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457.212 Statistics for Civil & Environmental Engineers
In-Class Material: Class 22
Hypothesis Testing (A&T: 6.3)

1. Hypothesis Testing

Decide which of two competing claims or statements about a parameter is true based on a sample.

(a) **Step 1**: state **Null Hypothesis** H_0
- usually represents *status quo* about a parameter
- a statement one wishes to test/examine based on a new sample and finding

(b) **Step 2**: formulate **Alternative Hypothesis** H_1
- usually represents *the question to be answered* or *theory to be tested*.
- a conjecture one can make based on a sample.
- the statement we should accept if the null hypothesis is rejected.
- note H_0 and H_1 are mutually (   )

(c) **Step 3**: try to reach one of the following conclusions:

- **Reject** H_0 in favor of H_1 because of sufficient evidence in the data or
- **Fail to reject** H_0 because of insufficient evidence in the data

Example: It has been believed that *p* = 0.10 and one conjectures that *p* might be "higher than 0.10" or "different from 0.10" from what he/she has seen in a sample.

H_0: $p = 0.10$

H_1: $p > 0.10$ (one-sided), $p < 0.10$ (one-sided), or $p \neq 0.10$ (two-sided)

(d) Test **errors**: Due to the limited size of a sample, there is a probability of making a wrong conclusion.

- **Type I Error:** Rejecting H_0 even though it is true.
  The probability of Type I Error: α, "**level of significance**"
  (interpreted as "probability of wrong rejection")
- **Type II Error:** Failing to reject H_0 because of insufficient evidence in the data.
  The probability of Type II Error: β

	H_0 is true	H_0 is false
Do not reject H_0	Correct Decision	Type II error
Reject H_0	Type I error	Correct Decision

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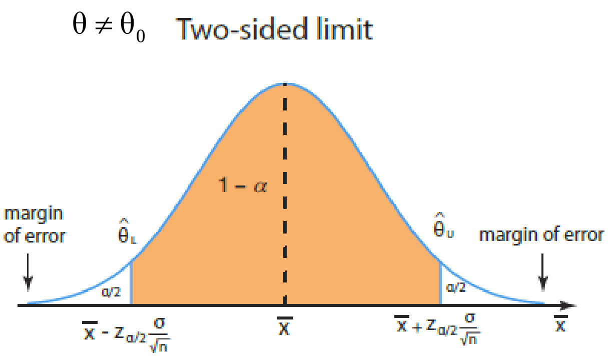
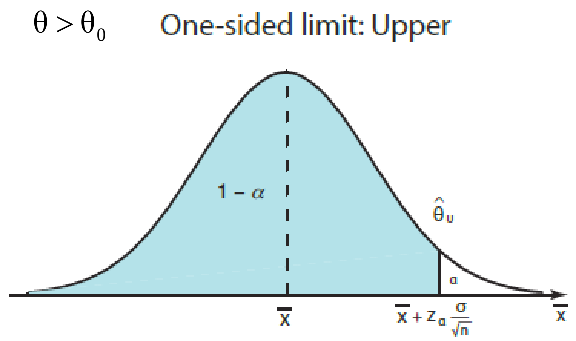
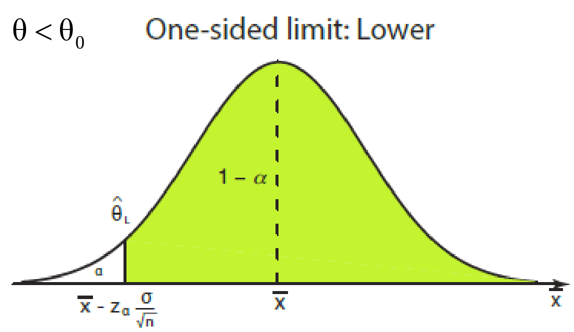
2. "Z-test" – Hypothesis Testing on Mean (with known σ)

cf. "T-test": The same test for unknown σ







Example 1 (A&T 6.5 - corrected): The specification for the yield strength of reinforcement bars ("rebars") requires that the mean value should be at least 38 psi. From the rebars delivered to the construction site, the engineer randomly selects 25 rebars and tested them for yield strengths.

The sample mean is 37.5 psi. The (exact) standard deviation of rebar strength is known to be 3.0 psi.

Since the engineer would be concerned only with rebars having mean yield strength lower than 38 psi, a ____-sided test is appropriate.

(a) What are the null and alternative hypotheses?

(b) With 5 % significance level, would the null hypothesis be accepted?

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Suppose the exact standard deviation is not known, so the testing should rely on the sample standard deviation 3.50 psi. Answer the above questions under this condition.

```
# t-test and z-test using sample
install.packages("BSDA") # for z.test
library(BSDA)

Ex22 = c(8.3, 5.6, 4.6, 9.3, 8.8, 6.5, 6.9, 7.1, 4.7, 9.2)
t.test(Ex22, alternative="greater", mu=5, conf.level=0.99)
t.test(Ex22, alternative="two.sided", mu=5, conf.level=0.99)
# one-sided: "greater" or "less"
# two-sided: "two.sided"
# If "p-value"(=probability of getting more extreme results given the null
   hypothesis is correct) is smaller than alpha, the null hypothesis is
   rejected

z.test(Ex22, alternative="greater", mu=5, sigma.x=3, conf.level=0.99)
```

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Example 2: Suppose an engineer is designing a sprinkler water supply system for a farm. To have an optimal water supply system, the flow rate of the sprinkler should be 20L/sec.

So, we are interested in testing whether or not the flow rate of the sprinkler is 20L/sec.

For hypothesis testing, we may express this formally as:

$H_0 : \mu = 20\text{L/sec}$
$H_1 : \mu \neq 20\text{L/sec}$

Suppose that n = 10 prototypes are tested to obtain the sample mean of flow rate of the sprinkler $\bar{X}$. The proposed testing method is that, if $18.5 < \bar{X} \leq 21.5$, we will not reject the null hypothesis H_0, and if either $\bar{X} \leq 18.5$ or $\bar{X} > 21.5$, we will reject the null hypothesis in favor of the alternative hypothesis H_1.

Assume that the standard deviation of the flow rate of the sprinkler is known as $\sigma = 2.5\text{L/sec}$, and the flow rate follows a distribution for which the conditions of the central limit theorem apply.

(a) What is the probability of Type I error?

(b) What is the probability of Type II error when the true mean is $\mu = 22\text{L/sec}$?

```
# Example 2(b)
1-pbinom(14, size=20, prob=0.55)


# Example 2(c)
pbinom(14, size=20, prob=0.8)


# Example 2(d)
1-pbinom(13, size=20, prob=0.55)
pbinom(13, size=20, prob=0.8)
```

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Example 3: It is known that a certain type of cement has a 55% chance of reaching the required strength after 28 days of "curing" process. To test whether a new and somewhat more expensive cement reaching the required strength better than before, 20 sample mixtures are made.

Proposed testing method: If more than 14 of those cement mixture reaches the required strength after 28 days of curing, the new cement will be considered *superior* to the one presently in use.

Let p denote the probability that the cement mixture will reach the required strength after 28 days.

(a) Null hypothesis and alternative hypothesis.

(b) Type I error α, or "level of significance"?

- The null hypothesis is being tested at $\alpha =$ level of significance.

(c) Type II error β (for an alternative hypothesis $p = 0.8$)?

(d) **New testing method**: If more than 13 surpass, the new cement is considered superior

$\alpha =$

$\beta =$

Note: The probability of committing both types of error can be reduced by increasing the sample size