Microbial kinetics

Today's lecture

- Monod kinetics
- Addressing decay
- Rate of substrate utilization associated with microbial growth

Monod equation

$$\mu_{syn} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{syn} = \hat{\mu}\frac{S}{K+S}$$

where μ_{syn} = specific growth rate due to synthesis (T^{-1})

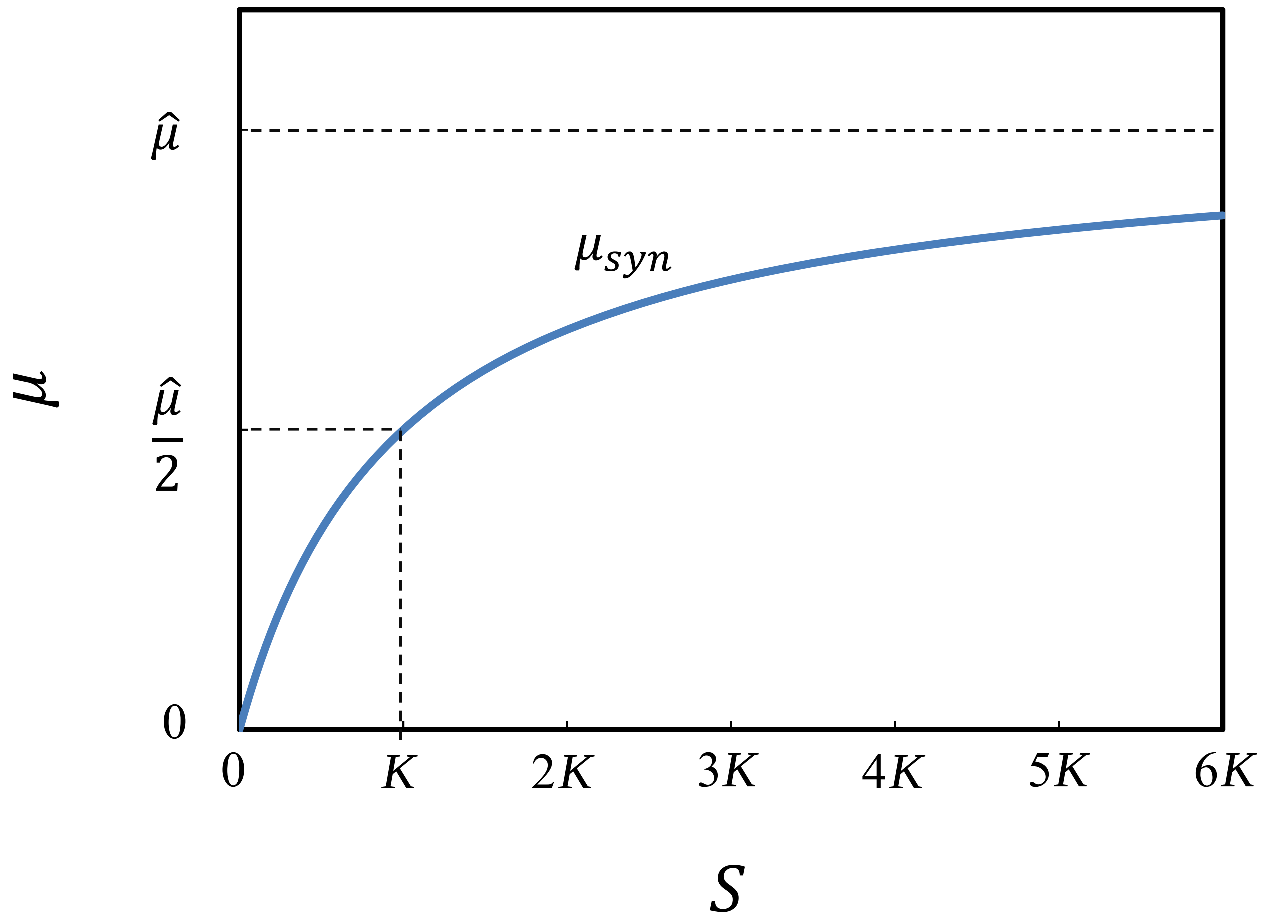
X_a = concentration of active biomass ($M_x L^{-3}$)

S = concentration of the rate-limiting substrate ($M_s L^{-3}$)

$\hat{\mu}$ = maximum specific growth rate (T^{-1})

K = half saturation coefficient ($M_s L^{-3}$)

Monod equation



Typical values for K

Process	K (mg substrate/L)
Aerobic: organic mixtures single organics nitrification	 50-150 mg COD/L 1-10 mg COD/L 0.4-2 mg NH_3-N/L
Anaerobic: denitrification methane fermentation: acetate, propionate sewage sludge	 0.06-0.20 mg NO_3^--N/L 600-900 mg COD/L 2000-3000 mg COD/L

Growth kinetics with decay

- As discussed in the previous lecture, we assume decay is proportional to cell biomass

$$\left(\frac{dX_a}{dt}\right)_{decay} = -bX_a$$

in the form of specific growth rate,

$$\mu_{dec} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{decay} = -b$$

where μ_{dec} = specific growth rate due to decay (T^{-1})

b = decay coefficient (T^{-1})

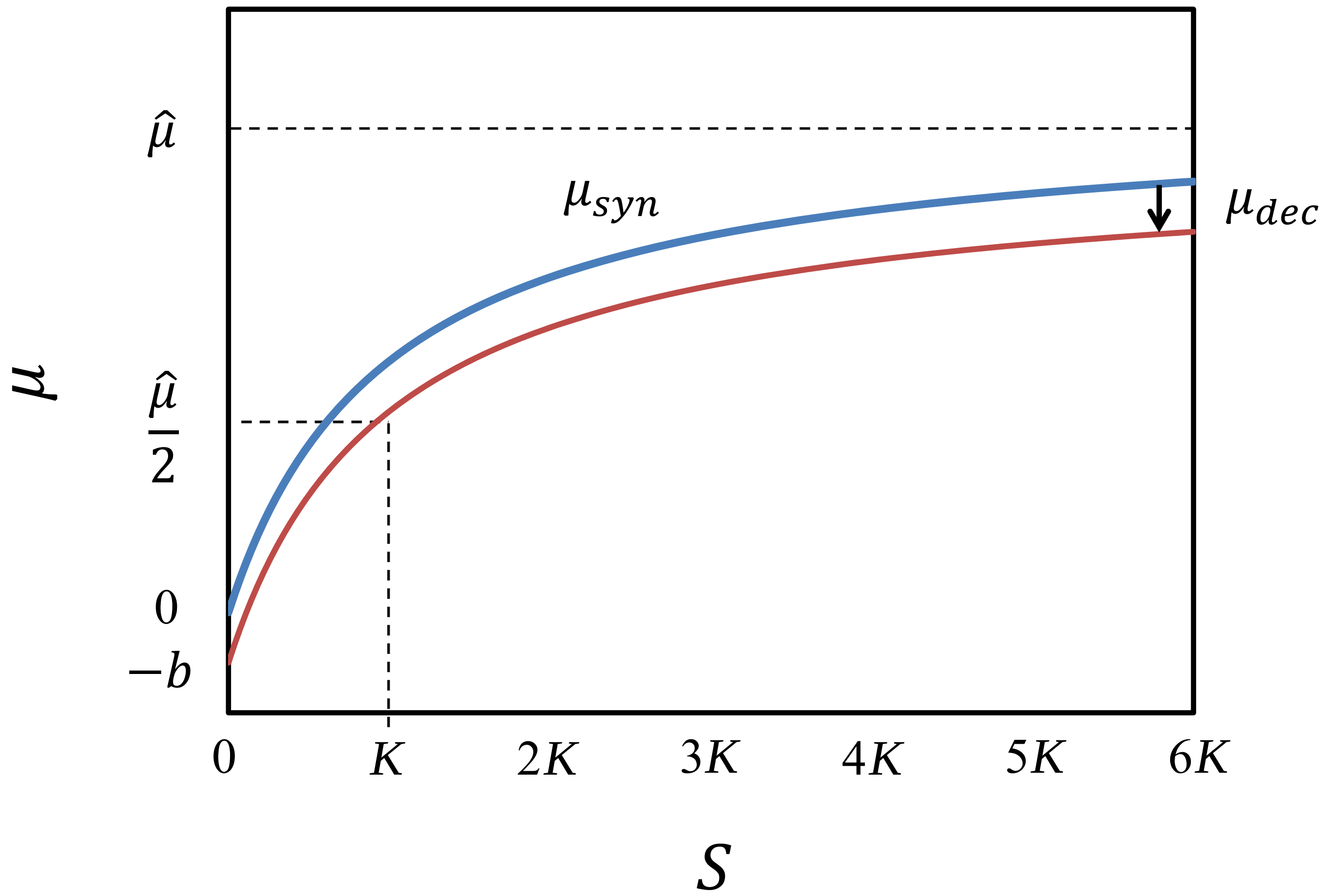
Overall bacterial growth kinetics

(Net growth) = (New growth) + (Decay)

$$\mu = \frac{1}{X_a} \cdot \frac{dX_a}{dt} = \mu_{syn} + \mu_{dec} = \hat{\mu}\frac{S}{K+S} - b$$

where μ = net specific growth rate (T^{-1})

Growth kinetics with decay



More on decay

$$\mu_{dec} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{decay} = -b$$

- Most fraction (f_d≈0.8) is oxidized
- The other fraction (1-f_d ≈0.2) is accumulated as inert biomass

Rate of oxidation (respiration): $$\left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{resp} = -f_d b$$

Rate of conversion to inert biomass:

$$\left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{inert} = -\frac{1}{X_a} \cdot \frac{dX_i}{dt} = -(1 - f_d)b$$

X_i = inert biomass ($M_x L^{-3}$)

Substrate utilization rate

Recall that,

$$Y = \frac{(g\ cells\ produced)}{(g\ substrate\ utilized)} = \frac{(dX_a/dt)_{syn}}{-dS/dt}$$

and

$$\mu_{syn} = \left(\frac{1}{X_a} \cdot \frac{dX_a}{dt}\right)_{syn} = \hat{\mu}\frac{S}{K+S}$$

So Monod equation can be also written as:

$$\frac{dS}{dt} = -\frac{1}{Y}\left(\frac{dX_a}{dt}\right)_{syn} = -\frac{\hat{\mu}}{Y}\frac{S}{K+S}X_a$$

Substrate utilization rate

Substrate utilization rate, r_{ut} $[M_s L^{-3} T^{-1}]$

$$r_{ut} = \frac{dS}{dt} = -\frac{\hat{q}S}{K+S} X_a$$

$\hat{q} = \hat{\mu}/Y$, max. specific rate of substrate utilization $(M_s M_x^{-1} T^{-1})$

Recall that,

$$Y = f_s^{\,0} \frac{M_c}{n_e \cdot (8\ g\ COD/e^-\ eq)}$$