0 and x = L.

Then, $\frac{\partial\phi}{\partial x} = 2a_m\frac{m\pi}{L}\sin\frac{2m\pi x}{L}$ and $\frac{\partial^2\phi}{\partial x^2} = 4a_m\frac{m^2\pi^2}{L^2}\cos\frac{2m\pi x}{L}$

Since, $\int_0^L \sin^2\frac{m\pi x}{L}dx = \int_0^L \cos^2\frac{m\pi x}{L}dx = \frac{L}{2}$ and $\int_0^L \left(1-\cos\frac{2m\pi x}{L}\right)^2 dx = \frac{3L}{2}$

Then

$$\boxed{\sigma_{cr} = \left(16EI_z\bar{z}^2\frac{m^2\pi^2}{L^2} + 4GJ + 3k\frac{L^2}{m^2\pi^2}\right)/(4I_o)}$$

When $\partial\sigma_{cr}/\partial m = 0$ minimum σ_{cr} occurs. This leads to

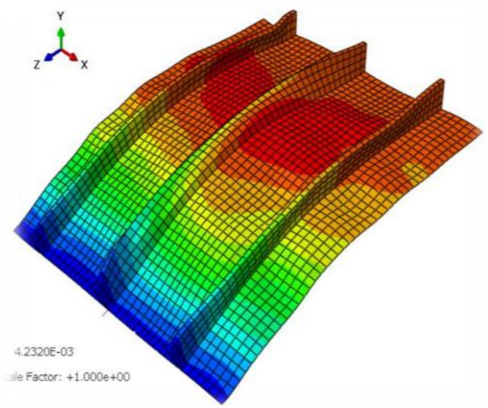
C.L.

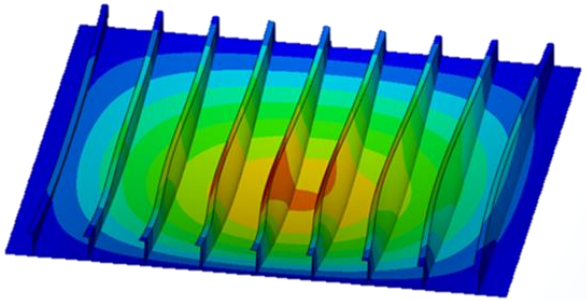
$$\boxed{\text{Integer} \quad m = \frac{L}{2\pi}\left(\frac{3k}{EI_z\bar{z}^2}\right)^{1/4} > 1}$$

S.S.

$$\text{Integer} \quad m = \frac{L}{\pi}\left(\frac{k}{EI_z\bar{z}^2}\right)^{1/4} > 1$$

Comparing this with that of simply supported stiffener, the term for k is 3 times greater than that for simply supported case. This means that the built-in ends give more stable ability and the buckling form is in higher order (i.e. large number of waves).





*Lecture 9.4:

Design Considerations

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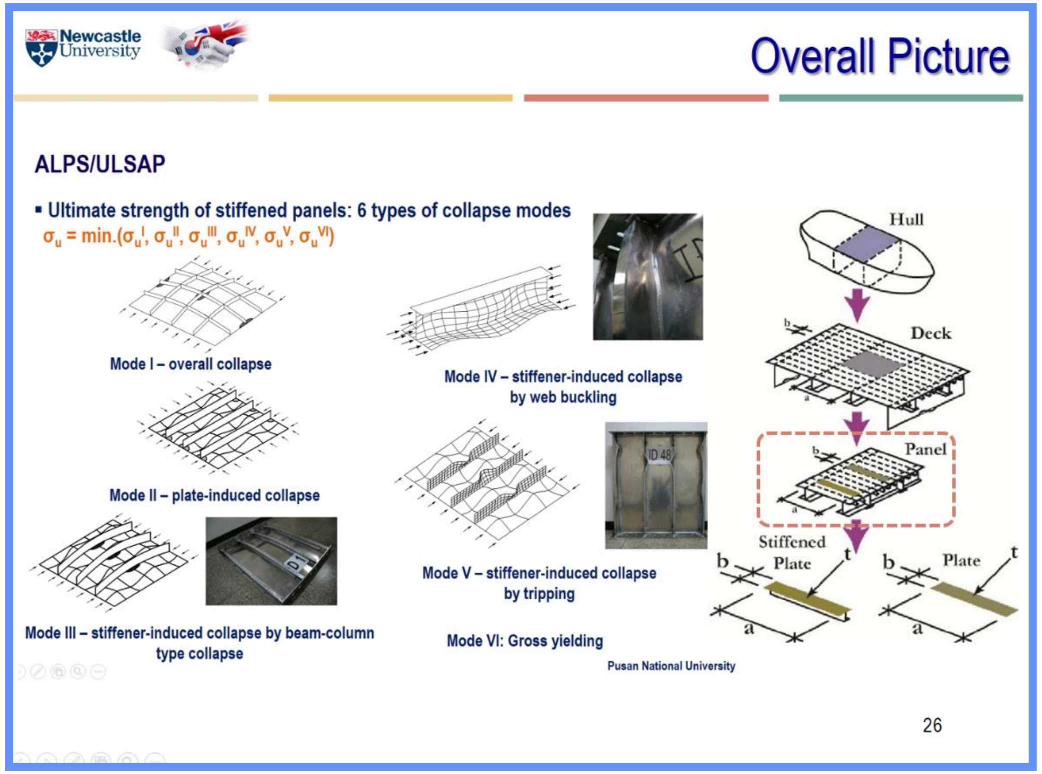
Design considerations of plate and stiffened panel

Local buckling load

Following calculations should be carried out to ensure whether the strength of the plate and stiffened panel under local buckling load are acceptable or not.

- Check column buckling strength of stiffened panel against plate-induced failure (PIF) or stiffener-induced failure (SIF)
- Check buckling strength of stiffeners against tripping
- Check torsional buckling strength of primary member against overall buckling
- Check bending strength of plate-stiffener combination (PSC) against local bending
- Check plate strength against local load
- Check bending and shear strength of primary members



Learning Outcomes (Review)

- We have investigated the Tripping of Stiffeners.

- Now we are able to:
  - Derive the formula for the critical stress of tripping a stiffener.
  - Calculate the critical stress of tripping a stiffener of rolled or built-up section.
  - Be aware of post-buckling behaviour of plate
    (This will be continued in Topic 9A)

- Details can be referred to topics 9 in the lecture notes.

Adv. Marine Structures / Adv. Structural Design & Analysis (Next Lecture)

[Theory of Plates and Grillages]

[Part I] Plastic Design of Structures

- Plastic theory of bending (Topic 1)
- Ultimate loads on beams (Topic 2)
- Collapse of frames and grillage structures (Topic 3)

[Part II] Elastic Plate Theory

- Basic (Topic 4)
- Simply supported plates under Sinusoidal Loading (Topic 5)
- Long clamped plates (Topic 6)
- **Short clamped plates (Topic 7)**
- Additional (Low aspect ratio plates, strength & permanent set)

[Part III] Buckling of Stiffened Panels

- Failure modes (Topic 8)
- Tripping (Topic 9) + Post-buckling strength of plate (Topic 9A)
- Post-buckling behaviour (Topic 10)

Kam Sa Hab Ni Da

감사합니다

Thank you!

Questions?

Aerial View of Korean Presidential Archives in Sejong city
(Construction Completed in 2014)