

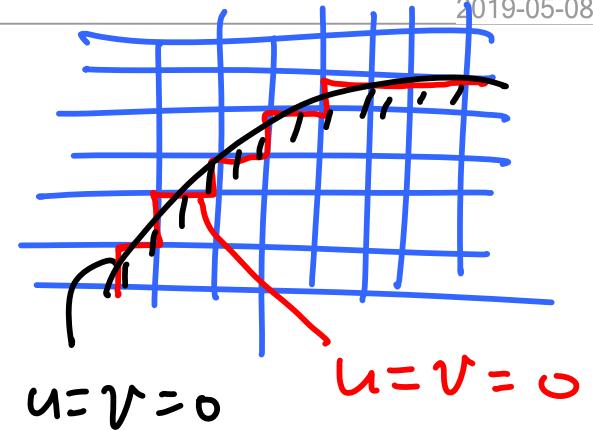
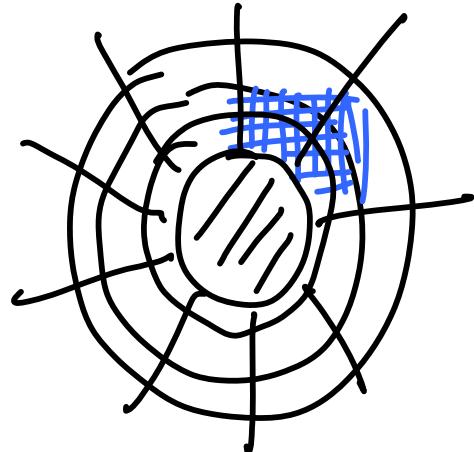
# Ch. 8 Complex geometries

노트 제목

2019-05-08

## 1. choice of grids

- ① stepwise approx. using regular grids
- ② overlapping grids like chimera grids

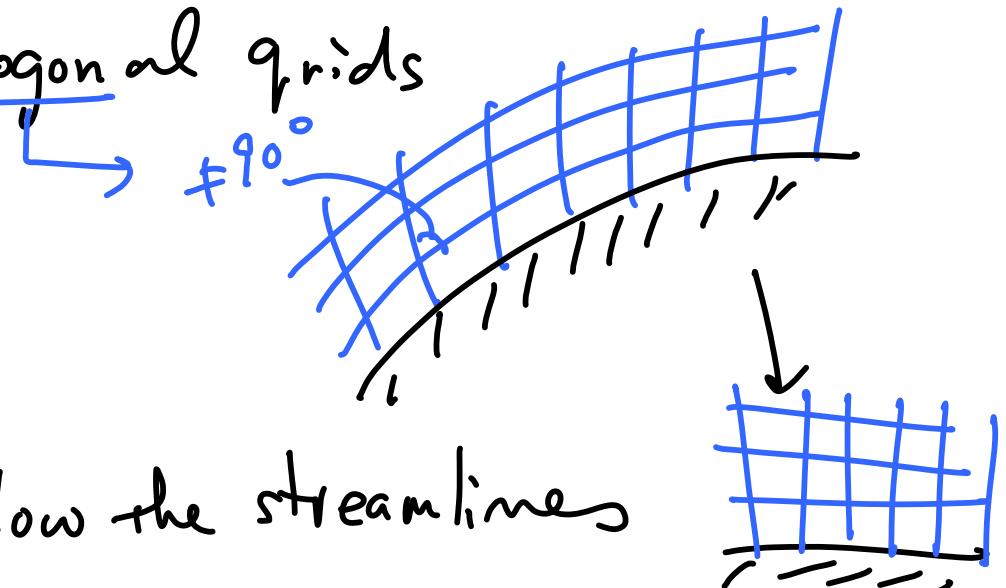


### ③ boundary-fitted non-orthogonal grids

advantage : can be adapted to  
any geometry.

good for b.c.

grid lines can follow the streamlines



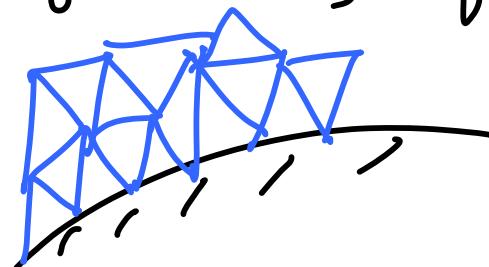
disadvantage : transformed egs contain more terms.

difficulty of programming

increase cost of solving egs.

### ④ unstructured grids

good for b.c.



Grid generation is difficult.

resulting in sparse matrix.

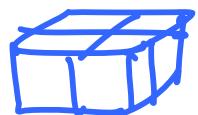
very popular.

2. Grid generation - very important issue    Thompson et al.  
(1985)

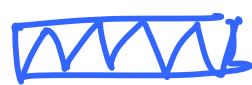
① make grids as nearly orthogonal as possible

② cell topology

in general, quadrilaterals in 2D



hexahedra in 3D are better than



triangles in 2D



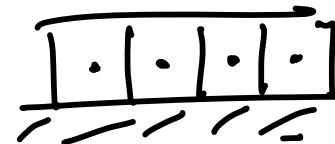
tetrahedra in 3D.



if i) midpoint rule integral approx.  
linear interpolation  
central difference

} are used

ii) very near the wall

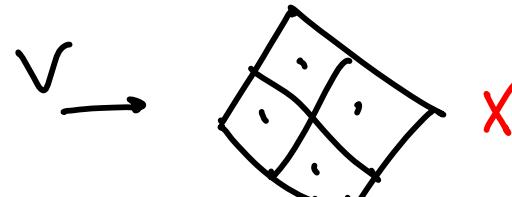
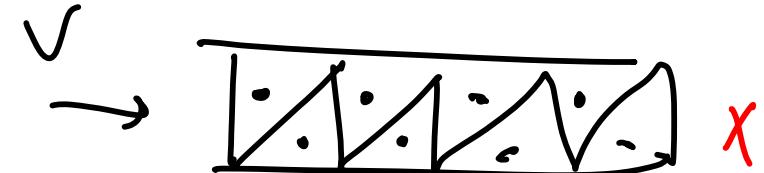
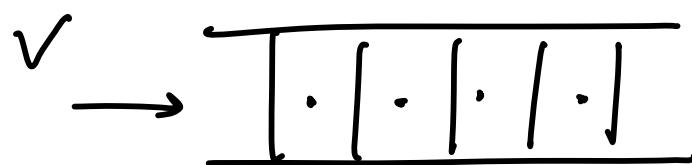


vs.

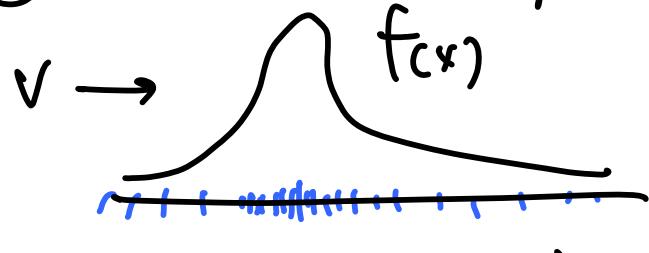


iii) for convection terms

(accuracy is improved if grid lines follow streamlines)

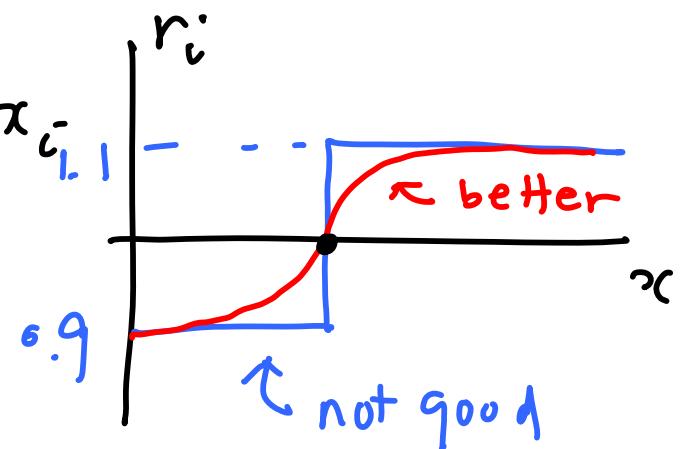


### ③ non-uniform grids

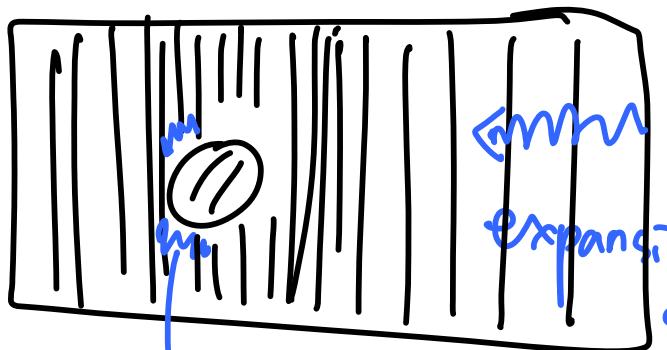


compression      expansion of grids

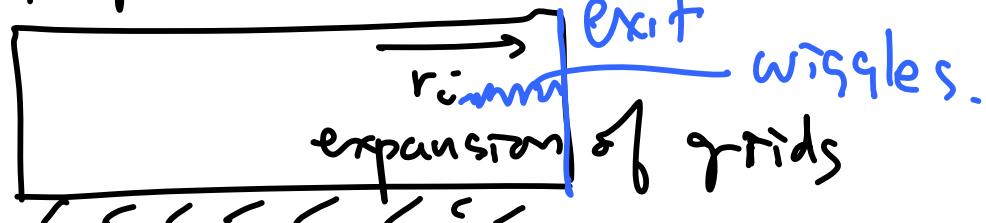
$$r_i = \partial x_{i+1} / \partial x_i$$



Hahn & Choi (1997, JCP)



wiggles w/ CD2      no wiggle w/ upwind

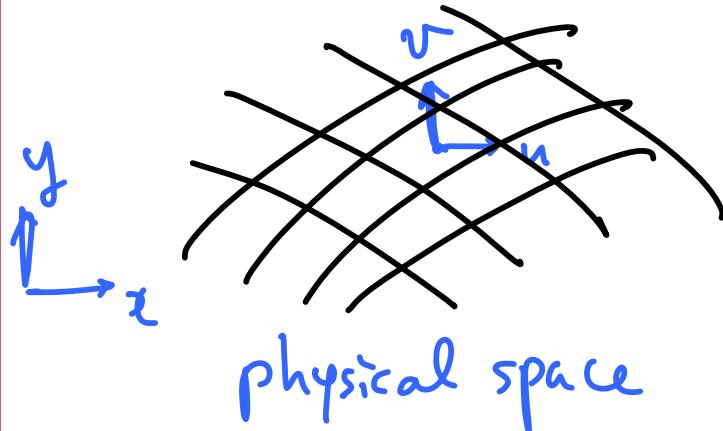


$r_i$  should be kept under control.

wiggles.

### 3. Choice of velocity components

#### ① Grid-oriented vel. components



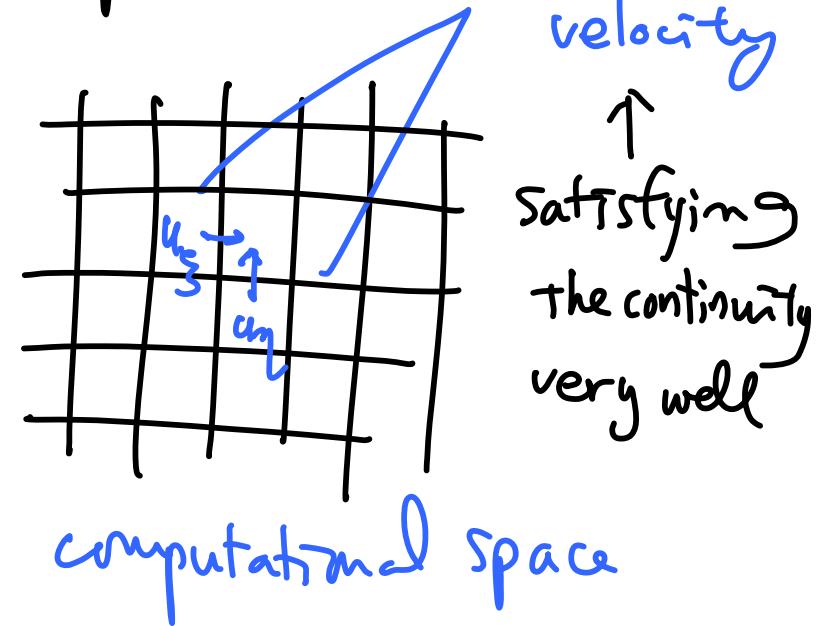
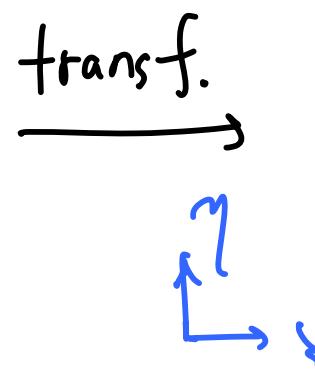
← contours

$$\omega_z \ 0$$

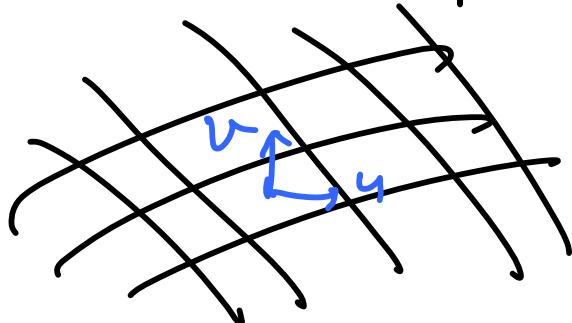
$$P \ X$$

quite complicated ← (e.g. Choi, Moim & Kim  
(1993, JFM))

transf.



② Cartesian vel. comps.



unstructured  $\rightarrow$  sparse matrix

generalized body-fitted grids

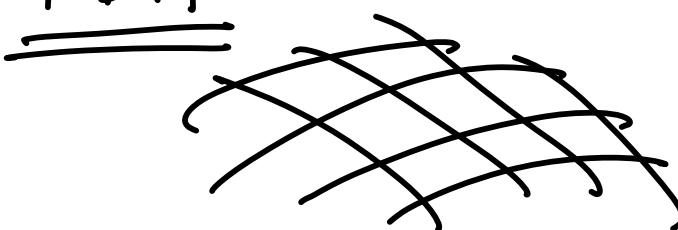
$$\hookrightarrow \frac{\partial u_i}{\partial x_j} \rightarrow \frac{\partial u_i}{\partial \xi_{1C}} \frac{\partial \xi_{1C}}{\partial x_j}$$

$\downarrow$   
non-sparse  
matrix.

4. choice of variable arrangement

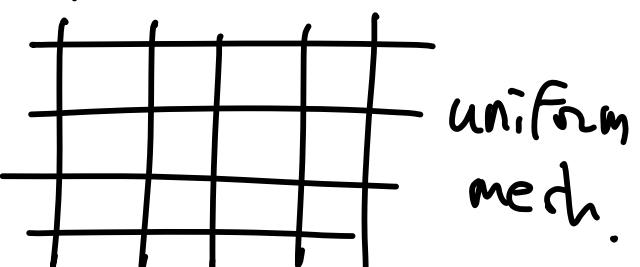
① staggered ② collocated

5. FDM  $\rightarrow$  coordinate transformation



transf.

$x_i$



$$x_i \rightarrow \xi_i$$

$$\frac{\partial \phi}{\partial x_i} \rightarrow \frac{\partial \phi}{\partial \xi_j} \frac{\partial \xi_j}{\partial x_i}$$

metric coeff.

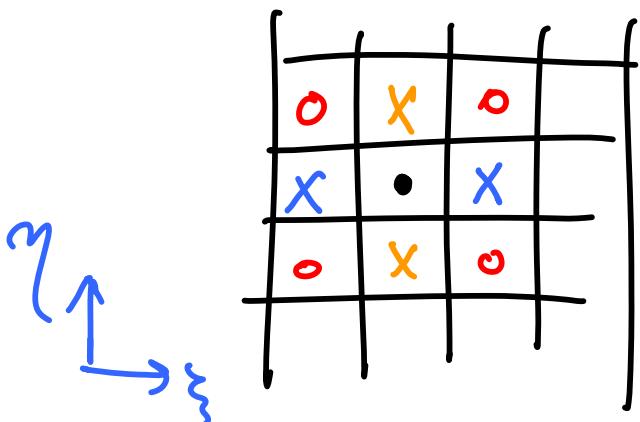
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \rightarrow \frac{\partial^2 \phi}{\partial \xi^2}, \frac{\partial^2 \phi}{\partial \eta^2}, \frac{\partial^2 \phi}{\partial \xi \partial \eta}$$

$$J = \det \left( \frac{\partial x_i}{\partial \xi_j} \right) : \text{Jacobian}$$

cross-derivative term

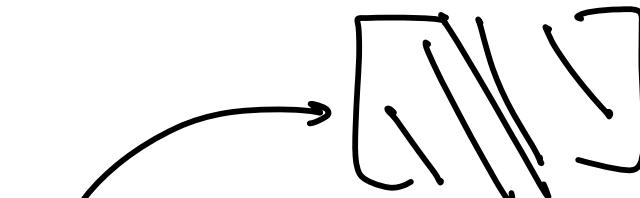
$= 0$  for orthogonal grids

$\neq 0$  " non- " "



$$\frac{\partial u_i}{\partial t} + \dots = \nabla^2 u_i \rightsquigarrow \text{ADI}$$

$$(1 - \alpha t A_x - \alpha t A_y) \hat{u}_j = \dots$$

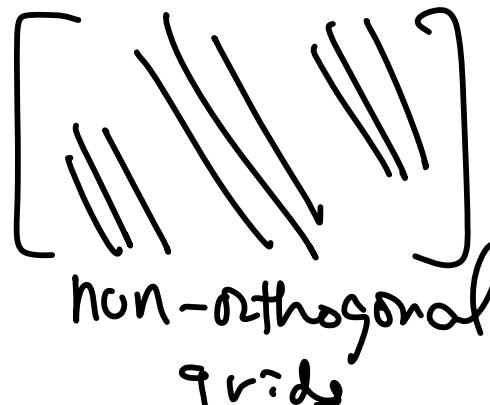


$$\rightarrow (I - \alpha A_x)(I - \alpha A_y) \hat{u}_i = \dots$$

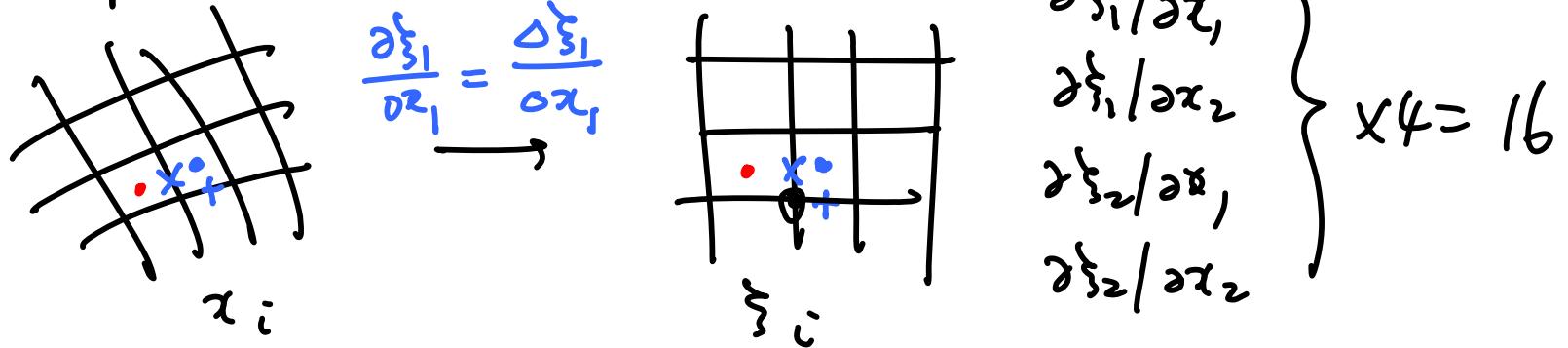
with  $\frac{\partial^2 \phi}{\partial x^2}$ , ADI is impossible!

Poisson eq.  $\nabla^2 \phi = r$  (no st!)  $\rightarrow$  (no ADI!)

$\hookrightarrow$  iteratively solve the discretized eqs.



- need to store metric coeffs.  $\frac{\partial \xi_i}{\partial x_j}$
- easy to discretize in the transformed space
- no interpolation of metric coeffs.



interpolation breaks the conservation (Thompson et al. 1987)

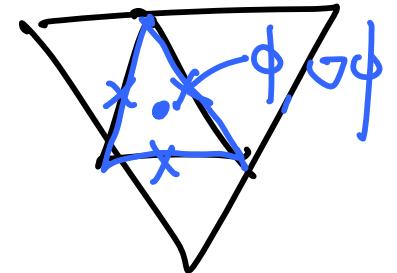
- skewness and large aspect ratio of grids give poor convergence or oscillations in the solution.



6. FVM

$$\frac{\partial}{\partial t} \int_{\Omega} \rho \phi d\Omega + \int_{\Omega} (\rho \phi \nabla \cdot \underline{u}) ds = \int_{\Gamma} \Gamma \nabla \phi \cdot \underline{n} ds + \int_{\Omega} \delta_{\phi} d\Omega$$

- no need to transform
- need to get  $\phi$  or  $\nabla \phi$  at cell face  
by interpolation or some other methods.



7. FEM - natural method for complex geometries.

FSI



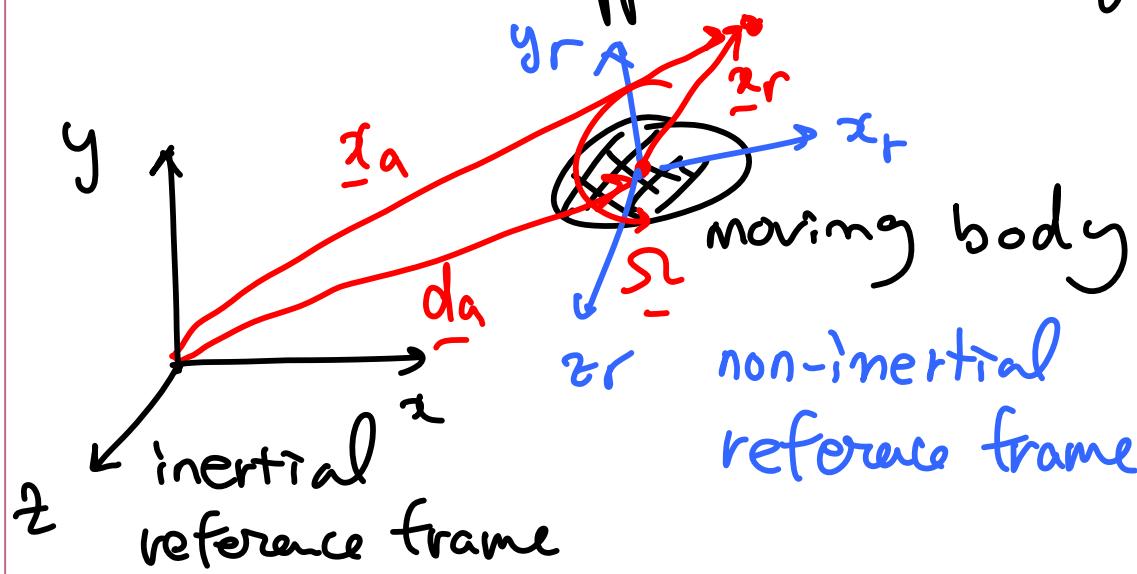
8. Pressure correction eq. — covered already

9. Axially-symmetric problems  $(r, \theta, z)$

centrifugal force  $-\underline{\Omega} \times \underline{\Omega} \times \underline{z}$

Coriolis force  $-2\underline{\Omega} \times \underline{u}_r$

appear in N-S eqs.



Term project

download

on eTL

May 28 & 22

9:30 - 10:45

tutorials

$N-S$  eqs in the non-inertial reference frame are

$$\frac{\partial \underline{u}_r}{\partial t} \Big|_r + \nabla \cdot (\underline{u}_r \underline{u}_r) = -\nabla p + \frac{1}{R_e} \nabla^2 \underline{u}_r - \underline{\Omega} \times \underline{R} \times \underline{x}_r$$

$$- 2 \underline{\Omega} \times \underline{u}_r - \frac{d\underline{R}}{dt} \times \underline{x}_r - \underline{R}^T \frac{d \underline{a}}{dt}$$

centrifugal

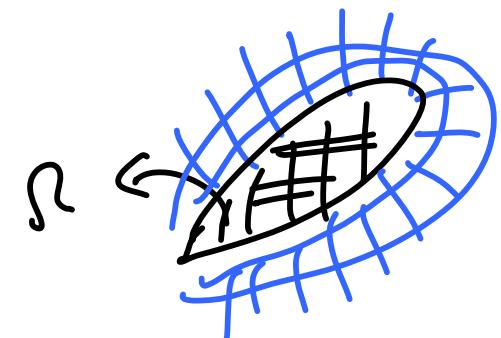
: non  
-conserv  
-tive  
form

where  $\underline{x}_a = \underline{R} \underline{x}_r + \underline{a}_a$  Coriolis.

$$\underline{u}_g = \underline{R} (\underline{u}_r + \underline{\Omega} \times \underline{x}_r + \underline{u}_s)$$

$$\underline{u}_s = \underline{R}^T \frac{d \underline{a}}{dt}$$

$$\frac{\partial \underline{u}_r}{\partial t} \Big|_r = \frac{\partial \underline{u}_r}{\partial t} \Big|_a + (\underline{\Omega} \times \underline{x}_r + \underline{u}_s) \cdot \nabla \underline{u}_r$$



Beddhu et al (JCP, ... ) - conservative form

$$\left\{ \begin{array}{l} \frac{\partial \underline{u}}{\partial t}|_r + \nabla \cdot [(\underline{u} - \underline{v}) \underline{u} + \underline{u} \underline{\omega}] = - \nabla p + \frac{1}{\rho_e} \nabla^2 \underline{u} \\ \nabla \cdot \underline{u} = 0 \end{array} \right.$$

where  $\underline{u} = \underline{u}_r + \underline{v} = R^T \underline{u}_a$  Kim & Choi;

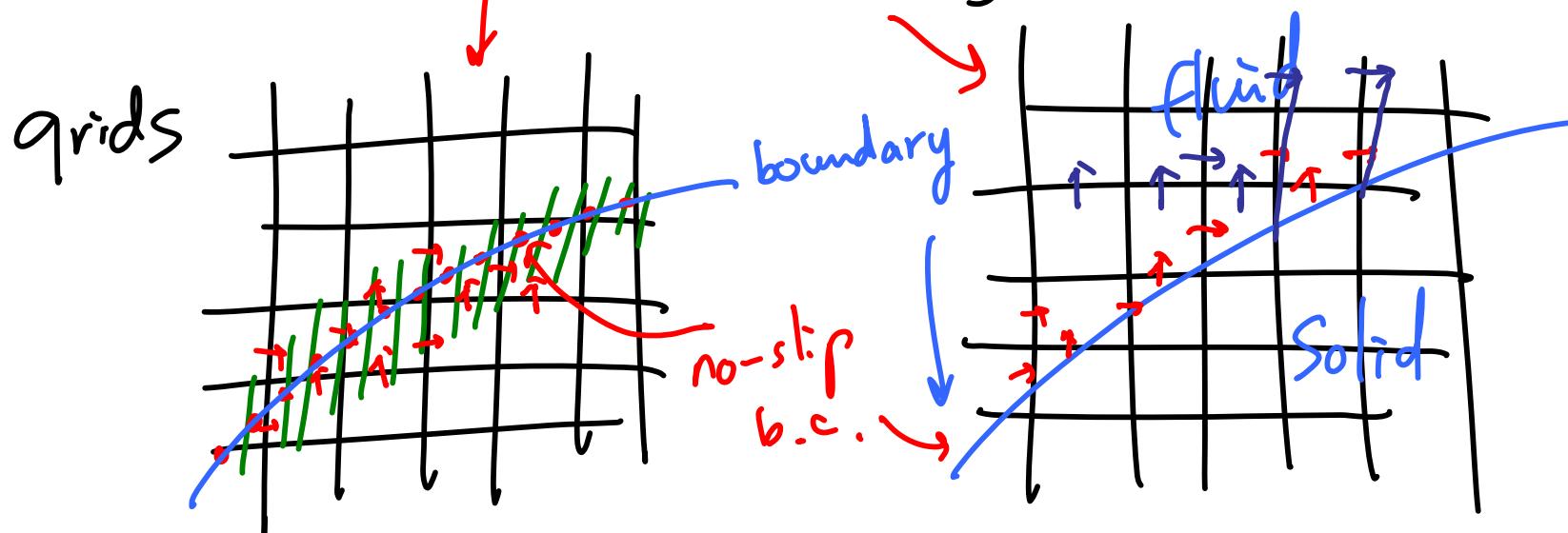
$$\underline{v} = \underline{\Omega} \times \underline{x}_r + \underline{u}_s \quad (\text{JCP, 2006})$$

$$\underline{u}_s = \underline{\Omega} \times \underline{x}_r$$

## \* Immersed boundary (IB) method

( Peskin : continuous forcing IB method

( Fadlun et al. : discrete forcing IB method (JCP 1995) 2000



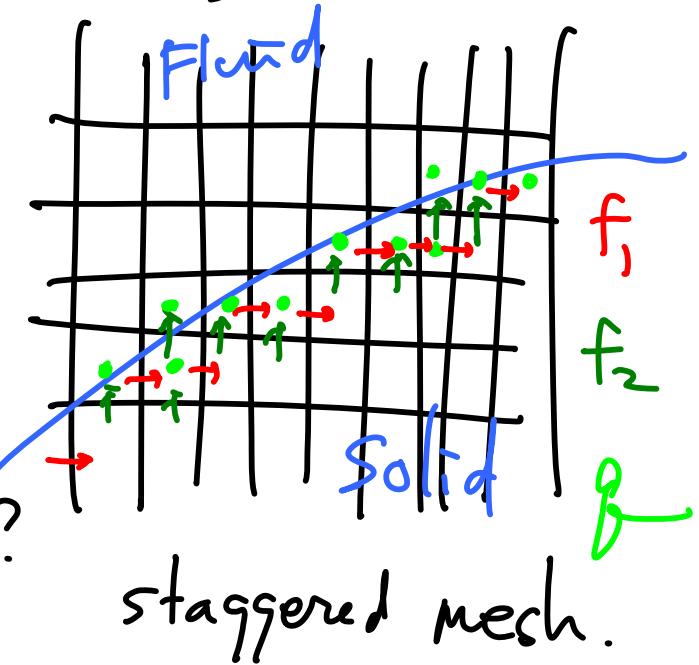
Kim, Kim & Choi (2001, JCP) - discrete forcing IB method

$$\left\{ \begin{array}{l} \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) = - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \nabla^2 u_i + f_i \\ \frac{\partial u_i}{\partial x_i} - g = 0 \end{array} \right.$$

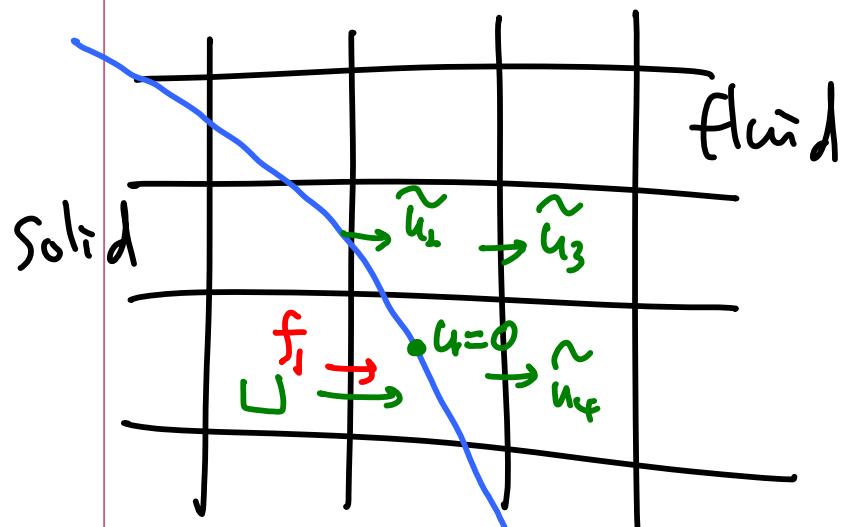
How to find  $f_i$  &  $g$  to satisfy  
the no-slip b.c.'s on the boundary?

Fractional step method (RK3 + CN)

$$\int \frac{\hat{u}_i^k - u_i^{k-1}}{\partial t} = \alpha_k L_i(\hat{u}^k) + \alpha_k L_i(u^{k-1}) - 2\alpha_k \frac{\partial p^{k-1}}{\partial x_i} - \gamma_k N_i(u^{k-1}) - \rho_k N_i(u^{k-2}) + f_i^k \quad k=1, 2, 3$$



$$\left. \begin{aligned} \nabla^2 \phi^k &= \frac{1}{2\alpha_{kot}} \left( \frac{\partial \hat{u}_i^k}{\partial x_i} - g^k \right) \\ u_i^k &= \hat{u}_i^k - 2\alpha_k ot \frac{\partial \phi^k}{\partial x_i} \\ p^k &= p^{k-1} + \phi^k - \frac{g_{tot}}{Re} \nabla^2 \phi^k \end{aligned} \right\}$$



$f_i \neq 0$  in solid part,  $f_i = 0$  in fluid part.

$$\left. \begin{aligned} \alpha_1 &= \frac{4}{15}, \quad \alpha_2 = \frac{1}{15}, \quad \alpha_3 = \frac{1}{6} \\ \gamma_1 &= \frac{8}{15}, \quad \gamma_2 = \frac{5}{12}, \quad \gamma_3 = \frac{3}{4} \\ \rho_1 &= 0, \quad \rho_2 = -\frac{11}{60}, \quad \rho_3 = -\frac{5}{12} \end{aligned} \right\}$$

A provisional velocity  $\hat{u}_i$  from explicit Euler method

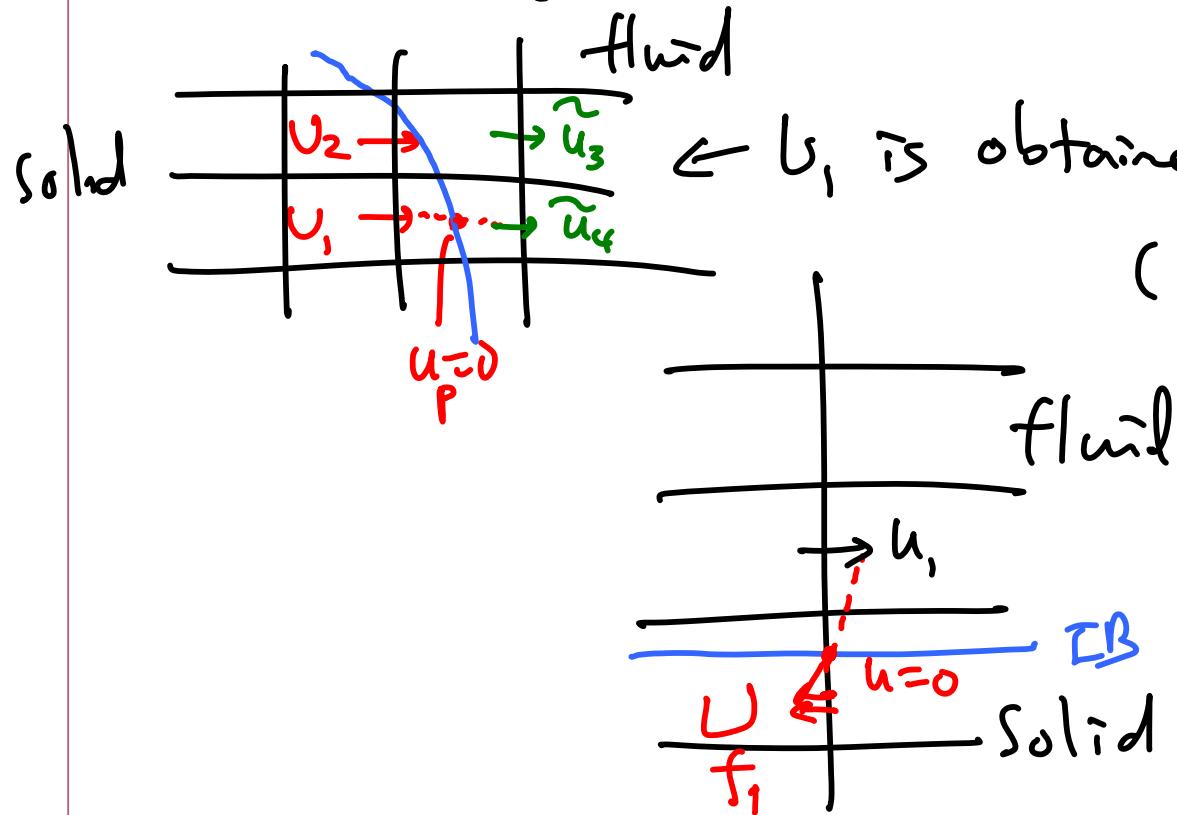
$$\frac{u_i^k - u_i^{k-1}}{ot} = 2\alpha_k L_i(\underline{u}^{k-1}) - 2\alpha_{1c} \frac{2p^{k-1}}{\partial x_i} - \gamma_k N_i(\underline{u}^{k-1}) - \beta_k N_i(\underline{u}^{k-2})$$

↪ obtain  $\hat{u}_2, \hat{u}_3$  &  $\hat{u}_4$ .

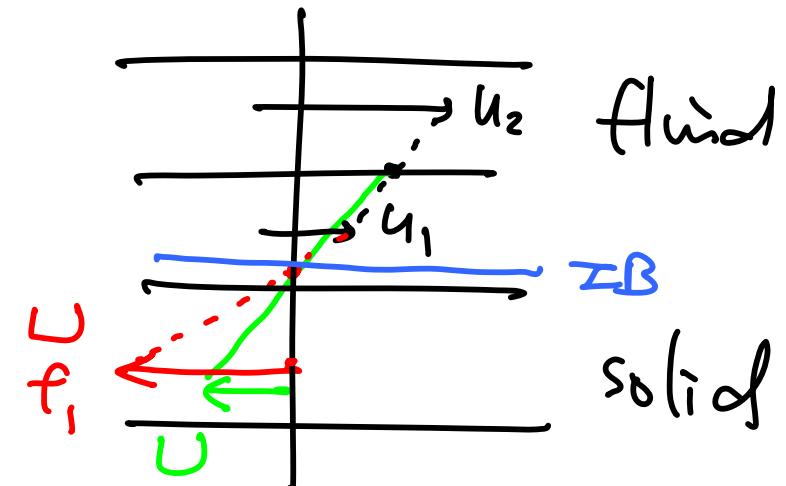
$\underline{U}$  is obtained from  $\tilde{u}_2$ ,  $\tilde{u}_3$  &  $\tilde{u}_4$  by satisfying  $u=0$   
 (bilinear interpolation) on IB.

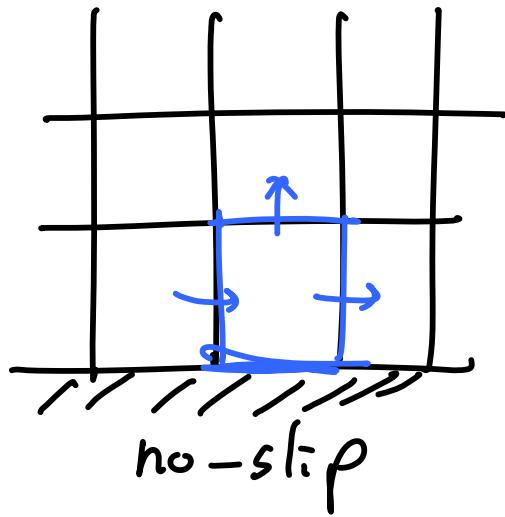
$$\rightarrow f_i^k = \frac{u_i - u_i^{k-1}}{\partial t} - 2\alpha_k L_i(\underline{u}^{k-1}) + 2\alpha_k \frac{\partial p^{k-1}}{\partial x_i} + \gamma_k N_i(\underline{u}^{k-1}) + \beta_k N_i(\underline{u}^{k-2})$$

for forcing points.



$u_i$  is obtained by  $u_p=0$  and  $\tilde{u}_4$   
 (linear interpolation)





Kim Dongjoo  
FVM unstructured mesh  
(JCP, 2000)

