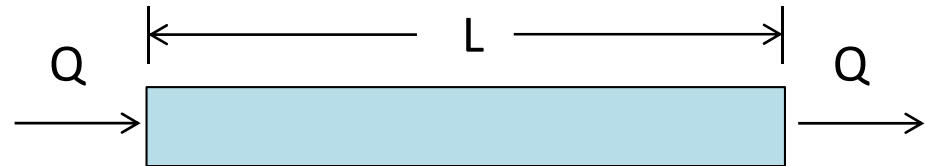


Reactors II

Today's lecture

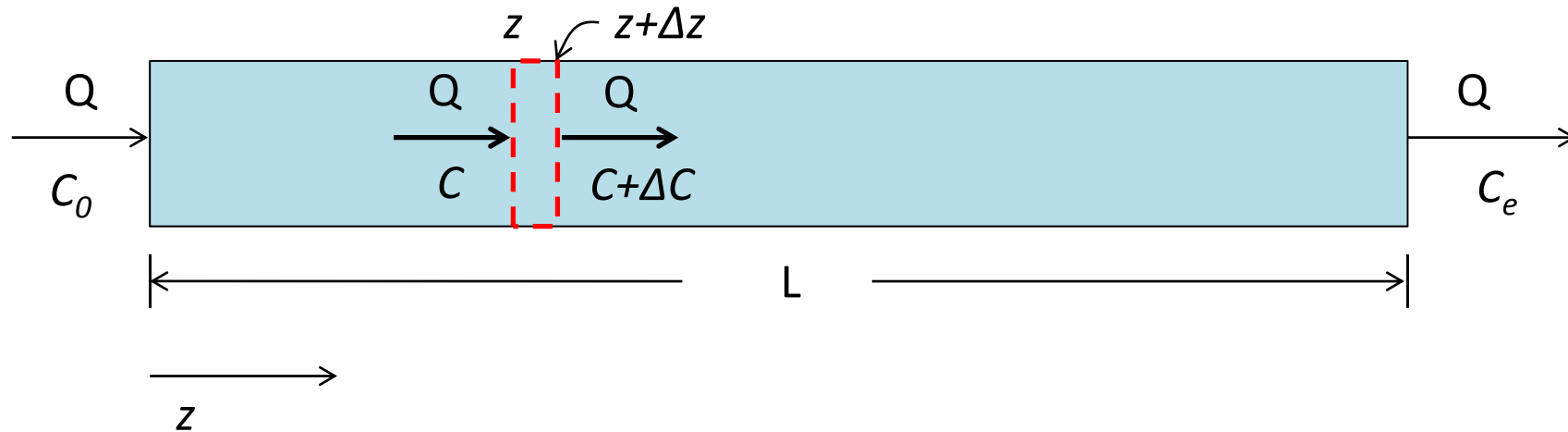
- **Plug flow reactor**
 - Concept
 - PFR analysis for 1st order reaction
 - PFR analysis for Monod kinetics
- **Continuous-stirred tank reactor**
 - CSTR analysis for 1st order reaction
 - PFR vs. CSTR
 - CSTR analysis for Monod kinetics

Reactor analysis: PFR



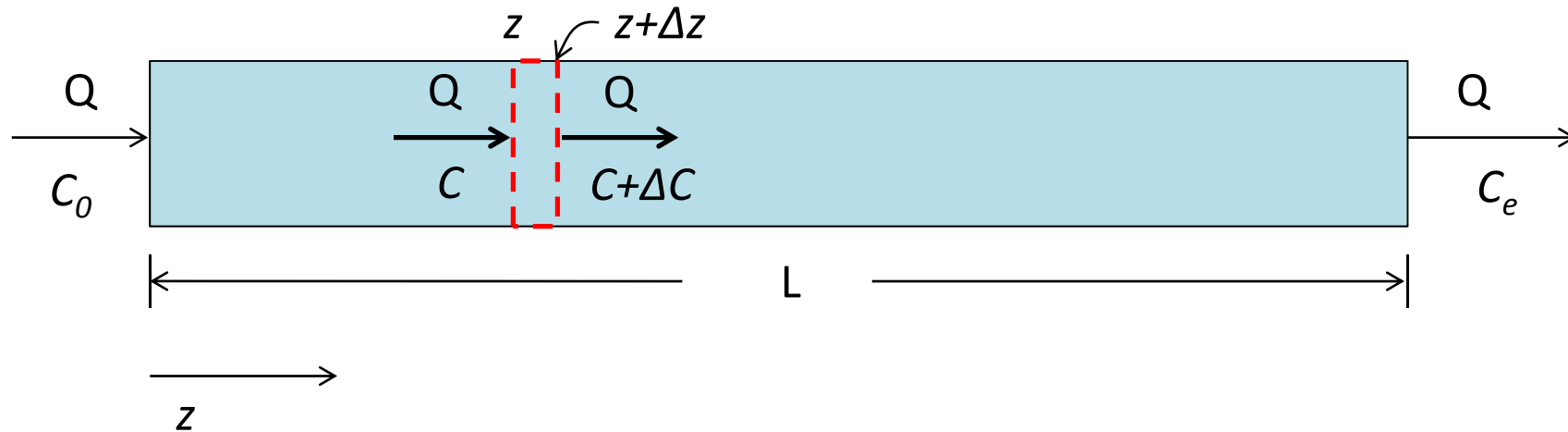
- Plug flow reactor (PFR)
 - assumption: no mixing in the direction of flow & completely mixed in the direction perpendicular to the flow
 - reactors get close to PFR as the length gets longer than the width and depth (e.g., rivers)

PFR, first-order reaction



- Take control volume as a thin plate perpendicular to the flow at $z=z$ with a dimension of Δz in z dir.

PFR, first-order reaction



$C/C_0 = e^{-k\theta}$ → Same form as the batch reactor (why??)

where, $\theta \equiv \frac{V}{Q} = \frac{A \cdot L}{A \cdot u} = \frac{L}{u}$

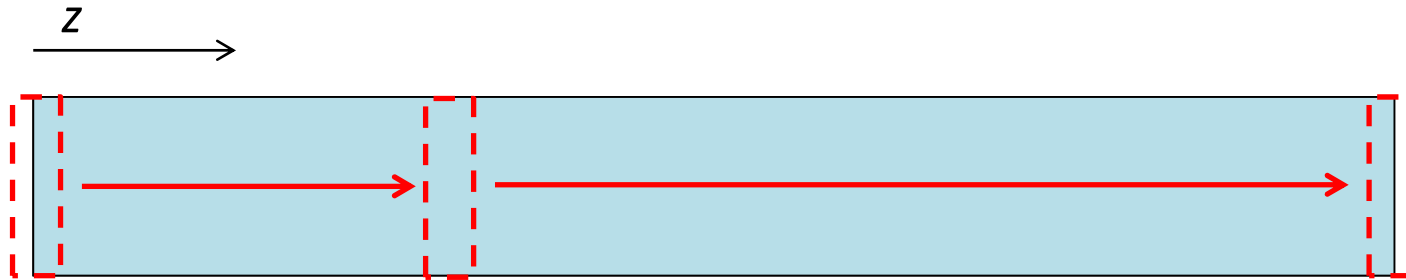
$\theta = \text{hydraulic retention time, HRT [T]}$

$V = \text{reactor volume [L}^3\text{]}$

$u = \text{flow velocity [LT}^{-1}\text{]}$

$A = \text{cross-sectional area [L}^2\text{]}$

PFR, first-order reaction



distance=0

time=0

$$C=C_0$$

distance=z

time=t

$$C=C_0e^{-kt}$$
$$=C_0e^{-kz/u}$$

distance=L

time=θ

$$C=C_0e^{-kθ}$$
$$=C_0e^{-kL/u}$$

- We model plug flow reactor as a movement of a “plug”
- The plug has a cross sectional area same as the reactor dimension and an infinitesimal dimension in z-dir (a thin plate)
- Complete mixing within the plug → batch reactor moving in the direction of flow

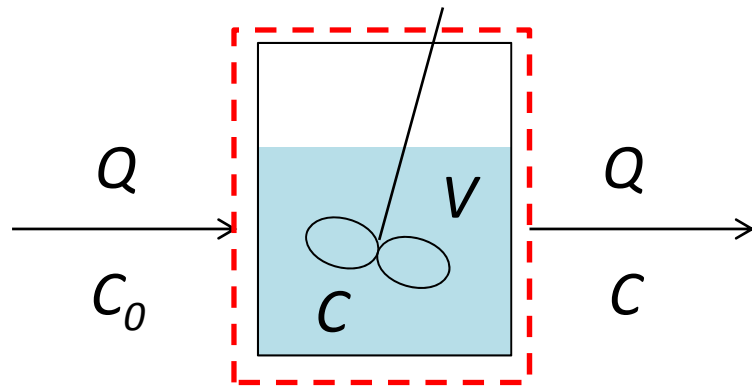
PFR, Monod – Semi-analytical solution

Assuming decay is negligible,

$$\frac{z}{u} = -\frac{1}{\hat{q}} \left\{ \left(\frac{K}{X_a^0 + YS^0} + \frac{1}{Y} \right) \ln(X_a^0 + YS^0 - YS) - \left(\frac{K}{X_a^0 + YS^0} \right) \ln \frac{SX_a^0}{S^0} - \frac{1}{Y} \ln X_a^0 \right\}$$

[5.26] in the textbook

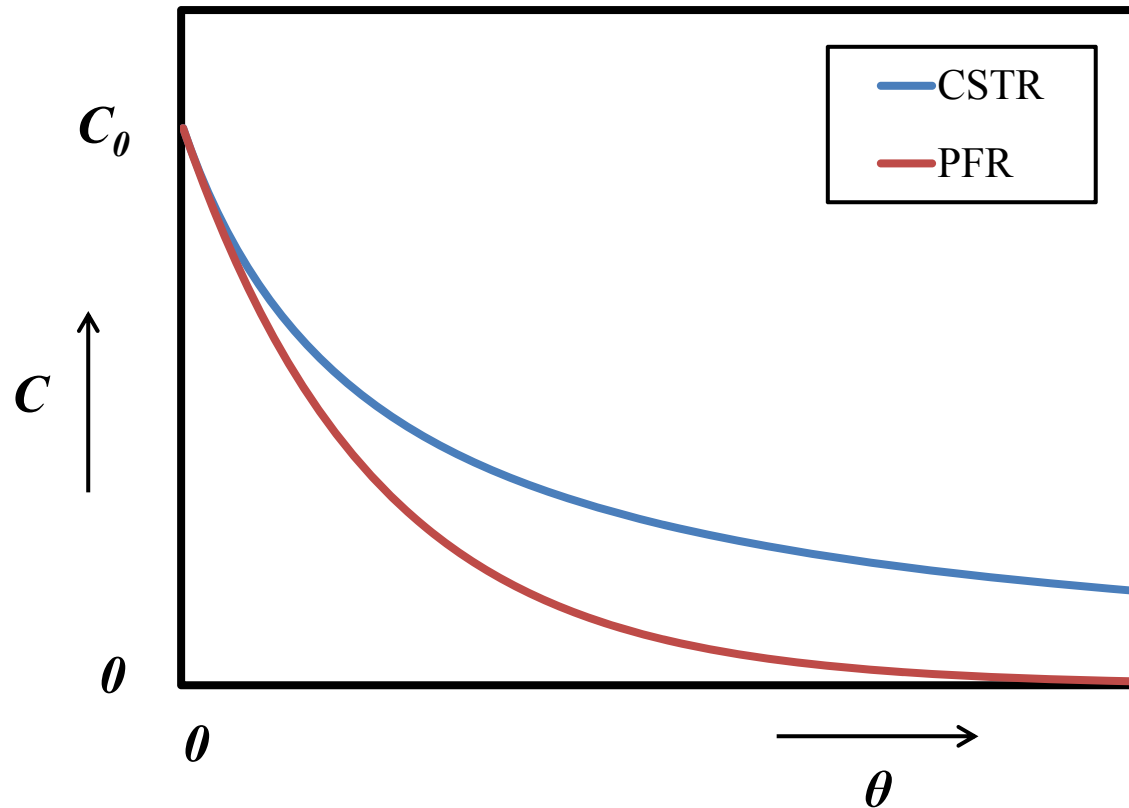
Reactor analysis: CSTR, 1st order



At steady state,

$$C = \frac{C_0}{1 + k\theta}$$

PFR vs. CSTR



PFR shows better performance esp. at high HRTs

For 1st order reaction,

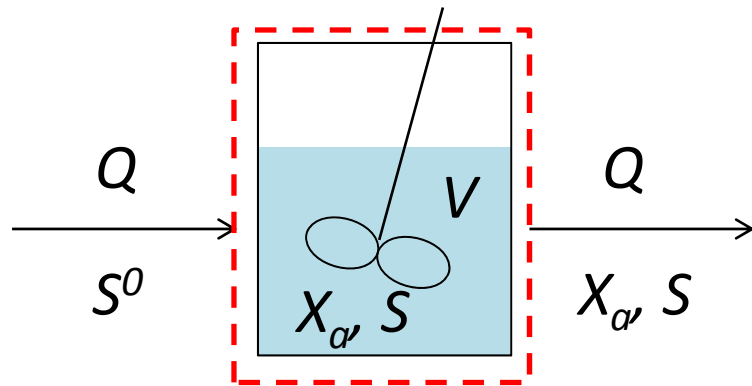
CSTR:

$$C = \frac{C_0}{1 + k\theta}$$

PFR:

$$C = C_0 e^{-k\theta}$$

Reactor analysis: CSTR, Monod kinetics



Assumption:

- Steady state
- $X_a = 0$ in the influent
(negligible influent biomass)

$$S = K \frac{1 + b\theta}{Y\hat{q}\theta - (1 + b\theta)}$$

No S_0 or X_a in the equation!

$$X_a = Y \frac{S^0 - S}{1 + b\theta}$$