

457.212 Statistics for Civil & Environmental Engineers
In-Class Material: Class 07
Elements of Probability Theory – Part II (A&T: 2.3)

4. Conditional probability and statistical independence (Continued)

(d) **Multiplication rule**

- $P(E_1 E_2) =$

hint: $P(E_1 | E_2) = \frac{P(E_1 E_2)}{P(E_2)}$

(e) All the probability rules should apply to conditional probabilities defined within the same reconstituted () ().

- $P(E_1 \cup E_2 | E_3) =$

- $P(E_1 E_2 | E_3) =$

- $P(\bar{E}_1 | E_2) = 1 -$

Thus,

- $P(E_1 E_2 E_3) =$

- $P(E_1 E_2 \cdots E_n) =$

(f) **Statistical independence** – the occurrence (or non-occurrence) of one event does not affect the probability of the other event.

Three equivalent expressions for “ E_1 and E_2 are statistically independent.”

- $P(E_1 | E_2) =$

- $P(E_1 E_2) =$

- $P(E_2 | E_1) =$

Extended to n events:

- $P\left(\bigcap_{i=1}^n E_i\right) =$

(g) “ E_1 and E_2 are **statistically independent (s.i.)**”

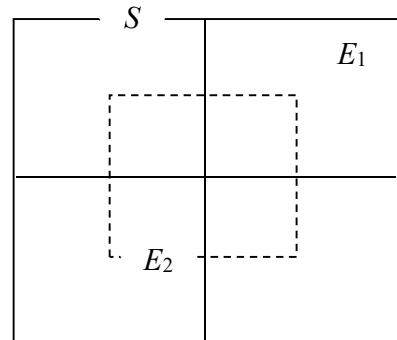
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(proof for the first case: as a practice regarding probability rules)

$$\begin{aligned}
 P(\overline{E_1}\overline{E_2}) &= P(\overline{E_1 \cup E_2}) \\
 &= 1 - P(E_1 \cup E_2) \\
 &= 1 - P(E_1) - P(E_2) + P(E_1 \cap E_2) \\
 &= [1 - P(E_1)][1 - P(E_2)] \\
 &= P(\overline{E_1})P(\overline{E_2})
 \end{aligned}$$

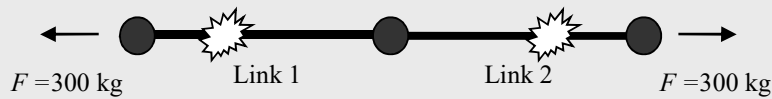
Note: In general, it is not straightforward to visualize “s.i.” using a Venn diagram, but here is a special example:

$$\begin{aligned}
 P(E_1) &= \\
 P(E_1 | E_2) &=
 \end{aligned}$$



Example: a baseball hitter whose batting average vs. lefty is the same as the total avg.

Example 1 (A&T 2.19): Consider a chain system consisting of two links subjected to a force $F = 300$ kg. Let E_1 and E_2 respectively denote the events that “Link 1 fails” and “Link 2 fails.” It is known that $P(E_1) = P(E_2) = 0.05$.

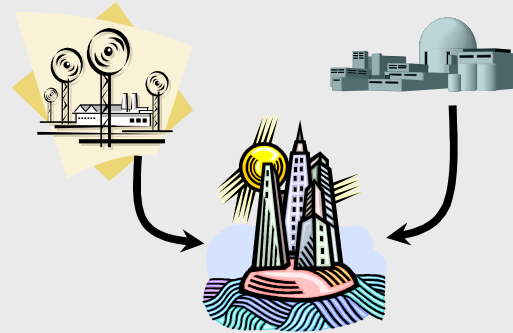


Compute the probability of the failure of the chain when

(a) the failures of the links are statistically independent of each other:

(b) the failures of the links have complete or total dependence between each other, i.e. if one fails, the other one always fails:

Example 2 (A&T 2.22): A city is powered by two generating plants – *Plant a* and *Plant b*. The city needs the capacities of both plants during peak hours. Otherwise, it will have a brownout or blackout. Events *A* and *B* denote the failure of *Plant a* and *Plant b*, respectively.



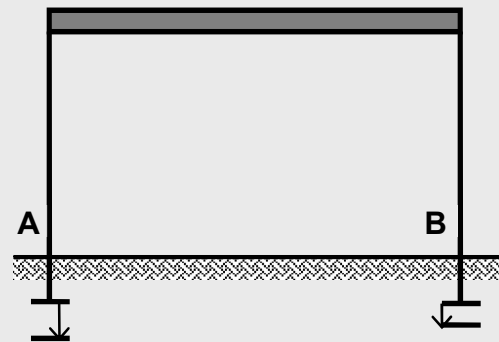
Given: $P(A) = 0.05$, $P(B) = 0.07$, and
 $P(AB) = 0.01$

- (a) If *Plant a* fails, what is the probability that *Plant b* fails?
- (b) If *Plant b* fails, what is the probability that *Plant a* fails?
- (c) What is the probability that there will be a brown/blackout in the city during the peak hour?

When there is a brown/blackout in the city during the peak hour, compute the following probabilities:

- (d) the brown/blackout caused solely by the failure of *Plant a*.
- (e) the brown/blackout caused solely by the failure of *Plant b*.
- (f) the brown/blackout caused by both plants.

Example 3: Consider a steel frame in the figure. E_A and E_B respectively denote the settlements at location A and B, respectively.



Given: $P(E_A) = 0.1$
 $P(E_B) = 0.1$
 $P(E_A | E_B) = 0.8$
 $P(E_B | E_A) = 0.8$

(a) Probability that there will be any settlements:

(b) Probability of a “differential” settlement:

Example 4: Suppose the foundation of a structure can fail either when the structural demand exceeds its bearing capacity (Event B) and/or when the settlement (Event S_t) occurs.



Given: $P(B) = 0.001$, $P(S_t) = 0.008$ and
 $P(B | S_t) = 0.1$

(a) Probability of the failure of foundation:

(What if s.i. assumed?)

(b) Probability that building has settlement, but no excessive demands in terms of bearing capacity?