Seoul National University
Dept. of Civil and Environmental Engineering

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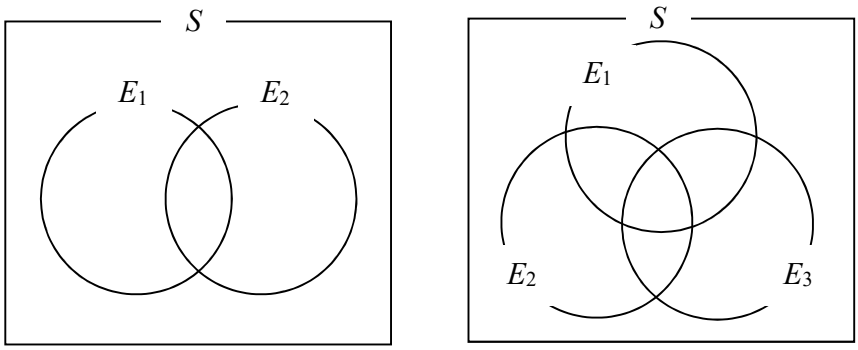
457.212 Statistics for Civil & Environmental Engineers

In-Class Material: Class 05

Elements of Set Theory – Part II (A&T: 2.1-2.2)

3. **Operations** of events

(a) **Union** of events



- **Union** of E_1 and E_2, ( ): An event that contains all the sample points that are in E_1 (/) E_2.
- Can be extended to the cases with more than two events - **Union** of $E_1, E_2, ..., E_n$, ( ) or ( ):

  An event that contains all the sample points that are in ( ) one of $E_1, E_2, ..., E_n$.
- Some notable cases:

  (1) $E \cup S =$

  (2) $E \cup \phi =$

  (3) $E \cup E =$

  (4) If $E_1 \subset E_2$, $E_1 \cup E_2 =$
- R function

```
E1 = c(1,2,3,4)
E2 = c(3,4,5,6)

E1U2 = union(E1, E2) # union of events (sets)
```

- CEE examples:

  (1) Concrete production may be hampered by shortage of water (E_1), sand (E_2), gravel (E_3) or cement (E_4).

  The event that concrete cannot be produced due to material shortage, E

  E =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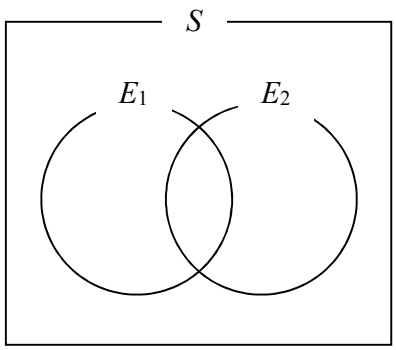
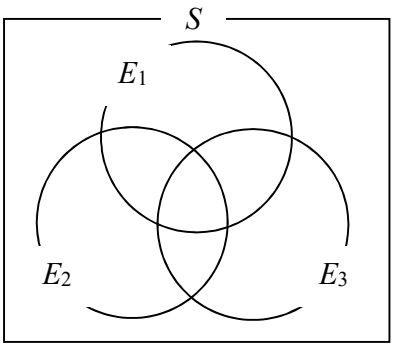
(2) During a hurricane event, a wall/roof panel can fail due to wind pressure (E_1) or missile-like flying objects (E_2). *Animation

The event that a wall panel fails, *E*

E =

(b) **Intersection** of events





- **Intersection** of E_1 and E_2, ( ): An event that contains all the sample points that are both in E_1 ( ) E_2.
- Can be extended to the cases with more than two events - **Intersection** of $E_1, E_2, ..., E_n$, ( ) or ( ): An event that contains all the sample points that belongs to ( ) one of $E_1, E_2, ..., E_n$.
- Some notable cases:

  (1) $E \bigcap S = E \cdot S =$

  (2) $E \bigcap \phi = E \cdot \phi =$

  (3) $E \bigcap E = E \cdot E =$

  (4) If $E_1 \subset E_2$, $E_1 \bigcap E_2 = E_1 \cdot E_2 =$

- R function

```
E12= intersect(E1, E2) # intersection of events (sets)
```

- CEE examples:

  (1) A city has *n* evacuation routes. Let E_i, i = 1,…,*n* denote the event that the i-th route is not available after a hazardous event.

  The event that people cannot evaculate, *E*

  E =

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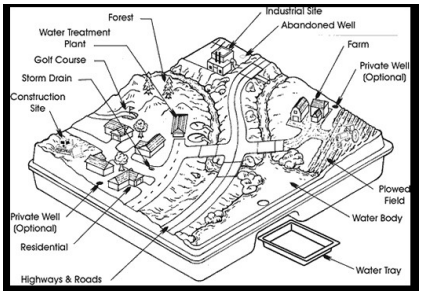
(2) The industrial site could accidentally discharge the pollutant X (Event E_1). The water treatment plant may not properly work (Event E_2). Joe will be exposed to the pollutant if he does not have his own filtering system (Event E_3).

The event that Joe will be exposed to the pollutant X, E

E =

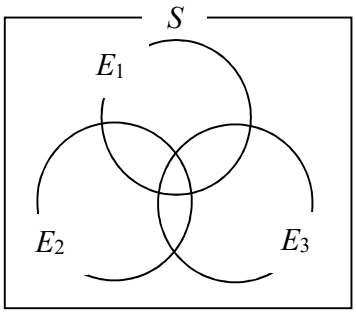


- Other R functions for set operations

```
S1 = c(1,2,3,4,5)
S2 = c(4,5,6,7)
S3 = c(5,4,3,2,1)

setdiff(S1, S2) # set difference of S1 and S2 (S1-S2)
setdiff(S2, S1) # set difference of S2 and S1 (S2-S1)

setequal(S1, S2) # logical value - equal: 1, unequal: 0
setequal(S1, S3)

is.element(S1, S2) # test if S1 contains S2's elements
is.element(S2, S1)
```

Note: Treat compound events obtained by union or intersection operations just like elementary events in the following operations.

Example: Intersection of the compound event $E_1 \cup E_2$ and E_3, i.e.



4. **Four basic rules** of operations of events

(a) **Commutative Rule**: the order of the events in union or intersection operations is ( ).

- $E_1 \bigcup E_2 =$
- $E_1 \cdot E_2 =$

Question: The events in which pair can be interchanged in $E_1 \bigcup E_2 \bigcap E_3$ and $(E_1 \bigcup E_2) \bigcap E_3$?

(b) **Associative Rule**: the union and intersection of three or more events are independent of the ( ).

- $(E_1 \bigcup E_2) \bigcup E_3 =$ =
- $(E_1 E_2) E_3 =$ =

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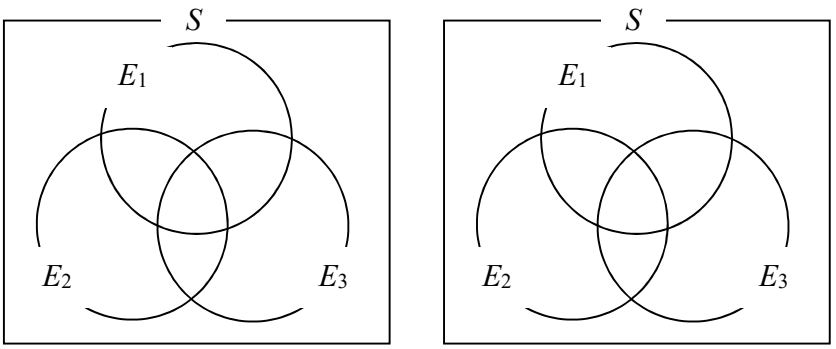
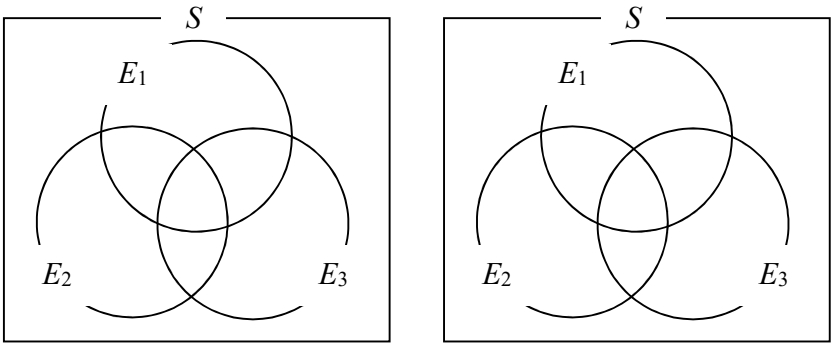
(c) **Distributive Rule**:

- $(E_1 \cup E_2)E_3$
  =

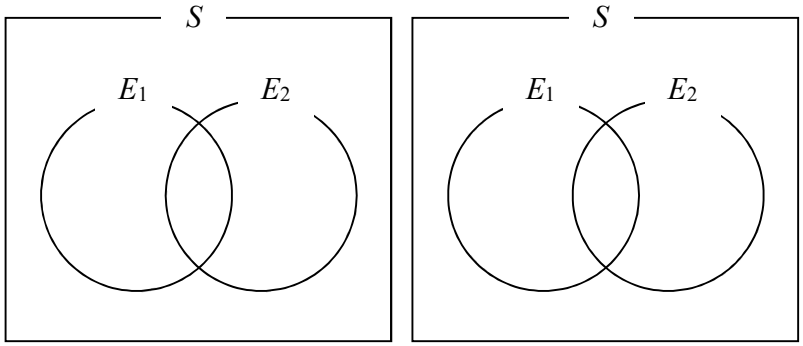


- $(E_1E_2) \cup E_3$
  =

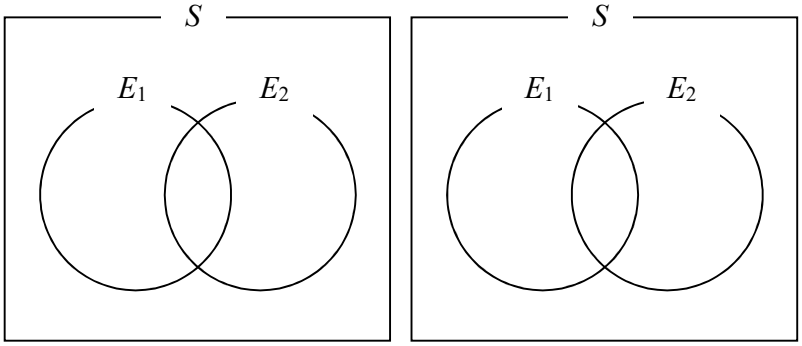


(d) **De Morgan's Rule**:

- $\overline{(E_1 \cup E_2)}$
  =



- $\overline{E_1 \cap E_2} = \overline{E_1E_2}$
  =



Note: Extension to cases with *n* events:

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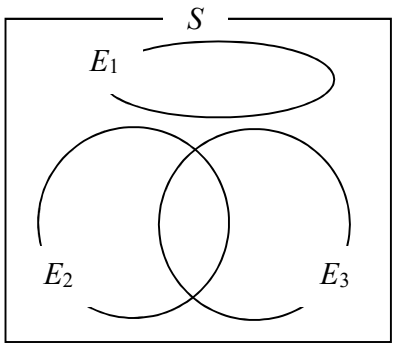
5. **Notable relationship** between events

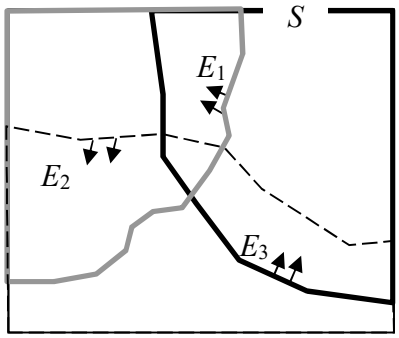


(a) **Mutually exclusive** events: when the intersection of the events are ( ) events.

$$E_1 \bigcap E_2 = E_1 E_2 =$$

- The events "cannot occur together."
- An obvious example:

Question: Which pairs are mutually exclusive?



(b) **Collective exhaustive** events: when the union of the events constitutes the ( )

$$E_1 \bigcup E_2 \bigcup \cdots \bigcup E_n = \bigcup_{i=1}^{n} E_i =$$

- Every sample point should belong to ( ) of these events.
- An obvious example:

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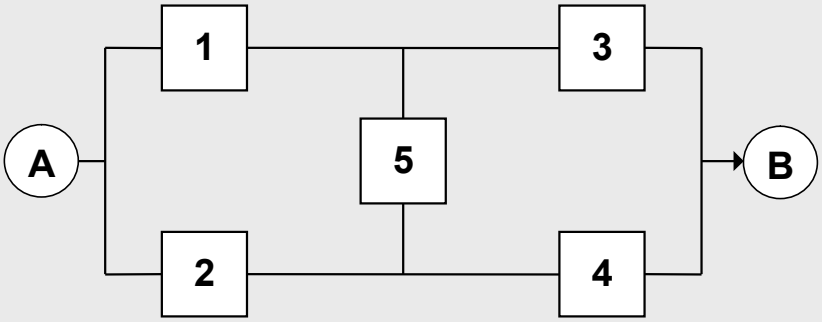
Example: Connectivity of an electrical substation after an earthquake event

Consider a simplified example of substation:



Let E_i, i = 1,…,5 denote the event that the i-th equipment fails during an earthquake event. Describe the event that the electricity can not travel from the point A to B in terms of E_i's.

$E_{\text{disconnect}}$ =

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Example: Data of structures on SNU campus – structure name, number of ground floors, basement floors, year of completion and age

Download the dataset 'snu_struct.txt' from the eTL website.

Question 1: Find a group of buildings that satisfy each of the following conditions:

- Construction completed in the 21st century
- 5 or more ground floors
- Basement floors exist

Question 2: Find buildings that have 5 or more ground floors **or** basement floors

Question 3: Find buildings that have 5 or more ground floors **and** basement floors

Question 4: Find buildings that were completed in the 21st century **and** have no basement floors

Question 5: Draw a Venn diagram for the sets identified in **Problem 1**

➢ The development of this example was made possible by the help of the Urban Design Laboratory at Seoul National University (Faculty advisor: Prof. Young Sang Kwon).