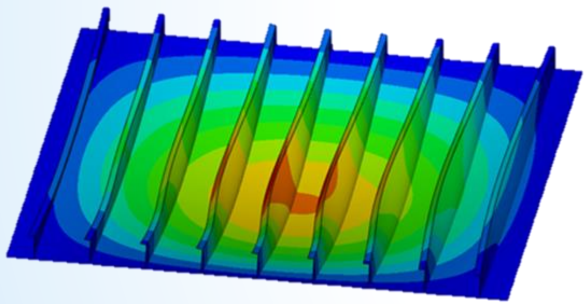
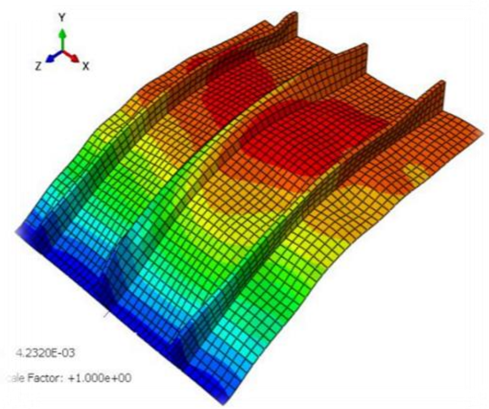
Topics in Ship Structures

(Advanced Local Structural Design & Analysis of Marine Structures)





* Buckling of Stiffened Panels (Topic 9A)

(Post-buckling Strength of Long Plate)

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Adv. Local Structural Design & Analysis of Marine Structures (Overview)

[Theory of Plates and Grillages]

[Part I] Plastic Design of Structures

- Plastic theory of bending (Topic 1)
- Ultimate loads on beams (Topic 2)
- Collapse of frames and grillage structures (Topic 3)

[Part II] Elastic Plate Theory under Pressure

- Basic (Topic 4)
- Simply supported plates under Sinusoidal Loading (Topic 5)
- Long clamped plates (Topic 6)
- Short clamped plates (Topic 7)
- Low aspect ratio plates, strength & permanent set (Topic 7A)

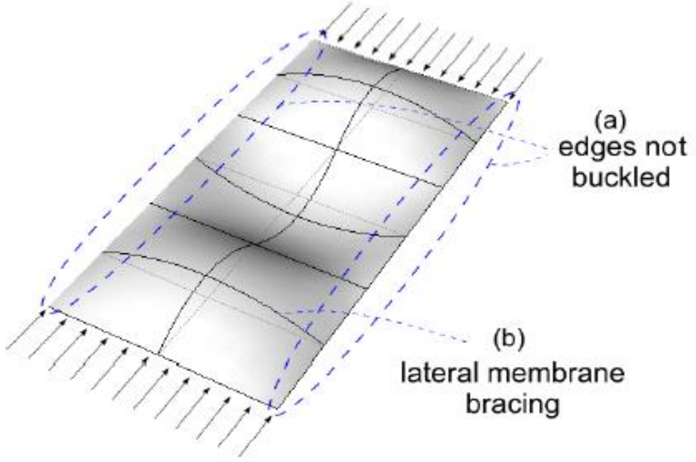
[Part III] Buckling of Stiffened Panels

- Failure modes (Topic 8)
- Tripping (Topic 9) + Post-buckling strength of plate (Topic 9A)
- Post-buckling behaviour (Topic 10)

Post-Buckling Strength of Long Plates (1/3)

In this lecture we will

- Discuss von-Karman's concept of post buckling behavior
- Compare to existing empirical formulations
  (i.e., Faulkner's and Paik's equations, and many Others)

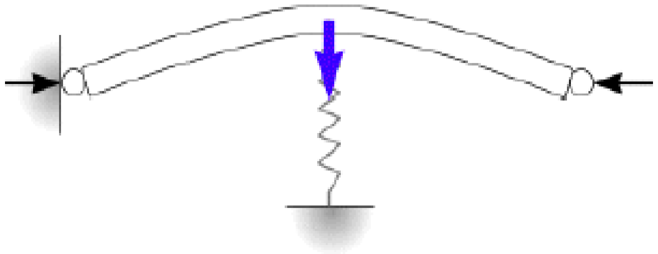


Long Plates/Post Buckling (edges simply supported)

- While columns have very little post-buckling strength, and certainly less than their buckling strength, plates (between frames) can exhibit higher strength after buckling than before. This is due to two factors:

  a) The long edges remain straight and intact due to frames (i.e., the part of the plate at that edges does not buckles)

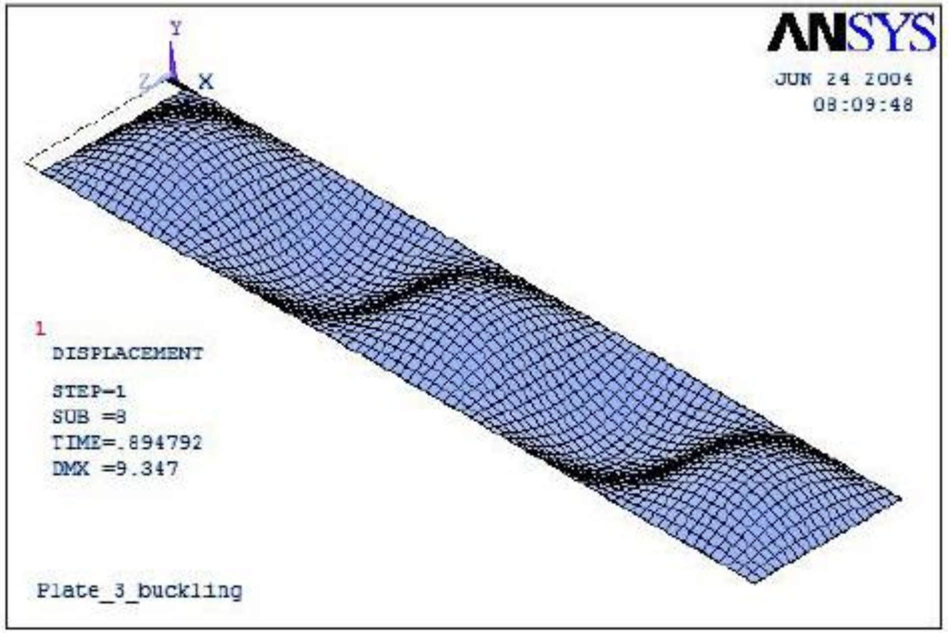
  b) There is a stabilising effect of the membrane stresses across the short dimension



Post-Buckling Strength of Long Plates (2/3)

Finite element plot of the buckled shape of a 2500mm x 500mm x 9mm plate. The model was a non-linear analysis, with an imposed end displacement, solved using the arc-length method.

The plot refers to 4.475mm end displacement (max. applied deflection was 5mm), which is well beyond the onset of buckling.

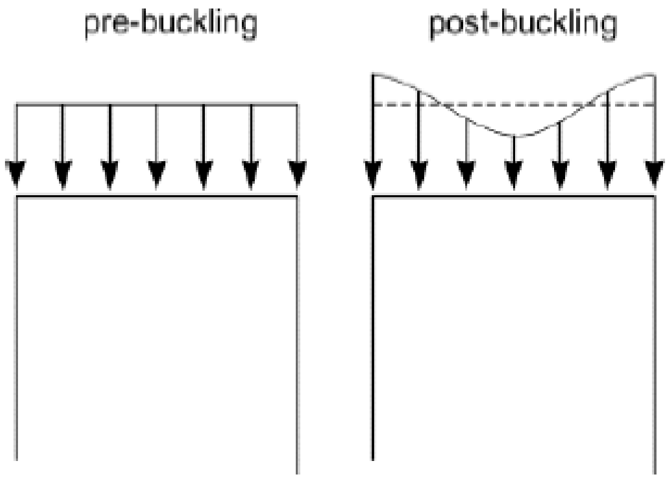


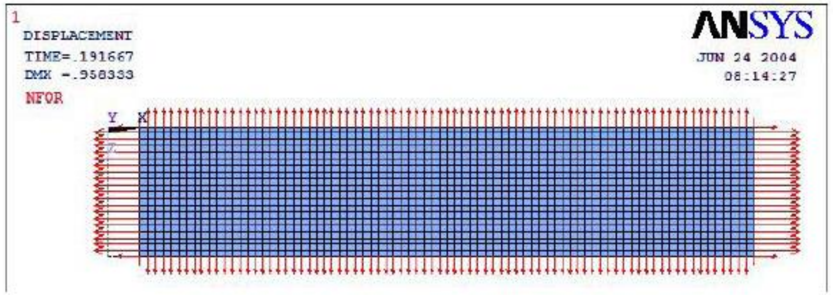
https://scholar.harvard.edu/files/vasios/files/ArcLength.pdf

https://mycourses.aalto.fi/pluginfile.php/394290/mod_resource/content/1/Lecture%208.pdf

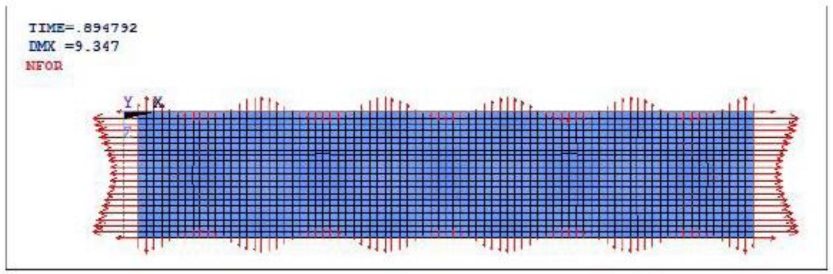
Post-Buckling Strength of Long Plates (3/3)

After buckling occurs the stress re-distribute themselves across the plate. This is shown in a sketch below. And in a finite element plot below that the ANSYS plot is actually showing the forces at each node pushing the surrounding boundary (which is why they are pushing out for a state of compression → See Reaction Force).





before buckling



after buckling

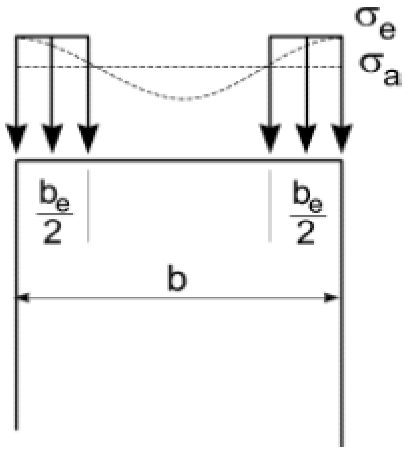
Effective Width (1/8)

Effective width

The redistribution of stresses can be treated by the 'effective width' concept.

- True stresses are replaced by two uniform zones, where the total zone width is b_e (note: the next plate is similar so the load patch is b_e wide, centered on the frame.)

- The true edge stress σ_e is the stress in b_e.



The total force can be expressed in a variety of ways;

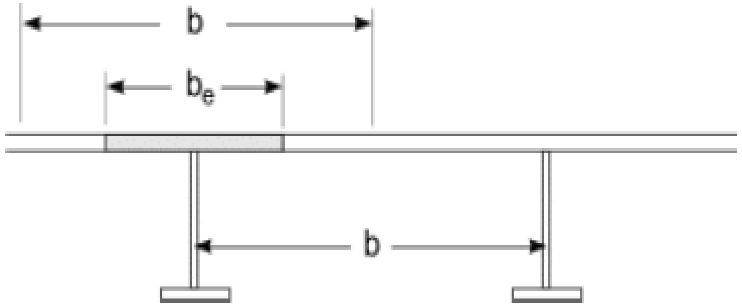
$$\boxed{F = \int_0^b \sigma dy = \sigma_e b_e = \sigma_a b} \qquad (1)$$

where σ_a is the average post-buckling stress. We can write;

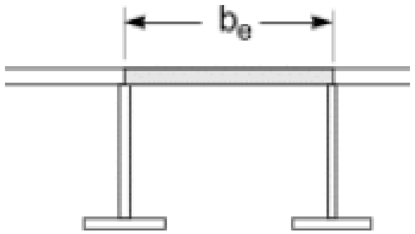
$$\boxed{\frac{b_e}{b} = \frac{\sigma_a}{\sigma_e}} \qquad (2)$$

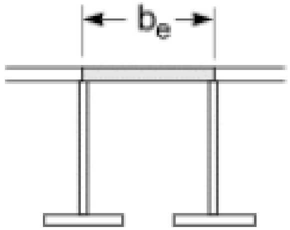
Why use this effective width concept?

The reason is that it will allow us to determine the edge stress, which is also the axial stress on the frame, and thus we will be able to determine the total post-buckling force.



Effective Width (2/8)





The way that we can find both σ_e and b_e is with a method proposed by von-Karman (1924).

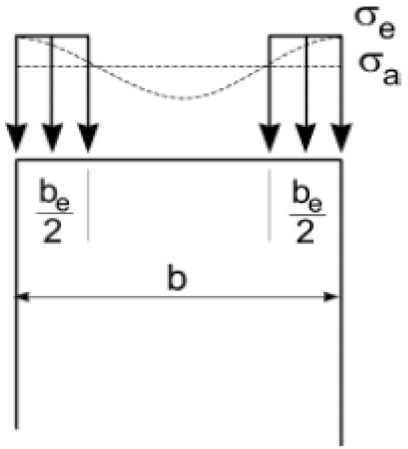
- Von-Karman was, at the time, working on aircraft structures, which had very similar construction to the ship structures that we are considering (plate over longitudinal ribs, in both the fuselage and wings).

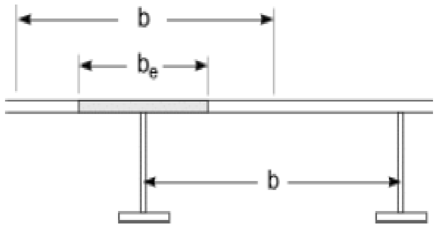
- Von-Karman proposed a very simple (elegant) way to find the edge stress along with the effective width. He suggested that *as the plate buckles, the edge stress (over b_e) would be the same as the Euler buckling stress for a plate of effective width (b_e).*

https://en.wikipedia.org/wiki/Theodore_von_K%C3%A1rm%C3%A1n

Effective Width (3/8)

- This idea turned out to be close to the truth, in effect, he proposed to see the plate as *two idea parts: the middle buckled part carrying NO LOAD*, and *a part (the effective part) near the frame carrying ALL the LOAD*.
- The 'effective' part would be *progressively narrowed by the progressive buckling of the middle part*.
- This is a key point. The idea is that plate buckling is a steady progressive process. When the stress first causes buckling, the plate only just starts to deform.





- If the stress is held at a level just above the buckling stress, the plate is only slightly deformed (barely noticeable). As the average stress increase, the middle part progressively sheds load to the sides.
- The whole process is actually steady and stable, right up to the point where the edge stress (the frame stress) reach yield. At least that's the idea, and it is quite close the reality.

Want to learn more? https://www.sciencedirect.com/science/article/pii/S0263823108000517?via%3Dihub

Buckling of Flat Plates (1/21)

Rectangular plates compressed in short edges with different edge conditions

- Based on energy method, the critical plate buckling stress in rectangular plate compressed in the short edges for different edge conditions can also be derived and presented in the form

$$\sigma_{E(plate)} = k\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2$$

where the buckling coefficient k varies with edge conditions and is shown in the following page.

$$k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

$$\frac{a}{b} \le \sqrt{m(m+1)}$$

m = buckling half-wave number

$$\sigma_{cr} = \frac{\pi^2 D}{b^2 t}\left(\frac{mb}{a} + \frac{a}{mb}\right)^2 = k\frac{\pi^2 D}{b^2 t}$$

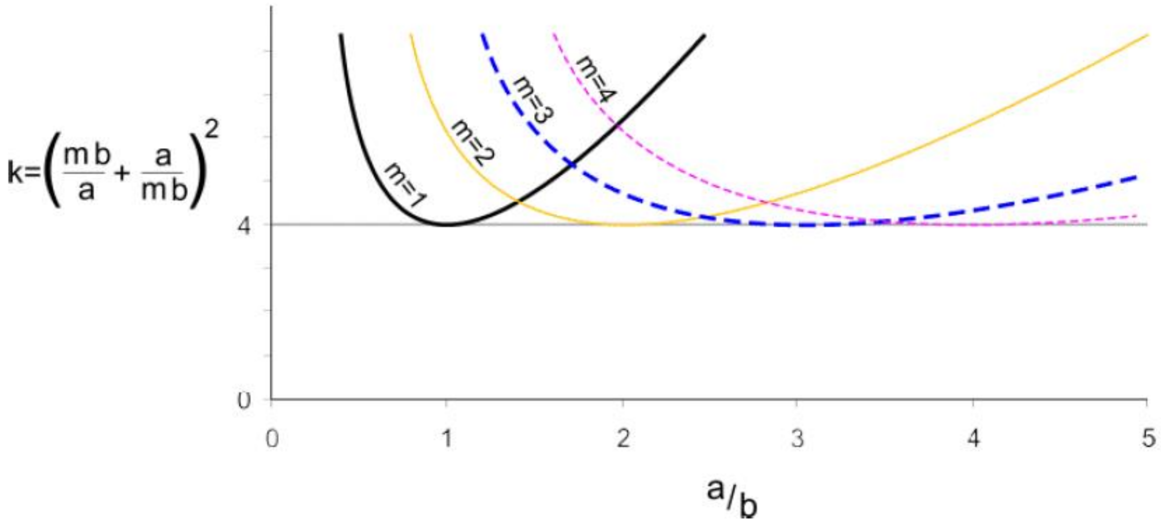
where

$$k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

$$D = \frac{Et^3}{12(1-\nu^2)}$$

we can write;

$$\sigma_{cr} = \frac{4\pi^2}{b^2 t}\frac{Et^3}{12(1-\nu^2)} = 3.62E\left(\frac{t}{b}\right)^2$$



Effective Width (4/8)

The von-Karman relationships are developed as follows:

Recall that
$$\frac{\sigma_{cr}}{\boxed{\sigma_Y}} = \frac{3.62}{\beta^2} \qquad (3)$$

where
$$\beta = \frac{b}{t}\sqrt{\frac{\sigma_Y}{E}} \qquad (4)$$

Or

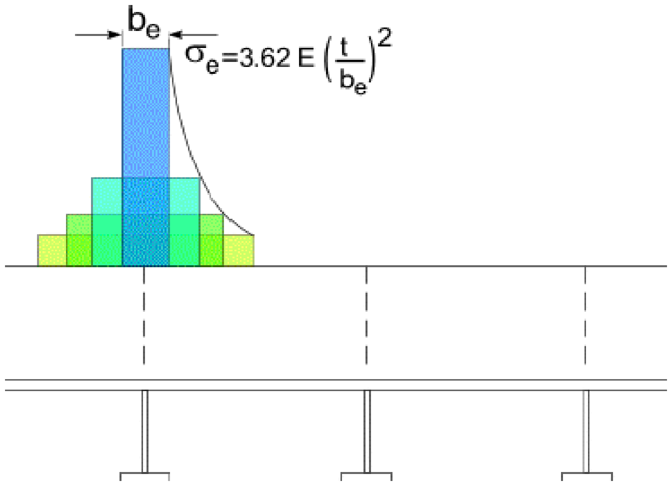
If we let b=b_e, we get:
$$\sigma_{crit} = 3.62\left(\frac{t}{b}\right)^2 E \qquad (5)$$

$$\sigma_e = 3.62\left(\frac{t}{b_e}\right)^2 E \qquad (6)$$

$$b_e = 1.9t\sqrt{\frac{E}{\sigma_e}} \qquad (7)$$

There are the equations relating $\sigma_{e\,(effective)}$ and $b_{e\,(effective)}$

Effective Width (5/8)



$$b_e = 1.9t\sqrt{\frac{E}{\sigma_e}} \qquad (7)$$

As the overall force on the deck increase and the effective width decreases, the von-Karman model will remain valid until the edge stress reaches yield. At this point, the frames will collapse. The minimum effective width can be found by substituting the yield stress into equation (7).

$$b_{em} = 1.9t\sqrt{\frac{E}{\sigma_Y}} \qquad (8)$$

For E = 207,000 MPa, Yield stress = 235 MPa we get; $\boxed{b_{em} = 56t}$

Effective Width (6/8)

- The plate buckles progressively and that the system can take more and more loads as the plate buckles. We can check this by calculating the force on the plate;

$$F = b_e \times t \times \sigma_e = b_e \times t \frac{3.62 t^2 E}{b_e^2} = \frac{3.62 t^3 E}{b_e} \qquad (9)$$

- Quite clearly, the force F will increases as b_e gets smaller.

- The limit situation occurs when $b_e = b_{em}$ and $\sigma_e = \sigma_Y$. At this point the force is;

$$\boxed{F_m = b_{em} \times t \times \sigma_Y} \qquad (10)$$

- The average stress on the plate in the limit condition is;

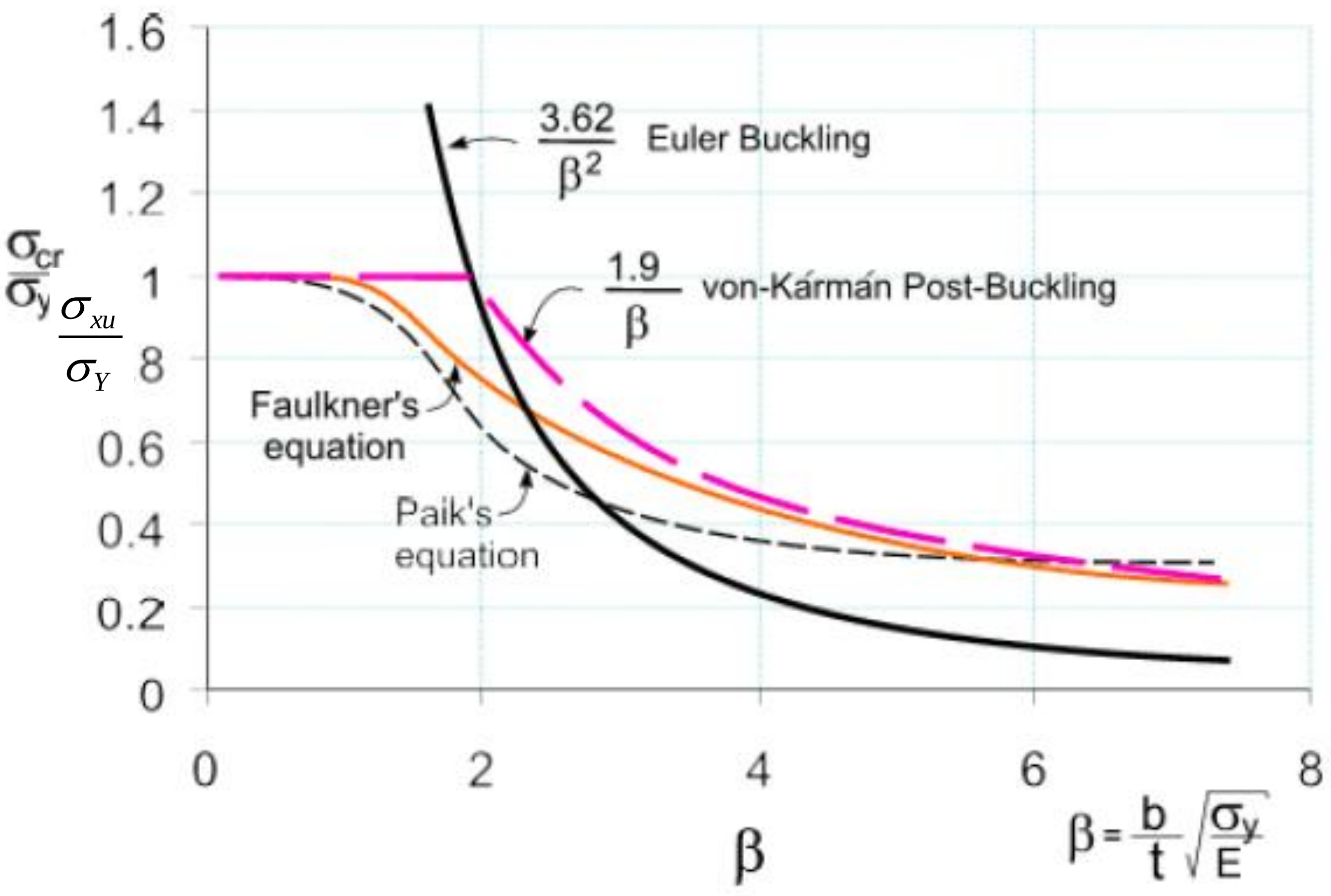
$$\sigma_m = \frac{F_m}{b \times t} = \frac{b_{em} \times t \times \sigma_Y}{b \times t} = \frac{1.9t\sqrt{\frac{E}{\sigma_Y}}\sigma_Y}{b} \qquad (11)$$

$$\frac{\sigma_m}{\sigma_Y} = 1.9\frac{t}{b}\sqrt{\frac{E}{\sigma_Y}} = \frac{1.9}{\beta} \qquad (12)$$

$$\boxed{b_{em} = 1.9t\sqrt{\frac{E}{\sigma_Y}}} \qquad (8)$$

Effective Width (7/8)

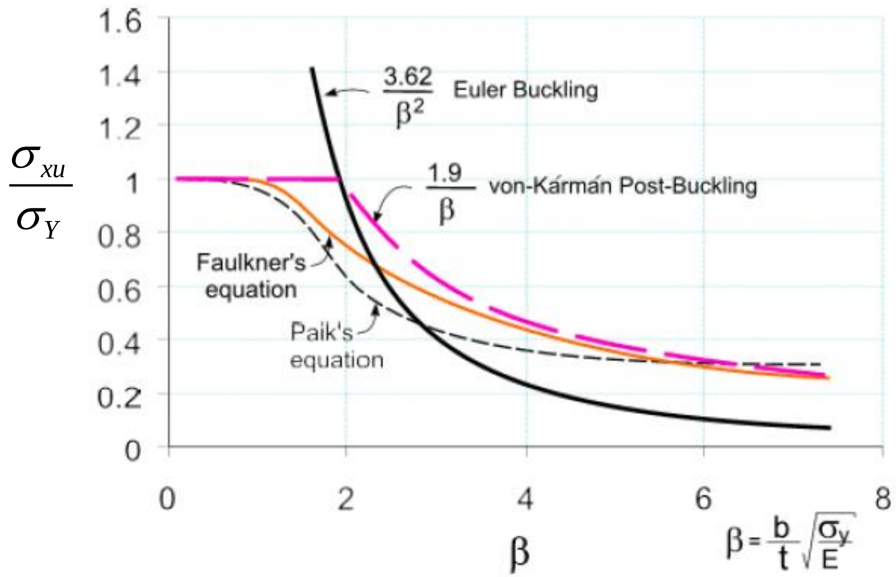
- The buckling (3) and post-buckling (12) capacities are shown below;



https://www.ce.jhu.edu/cfs/cfslibrary/SSRC%20Guide%202009%20Ch%204%20plates%20Schafer%20version.pdf

Effective Width (8/8)

- The buckling (3) and post-buckling (12) capacities are shown below;



Faulkner (Prof. at Glasgow), on the basis of experiments proposed a slightly modified equations:

$$\frac{\sigma_u}{\sigma_Y} = \frac{2}{\beta} - \frac{1}{\beta^2} \qquad (13)$$

- Paik (Prof at PNU, Korea), proposed an even more conservative equation, taking imperfections and initial stresses into account, and validated with experimental and numerical data. Paik's equation is;

$$\frac{\sigma_u}{\sigma_Y} = \begin{cases} -0.32\beta^4 + 0.002\beta^2 + 1 & \text{for } \beta \le 1.5 \\ 1.247/\beta & \text{for } 1.5 \le \beta \le 3.0 \\ 1.248/\beta^2 + 0.283 & \text{for } \beta > 3.0 \end{cases} \qquad (14)$$

Design application (WSD vs CBSD vs ULSD) (6/6)

Demand (D) and Capacity (C) used for typical design method

Design method	WSD	CBSD	ULSD
Demand	Design working stress (action effect)	Design working stress (action effect) or action (load)	Design working stress (action effect) or action (load)
Capacity	Allowable stress	Design critical buckling strength	Design ultimate strength

Note: **WSD** = working stress design; **CBSD** = critical bucking strength-based design; **ULSD** = ultimate-state design. "Design" implies that associated uncertainties are taken into account

Basic structural design concept

Capacity (C) > Demand (D)

Safety Factor (S.F.) > 1.0

(In general, we consider partial safety factors)

Example 1

Problems.

1. Find **the elastic buckling strength, critical buckling strength, and ultimate strength** for the following plates:

2. After we calculate ultimate strength, check the effective width (b_e)

(Note: plasticity correction can be performed by Johnson-Ostenfeld formula and ultimate strength can be calculated by Falkner and Paik's formulations.

	Plate 1	Plate 2	Plate 3
b [mm]	600	800	1200
t [mm]	20	15	10
σ_Y [MPa]	235	300	360
E [MPa]	207000	207000	207000

$$\sigma_{cr} = \begin{cases} \sigma_E & \text{for } \sigma_E < 0.5\sigma_Y \\ \sigma_Y\left(1 - \dfrac{\sigma_Y}{4\sigma_E}\right) & \text{for } \sigma_E \geq 0.5\sigma_Y \end{cases} \quad \text{(J-O)}$$

$$\frac{\sigma_u}{\sigma_Y} = \begin{cases} 1.0 & \text{for } \beta < 1.0 \\ \dfrac{2}{\beta} - \dfrac{1}{\beta^2} & \text{for } \beta \geq 1.0 \end{cases} \quad \text{(Faulkner)}$$

$$\frac{\sigma_u}{\sigma_Y} = \begin{cases} -0.32\beta^4 + 0.002\beta^2 + 1 & \text{for } \beta \leq 1.5 \\ 1.247/\beta & \text{for } 1.5 \leq \beta \leq 3.0 \\ 1.248/\beta^2 + 0.283 & \text{for } \beta > 3.0 \end{cases} \quad \text{(Paik)}$$

Example 2

Problems.

The plate located in deck is surrounded by support members under longitudinal compressive load of 1,764 kN. Based on WSD, CBSD and ULSD design concepts, determine the plate thicknesses.

(Note: BC = Simply supported, plate length (a) = 3200mm, plate breadth (b) = 800mm, Yield stress = 352.8MPa, E = 205.8GPa, v = 0.3)

Learning Outcomes (Review)

- We have investigated the Post-buckling behaviour of plate.

- Now we are able to:

  - Derive the formula for the critical stress of tripping a stiffener (Topic 9).
  - Calculate the critical stress of tripping a stiffener of rolled or built-up section (Topic 9).
  - Be aware of post-buckling behaviour of plate (Topic 9A)
    (This will be continued in Topic 10)

- Details can be referred to topics 9A in the lecture notes.

Adv. Marine Structures / Adv. Structural Design & Analysis (Next class)

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Thank you!

Questions?

Aerial View of Korean Presidential Archives in Sejong city
(Construction Completed in 2014)