Chemostat performance analysis – example question

1) S_{min}, θ_x^{min}, θ_x

$$S_{\min} = K\frac{b}{Y\hat{q}-b} = (20\ mg\ BOD_L/L)\frac{0.15/d}{(0.42\ g\ VSS_a/g\ BOD_L)(20\ g\ BOD_L/g\,VSS_a-d)-0.15/d}$$

$$= 0.36\ mg\ BOD_L/L$$

$$\theta_x^{\min} = \frac{K+S^0}{S^0(Y\hat{q}-b)-bK} = \frac{(20+500)\ mg\ BOD_L/L}{(50\ mg\ BOD_L/L)(0.42\cdot 20/d-0.15/d)-(20\ mg\ BOD_L/L)(0.15/d)}$$

$$= 0.126\ d$$

$$\theta_x = \theta = \frac{V}{Q} = \frac{2000m^3}{1000\ m^3/d} = 2\ d$$

$$\frac{\theta_x}{\theta_x^{\min}} = SF = \frac{2\ d}{0.126\ d} = 16$$ (SF of 16 for washout)

2) Effluent VSS, COD, BOD_L

Firstly, we need to determine the effluent substrate and active biomass concentrations:

$$S = K\frac{1+b\theta_x}{Y\hat{q}\theta_x-(1+b\theta_x)}$$

$$= (20\ mg\ BOD_L/L)\frac{1+(0.15/d)(2\ d)}{(0.42\ g\ VSS_a/g\ BOD_L)(20\ g\ BOD_L/g\ VSS_a-d)(2\ d)-(1+(0.15/d)(2\ d))}$$

$$= 1.7\ mg\ BOD_L/L$$

$$X_a = Y(S^0-S)\frac{1}{1+b\theta_x} = (0.42\ g\ VSS_a/g\ BOD_L)(500-1.7\ mg\ BOD_L/L)\frac{1}{1+(0.15/d)(2\ d)}$$

$$= 161\ mg\ VSS_a/L$$

Calculate the effluent inert VSS concentration:

$$X_i = X_i^0 + X_a(1-f_d)b\theta_x$$

$$= 50\ mg\ VSS_i/L + (161\ mg\ VSS_a/L)(1-0.8g\ VSS_i/g\ VSS_a)(0.15/d)(2\ d)$$

$$= 60\ mg\ VSS_i/L$$

Now,

$$X_v = X_a + X_i = 161 + 60 = 221\ mg\ VSS/L$$

Think about the composition of effluent COD & BOD_L:

Effluent COD = remaining substrate + SMP + all VSS (active biomass + inert)

Effluent BOD_L = remaining substrate + SMP + active and biodegradable biomass

* Effluent bsCOD = Effluent $sBOD_L$ = remaining substrate + SMP

  Effluent bpCOD = all VSS

  Effluent $pBOD_L$ = active and biodegradable biomass

Calculate the effluent SMP

- let's first calculate the individual terms for Eqs. [3.38] & [3.39]

$$r_{ut} = -\frac{\hat{q}S}{K+S}X_a = \frac{dS}{dt} = -\frac{S^0-S}{\theta} = -\frac{(500-1.7)\ mg\ BOD_L/L}{2\ d} = -249\ mg\ BOD_L/L-d$$

$$\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1 r_{ut}\theta = 1.8 \cdot 161 \cdot 2 + 100 + 0.12 \cdot (-249) \cdot 2 = 620\ mg\ BOD_L/L$$

$$4K_{UAP}k_1 r_{ut}\theta = 4 \cdot 100 \cdot 0.12 \cdot (-249) \cdot 2 = -23900\ (mg\ BOD_L/L)^2$$

$$K_{BAP} + (\hat{q}_{BAP} - k_2)X_a\theta = 85 + (0.1-0.09) \cdot 161 \cdot 2 = 88.2\ mg\ BOD_L/L$$

$$4K_{BAP}k_2 X_a\theta = 4 \cdot 85 \cdot 0.09 \cdot 161 \cdot 2 = 9850\ (mg\ BOD_L/L)^2$$

$$UAP = 620 + \frac{\sqrt{(620)^2 + 23900}}{2} = 9.5\ mg\ BOD_L/L$$

$$BAP = \frac{88.2 + \sqrt{(88.2)^2 + 9850}}{2} = 22.3\ mg\ BOD_L/L$$

$SMP = UAP + BAP = 9.5 + 22.3 = 31.8\, mg\, BOD_L/L$

Biomass COD: recall that the COD value for a cell formula of $C_5H_7O_2N$ was 1.42 g COD/g cells

In sum,

$Effluent\ COD = Substrate + SMP + Biomass\ COD$

$= 1.7 + 31.8 + (1.42\, g\ COD/g\ VSS) X_v = 1.7 + 31.8 + 1.42 \cdot 221$

$= 1.7 + 31.8 + 313.8 = 347\, mg\ COD/L$

* Biomass accounts for most of COD – this COD can be removed by settling (but good settling property should be guaranteed)

* SMP account for most of soluble COD

$Effluent\ BOD_L = Substrate + SMP + active$ and $biodegradable\ biomass$

$= 1.7 + 31.8 + (1.42\, g\ COD/g\ VSS) \cdot X_a \cdot f_d = 216\, mg\ BOD_L/L$

3) N and P

The N and P consumption rates,

$$r_N = (0.124\, g\ N/g\ VSS) \cdot (0.42\, g\ VSS/g\ \ BOD_L) \cdot (-249\, mg\ BOD_L/L - d) \cdot \frac{1 + (1 - 0.8) \cdot 0.15 \cdot 2}{1 + 0.15 \cdot 2}$$

$= -10.6\, mg\ N/L - d$

$r_P = r_N \cdot 0.2\, g\,P/g\,N = -10.6 \cdot 0.2 = -2.1\, mg\ \ P/L - d$

The effluent N and P concentrations

$C_N = C_N^0 + r_N \theta = 50\, mg\ N/L - (10.6\, mg\ N/L - d) \cdot 2\, d = 28.8\, mg\ NH_4^+ - N/L$

$C_P = C_P^0 + r_P \theta = 10\, mg\ P/L - (2.1\, mg\ P/L - d) \cdot 2\, d = 5.8\, mg\ PO_4^{3-} - P/L$

(the amount of nutrients did not limit the biological activity in the reactor)

4) O_2

The acceptor consumption in the reactor,

$$\frac{\Delta S_a}{\Delta t} = (1\ g\ O_2/g\ COD) \cdot (1000\ m^3/d)$$

$$\cdot\ [500 - 1.7 - 31.8 + 1.42(50 - 221)]\ mg\ COD/L \cdot 10^3\ L/m^3 \cdot 10^{-3} g/mg$$

$$= 2.24 \times 10^5\ g\ O_2/d$$

To support the acceptor consumption, O2 should be supplied to the reactor with a rate of:

$$R_{O_2} = 2.24 \times 10^5\ g\ O_2/d - (1000\ m^3/d) \cdot (6 - 2)\ mg/L \cdot 10^3\ L/m^3 \cdot 10^{-3}\ g/mg$$

$$= 2.20 \times 10^5\ g\ O_2/d$$

(O_2 supplied by the influent DO is very small compared to the O_2 requirement – aeration is essential)

5) Effect of hydrolysis

i) effluent particulate BOD, S_p

$$S_p = \frac{S_p^0}{1 + k_{hyd}\theta} = \frac{100\ mg\ COD/L}{1 + (0.2/d)(2\ d)} = 71\ mg\ COD/L$$

ii) effluent soluble BOD, S: no change, 1.7 mg BOD_L/L

iii) effective S^0 considering S_p, $S^{0'}$:

$$S^{0\prime} = S^0 + k_{hyd} S_p \theta = 500\ mg\ COD/L + (0.2/d)(71\ mg\ COD/L)(2\ d) = 528\ mg\ BOD_L/L$$

(error in the textbook!)

iv) Effluent VSS

$$X_a = Y(S^0 - S)\frac{1}{1+b\theta_x} = 0.42 \cdot (528 - 1.7)\frac{1}{1+0.15 \cdot 2} = 170\ mg\ VSS/L$$

(slight increase from 161 mg VSS/L without particulate BOD)

$$X_i = X_i^0 + X_a(1-f_d)b\theta_x = 50\ mg\ VSS/L + (170\ mg\ VSS/L) \cdot (1-0.8) \cdot (0.15/d) \cdot (2\ d)$$

$$= 60\ mg\ VSS/L$$

(didn't change much – slight increase happened, but not enough to have increase in significant numbers)

$$X_v = X_a + X_i + S_p = (170+60)\ mg\ VSS/L + \frac{71\ mg\ COD/L}{1.42\ mg\ COD/mg\ VSS} = 280\ mg\ VSS/L$$

(Assumed that the particulate COD has the same formula, $C_5H_7O_2N$, as the biomass)

v) SMP: let's skip the calculation and obtain value from the text:

SMP = 32.6 mg BOD_L/L

(slight increase from 31.8 mg BOD_L/L because of increased biomass – BAP increases)

vi) Effluent COD & BOD_L

$$Effluent\ COD = S + SMP + 1.42 \cdot X_v = 1.7 + 32.6 + 1.42 \cdot 280 = 432\ mg\ COD/L$$

$$Effluent\ BOD_L = S + SMP + 1.42 \cdot f_d \cdot X_a + S_p = 1.7 + 32.7 + 1.42 \cdot 0.8 \cdot 170 + 71 =$$

$$= 299\ mg\ BOD_L/L$$