

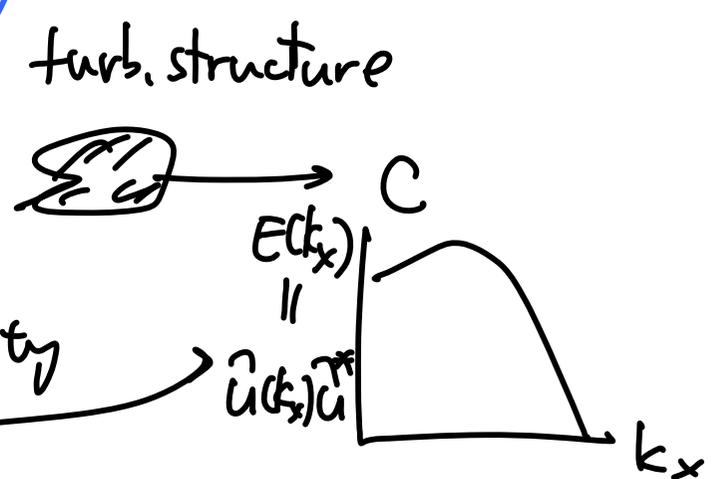
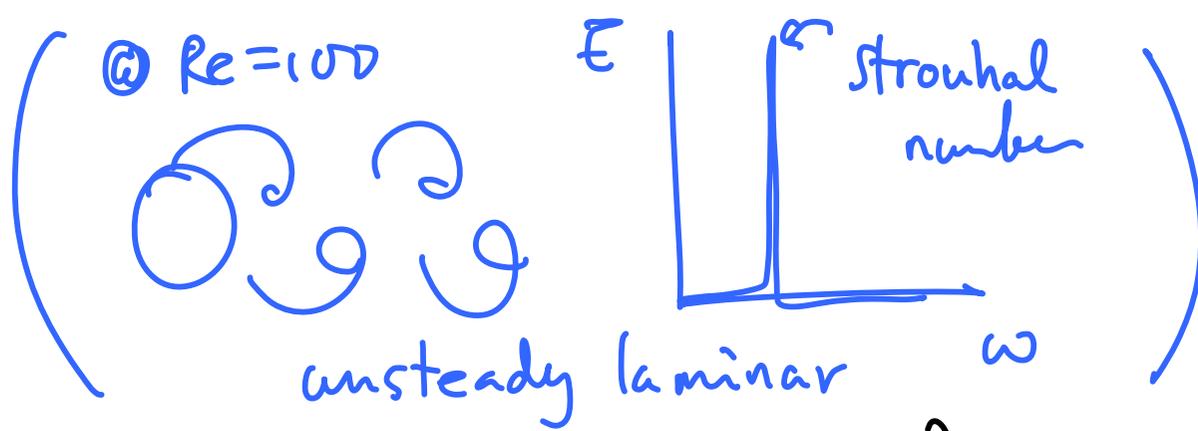
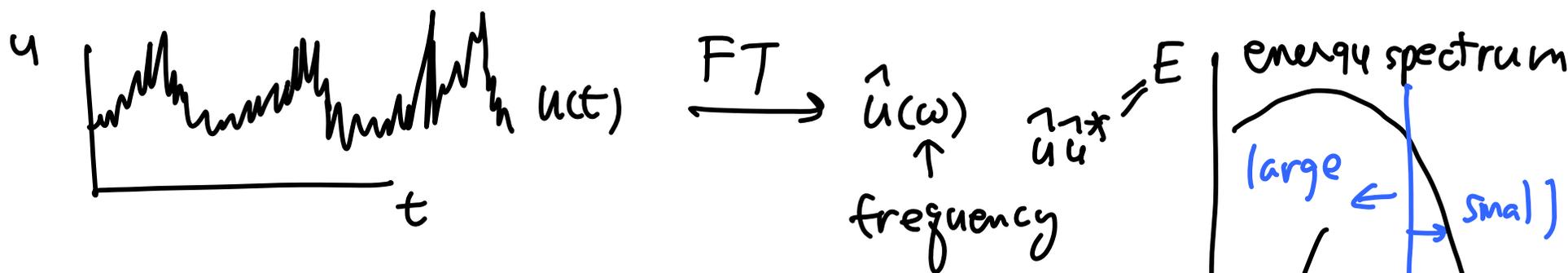
Ch. 9 Turbulent flows

are unsteady 3-D.

contain a variety of spatial and temporal scales
from large to small motions.



which makes the prediction of turbulent flow
very difficult.



Taylor hypothesis $\frac{\partial}{\partial t} \rightarrow C \frac{\partial}{\partial x}$
 $u(t) \rightarrow u(x) \xrightarrow{FT} \hat{u}(k_x)$
 convection velocity

* Prediction methods for turbulent flow ^{↑ wavenumber}

① RANS (Reynolds averaged Navier-Stokes eq.) technique

② LES (Large eddy simulation)

③ DNS (Direct numerical simulation)

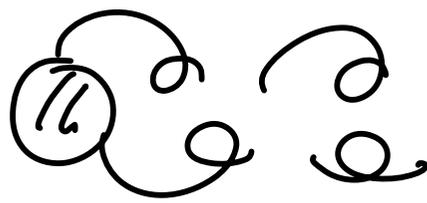
④ Wall-modelled LES or hybrid LES/RANS

* Direct numerical simulation

↳ no turbulence model

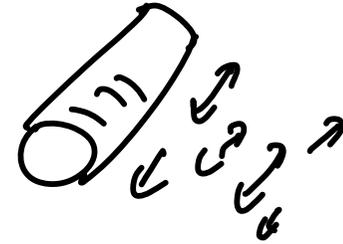
$$\rightarrow \begin{cases} \frac{\partial u_i}{\partial x_i} = 0 \\ \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i \end{cases}$$

should be unsteady & 3D.

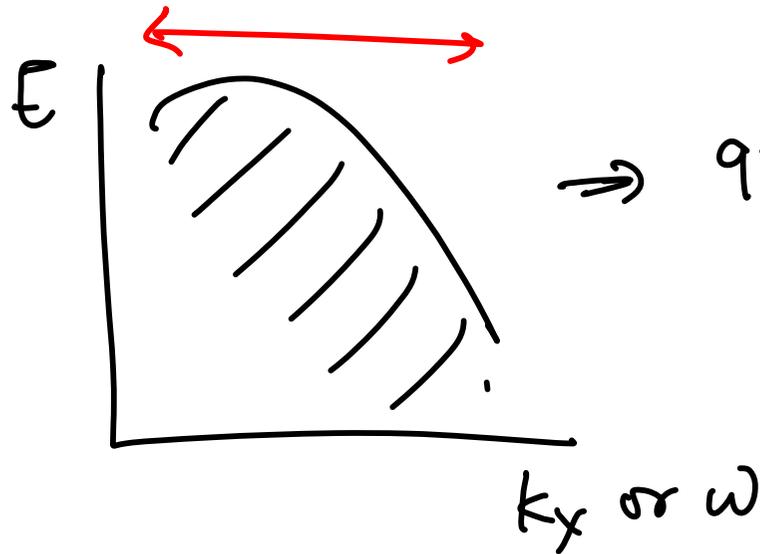


mean $\xrightarrow{\quad}$ 2D \leftarrow RANS

DNS \rightarrow 3D



unsteady.



\Rightarrow grid resolves all turbulence scales!

\hookrightarrow DNS is highly accurate,

\downarrow

(since late 1980's)

Kim, Moim & Moser (1987)

Prediction methods for turbulent flow

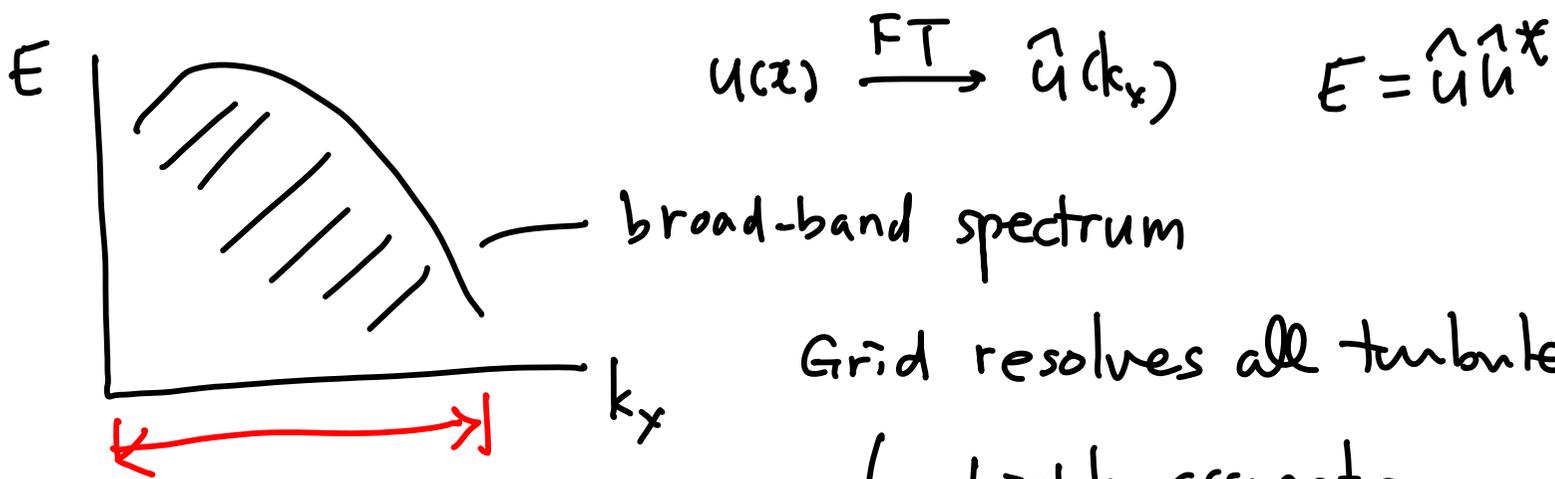
- ① RANS (Reynolds averaged Navier-Stokes eq)
- ② LES (Large eddy simulation)
- ③ DNS (Direct numerical simulation)
- ④ wall-modelled LES or Hybrid RANS/LES

* Direct numerical simulation (DNS) - no turbulence model

$$\left\{ \begin{array}{l} \frac{\partial u_i}{\partial x_i} = 0 \\ \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i \end{array} \right.$$

unsteady
3-D





— broad-band spectrum

Grid resolves all turbulence scales!

↳ highly accurate

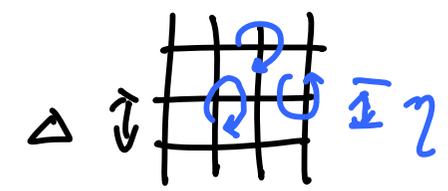
DNS ← since 1980's

- Number of grid point requirement

smallest length scale in turbulence - Kolmogorov length scale,

DNS should resolve the Kolmogorov scale

η

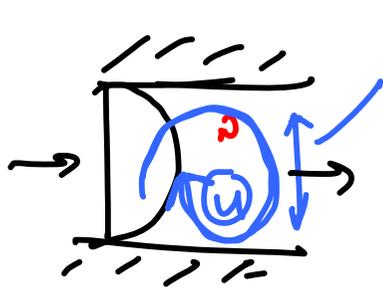
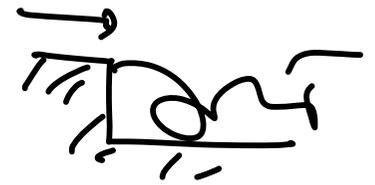


$$\Delta \sim \eta = (\nu^3 / \epsilon)^{1/4}$$

grid size

ν : kinematic viscosity
 ϵ : dissipation rate.

in real situation, $\Delta \sim O(1) \eta = 1 \sim 10 \eta$.



l : largest turbulent length scale

u : " " " velocity "

N : number of grid pts in 1-direction

$$N = \frac{l}{\Delta} \sim \frac{l}{\eta} = l (\epsilon / \nu^3)^{1/4} \sim l (P / \nu^3)^{1/4}$$

production

↓
 $(P \sim \epsilon)$

$$P \sim -\overline{u_i u_j} \frac{\partial \bar{u}_i}{\partial x_j} \sim u^2 \cdot \frac{u}{l} \sim u^3 / l$$

$$N \sim l (u^3/l/\nu^3)^{1/4} \sim \left(\frac{u^3 l^3}{\nu^3}\right)^{1/4} \sim Re_l^{3/4} \quad (Re_l = \frac{ul}{\nu})$$

total # of grid pts. in 3D

$$N^3 \sim Re_l^{9/4}$$

→ computationally very expensive
↓
rarely used for eng. applications

but very useful for academic purposes.

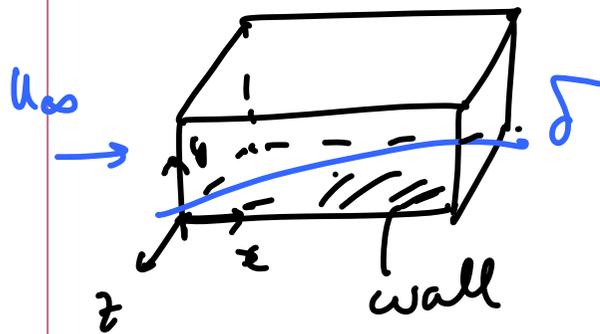
DNS → $u_i(x, t)$

$p(x, t)$

$\overline{p'u'}$

- development of turb. models
- understanding turb. physics
- development of turb. control

- # of grid pts requirement (Choi & Moim, 2012, PoF)



turb. boundary layer flow

$$Re_{Lx} = \frac{u_{\infty} L_x}{\nu}, \quad Re_{\delta x} = \frac{u_{\infty} \delta_{Lx}}{\nu}, \quad Re_{\tau_{Lx}} = \frac{u_{\tau_{Lx}} L_x}{\nu}$$

$$(u_{\tau_{Lx}} = \sqrt{\tau_{w_{Lx}} / \rho})$$

$$N^3 \sim Re_{Lx}^{3^{7/14}} \sim Re_{\delta_{Lx}}^{3^{9/12}} \sim Re_{\tau_{Lx}}^{3^{7/11}}$$

$$u_{\infty} = 15 \text{ m/s}$$

$$L_x = 10 \text{ m}$$

$$Re_{Lx} = \frac{15 \times 10}{1.5 \times 10^{-5}} = 10^7$$

$$N^3 \sim (10^7)^{3^{7/14}} = 10^{18.5} \quad !$$

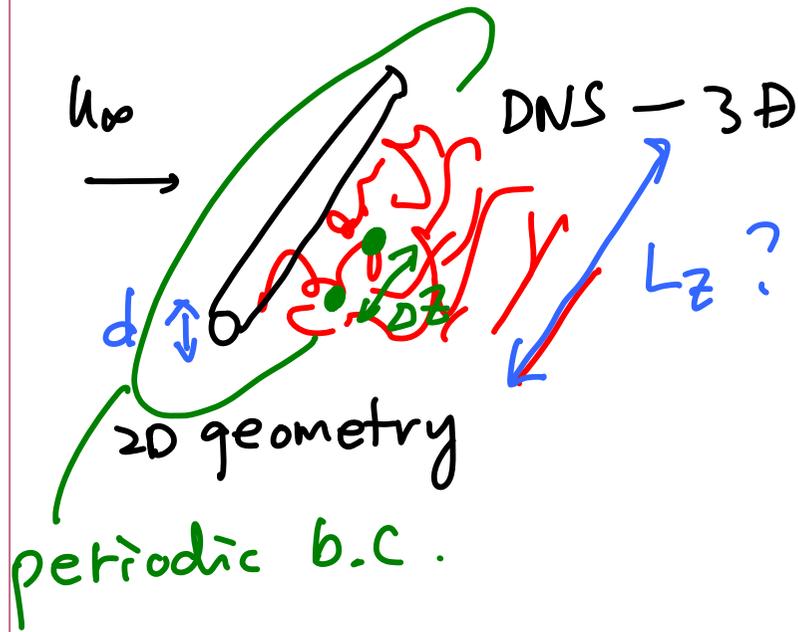
largest # of pts

$$N^3 \sim 10^{11} \quad (2016)$$

$$N^3 \sim 10^6 \quad (1989)$$

impossible!

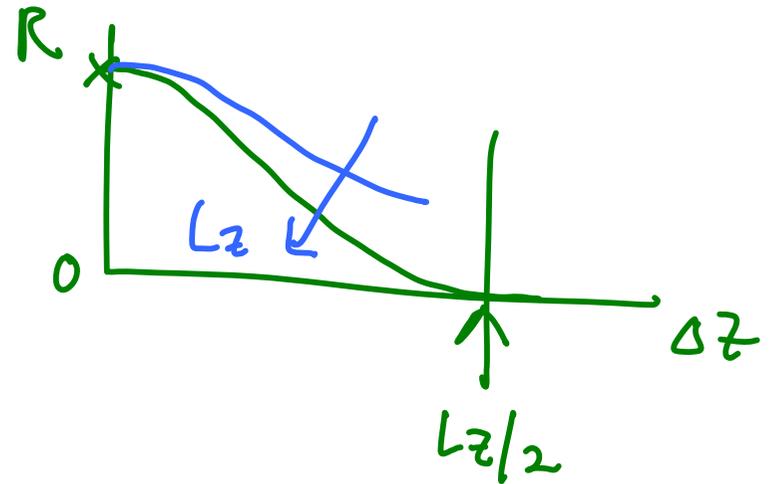
$$10^{11} \rightarrow \begin{matrix} u_i(\underline{x}, t) \\ p(\underline{x}, t) \end{matrix}$$



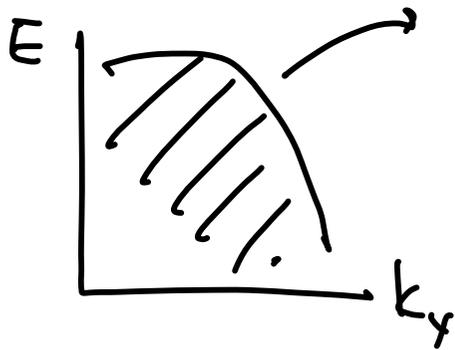
$$L_z/d = ?$$

two-point corr.

$$R(\Delta z) = \frac{u(x, y, z, t) u(x, y, z + \Delta z, t)}{u(x, y, z, t) u(x, y, z, t)}$$



* RANS



RANS models all turbulence scales.

$$u_i = \overline{u_i} + u_i'$$

\uparrow instantaneous velocity \uparrow mean (time-averaged) velocity
 \uparrow fluctuating velocity

: Reynolds decomposition

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0 \Rightarrow \frac{\partial \overline{u_i}}{\partial x_i} = 0 \rightarrow -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_i}$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} u_i u_j = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i \rightarrow \nu \nabla^2 \overline{u_i}$$

$$\hookrightarrow \frac{\partial}{\partial x_j} (\overline{u_i + u_i'})(\overline{u_j + u_j'}) = \frac{\partial}{\partial x_j} (\overline{u_i u_j} + \overline{u_i' u_j'})$$

$$\overline{u_i'} = \overline{u_j'} = 0$$

$$\downarrow$$

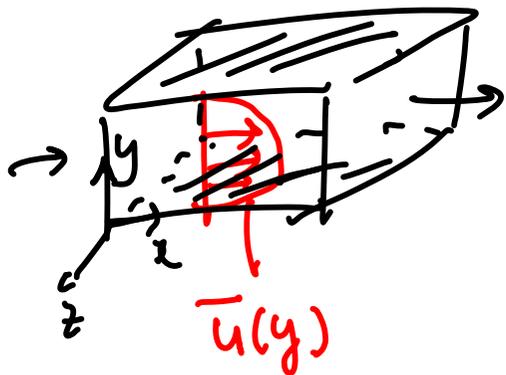
$$\left(\overline{u_i' u_j} = \overline{u_i u_j'} = 0 \right)$$

$$\Rightarrow \begin{cases} \frac{\partial \bar{u}_i}{\partial x_i} = 0 & \bar{u}_i, \bar{p} \\ \frac{\partial}{\partial x_j} \bar{u}_i \bar{u}_j = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \bar{u}_i}{\partial x_j} - \overline{u_i' u_j'} \right) \end{cases}$$

Reynolds stress

Reynolds stress is obtained by modeling whole turb. scales.

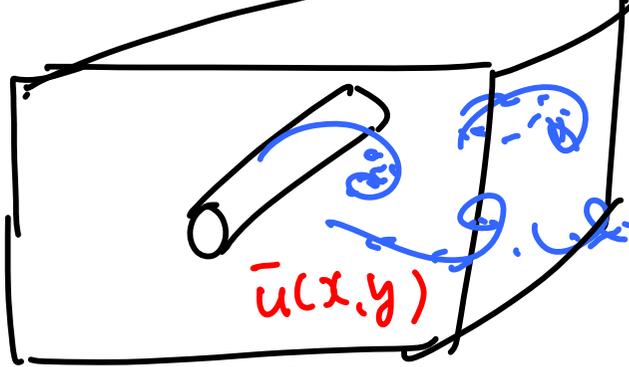
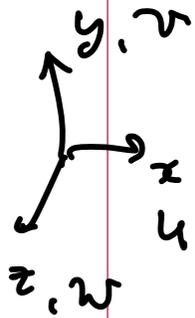
turbulent channel flow



DNS : unsteady, 3-D $\rightarrow N^3 \sim \mathcal{O}(10^6)$

RANS : steady, 1D $\rightarrow N \sim \mathcal{O}(10)$

Flow over a circular cylinder



$$\bar{u}(x,y)$$

$$\bar{v}(x,y)$$

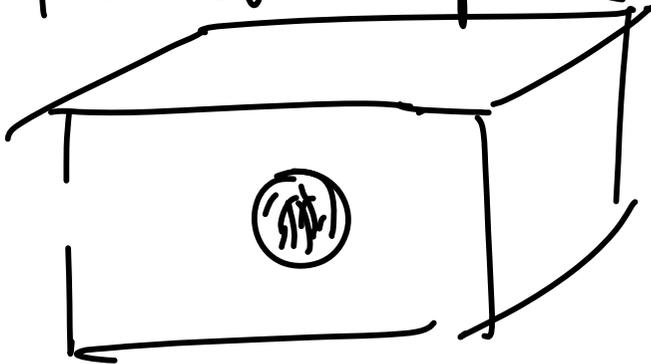
$$\bar{w} = 0$$

DNS : unsteady, 3-D

RANS : steady, 2-D

→ unsteady, 2-D

Flow over a sphere

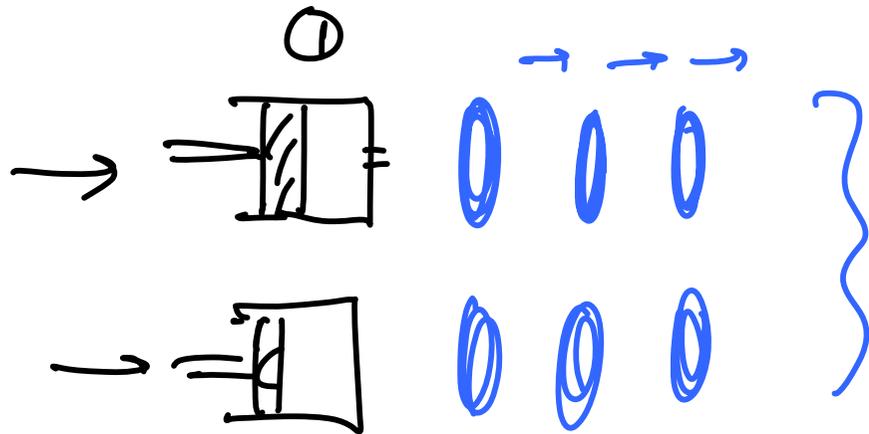


DNS : unsteady, 3-D

RANS : steady, 3-D

↳ unsteady 3-D

unsteady RANS : $u_i = \langle u_i \rangle + u_i'$: ensemble averaging



unsteady RANS

$$\frac{\partial \langle u_i \rangle}{\partial t} + \frac{\partial}{\partial x_j} \langle u_i \rangle \langle u_j \rangle$$

$$= -\frac{1}{\rho} \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \langle u_i \rangle}{\partial x_j} - \langle u_i' u_j' \rangle \right)$$

RANS : $N^3 \sim Re^0$

inaccurate for the prediction of massively separated flow or flow of high curvature.

useful for eng. purposes.

- How to model the Reynolds stress term $\langle u'_i u'_j \rangle$?

Boussinesq eddy viscosity hypothesis

laminar flow: $\tau_{ij} - \frac{1}{3} \delta_{ij} \tau_{kk} = 2\mu \tau_{ij}$

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

turbulent flow: $\frac{\partial}{\partial x_j} \left(\nu \frac{\partial \langle u_i \rangle}{\partial x_j} - \langle u'_i u'_j \rangle \right)$

$$\Rightarrow -\langle u'_i u'_j \rangle + \frac{2}{3} k \delta_{ij} = 2\nu_T \langle S_{ij} \rangle$$

$$\langle S_{ij} \rangle = \frac{1}{2} \left(\frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right)$$

fluid property

eddy viscosity
"flow" property

$k = \frac{1}{2} \langle u'_i u'_i \rangle$
turbulent kinetic energy.

• Eddy viscosity ν_T [$L^2 T^{-1}$] (m^2/s)

$$\nu_T \sim l (\text{length scale}) \times u (\text{velocity scale})$$

Eddy viscosity ν_T [$L^2 T^{-1}$] (m^2/s)

노트 제목

2019-05-29

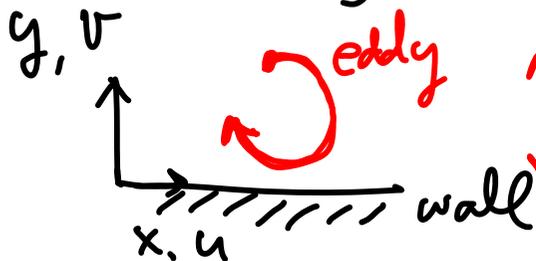
$$-\langle u_i' u_j' \rangle + \frac{2}{3} k \delta_{ij} = 2 \nu_T \langle s_{ij} \rangle$$

$\nu_T \sim l$ (length scale) $\times u$ (velocity scale)

what are the most relevant length & velocity scales?

① zero-equation model: no transport eq.

mixing length model (Prandtl 1925)

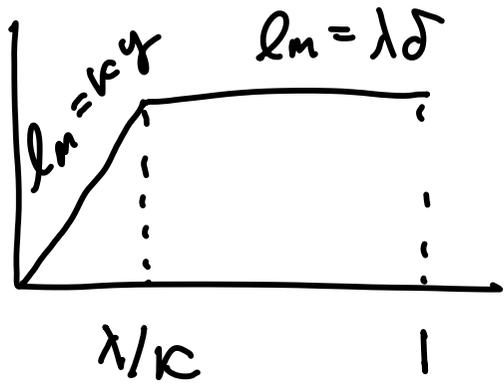


l_m : mixing length

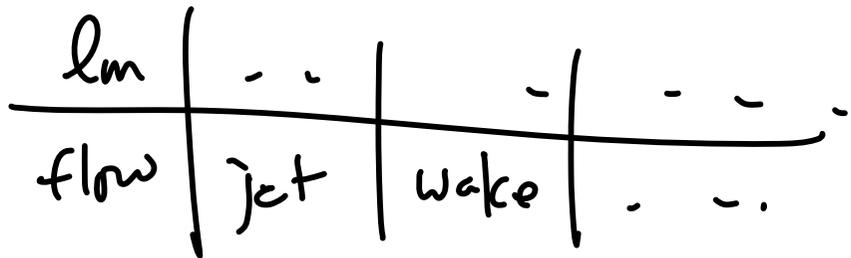
$$\nu_T = l u$$

$$= l_m \cdot l_m \frac{\partial \langle u \rangle}{\partial y} = l_m^2 \frac{\partial \langle u \rangle}{\partial y} > 0$$

boundary layer flow l_m



y/δ ← boundary layer thickness



② one-equation model : one transport eq.

i) k-eg : modeling the velocity scale only

$$u \sim \sqrt{k} \quad , \quad \nu_T = \zeta \mu_l \sqrt{k}$$

$$k = \frac{1}{2} \langle u'_i u'_i \rangle$$

$$u_i' \left[\frac{\partial u_i'}{\partial t} + \dots \right] \quad \frac{\partial u_i}{\partial t} + \dots$$

$$\left\langle \frac{\partial}{\partial t} \frac{1}{2} u_i' u_i' \right\rangle \rightarrow \frac{\partial k}{\partial t} + \dots$$

$\left\langle \frac{\partial u_i}{\partial t} + \dots \right\rangle$

$$\Rightarrow \left(\frac{\partial k}{\partial t} + \langle u_j \rangle \frac{\partial k}{\partial x_j} \right) = \frac{\partial}{\partial x_j} \left(\frac{\nu_T}{\sigma_k} \frac{\partial k}{\partial x_j} \right) + \nu_T \left(\frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) \frac{\partial \langle u_i \rangle}{\partial x_j} - c_D \frac{k^2}{l} \quad \text{Aerospace Science meeting (1992) # 8666}$$

$l = l_m \leftarrow \text{limitation}$

ii) Spalart - Almaras model

$$\nu_T = \tilde{\nu} f_{\nu}$$

$$\frac{\partial \tilde{\nu}}{\partial t} + \langle u_j \rangle \frac{\partial \tilde{\nu}}{\partial x_j} = c_{b1} \tilde{\nu} \tilde{\nu} + \frac{1}{\sigma} \left\{ \nabla \cdot [(\nu + \tilde{\nu}) \nabla \tilde{\nu}] + c_{b2} (\nabla \tilde{\nu})^2 \right\}$$

$$- c_{w1} f_w \left[\frac{\tilde{\nu}}{d} \right]^2$$

$$f_{\nu_1} = \frac{\kappa^3}{\kappa^3 + c_{\nu_1}^3}, \quad \kappa = \tilde{\nu} / \nu, \quad d: \text{ wall distance}$$

very popular for hybrid RANS/LES.

③ Two-equation model: two transport eqs.

$$\nu_T = l \times u$$

$$u \sim \sqrt{k}, \quad l? \Rightarrow \mathcal{E} = 2\nu \langle s_{ij}' s_{ij}' \rangle \sim \mathcal{P} \sim \frac{u^3}{l} \sim \frac{k^{3/2}}{l}$$

↓ dissipation rate
↙ production

$$s_{ij}' = \frac{1}{2} \left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i} \right)$$

$$\nu_T = C_{\mu} \frac{k^2}{\mathcal{E}}$$

$$\left(\nu_T \sim \frac{k^{3/2}}{\mathcal{E}} \cdot \sqrt{k} \sim \frac{k^2}{\mathcal{E}} \right)$$

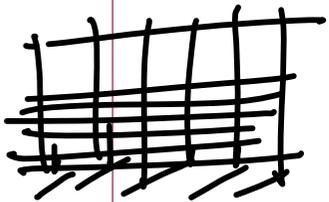
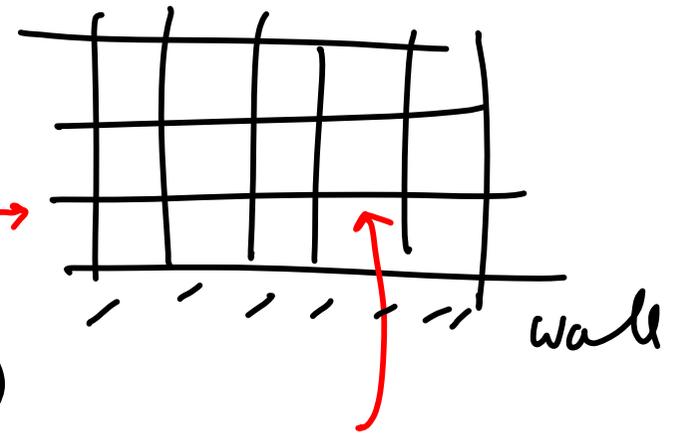
$$\begin{aligned}
 k\text{-eq: } \frac{\partial k}{\partial t} + \langle u_j \rangle \frac{\partial k}{\partial x_j} &= \frac{\partial}{\partial x_j} \left(\frac{\nu_T}{\sigma_k} \frac{\partial k}{\partial x_j} \right) + \nu_T \left(\frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) \frac{\partial \langle u_i \rangle}{\partial x_j} - \epsilon \\
 \epsilon\text{-eq: } \frac{\partial \epsilon}{\partial t} + \langle u_j \rangle \frac{\partial \epsilon}{\partial x_j} &= \frac{\partial}{\partial x_j} \left(\frac{\nu_T}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial x_j} \right) + c_{1\epsilon} \frac{\epsilon}{k} \nu_T \left(\frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right) - c_{2\epsilon} \frac{\epsilon^2}{k}
 \end{aligned}$$

standard k-ε model

b.c's high Re number : $\frac{\partial k}{\partial n} = 0$

$$\epsilon = c_\mu^{3/4} k^{3/2} / (\kappa y)$$

together with log law for $\langle u \rangle$



low Re number : $\langle u \rangle = 0$
 $k = 0$ $\frac{\partial \epsilon}{\partial n} = 0$ @ wall

Other turbulence models

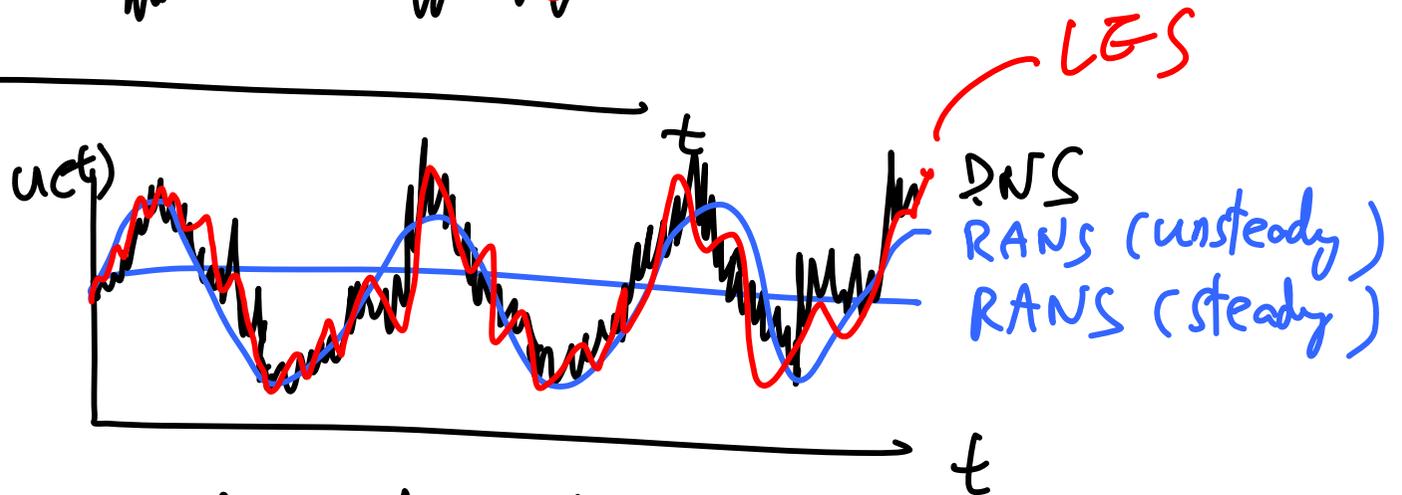
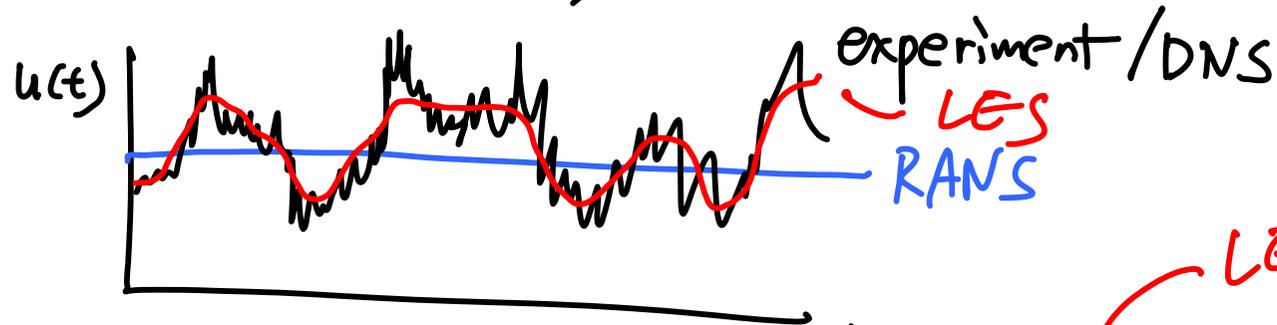
- SST k - ω model (Menter, 1993) ∇
- k - ε - ν^2 - f model (Durbin, 199-)
- Reynolds stress model (Launder, Reece & Rodi, 1984)

$$\langle u_i' u_j' \rangle \quad \overline{u'^2}, \overline{u'w'}, \overline{v'^2}, \overline{w'v'}, \overline{w'^2}, \overline{u'v'}, \varepsilon$$

\Rightarrow RANS models have been widely used for the prediction of turbulent flows in engineering applications.

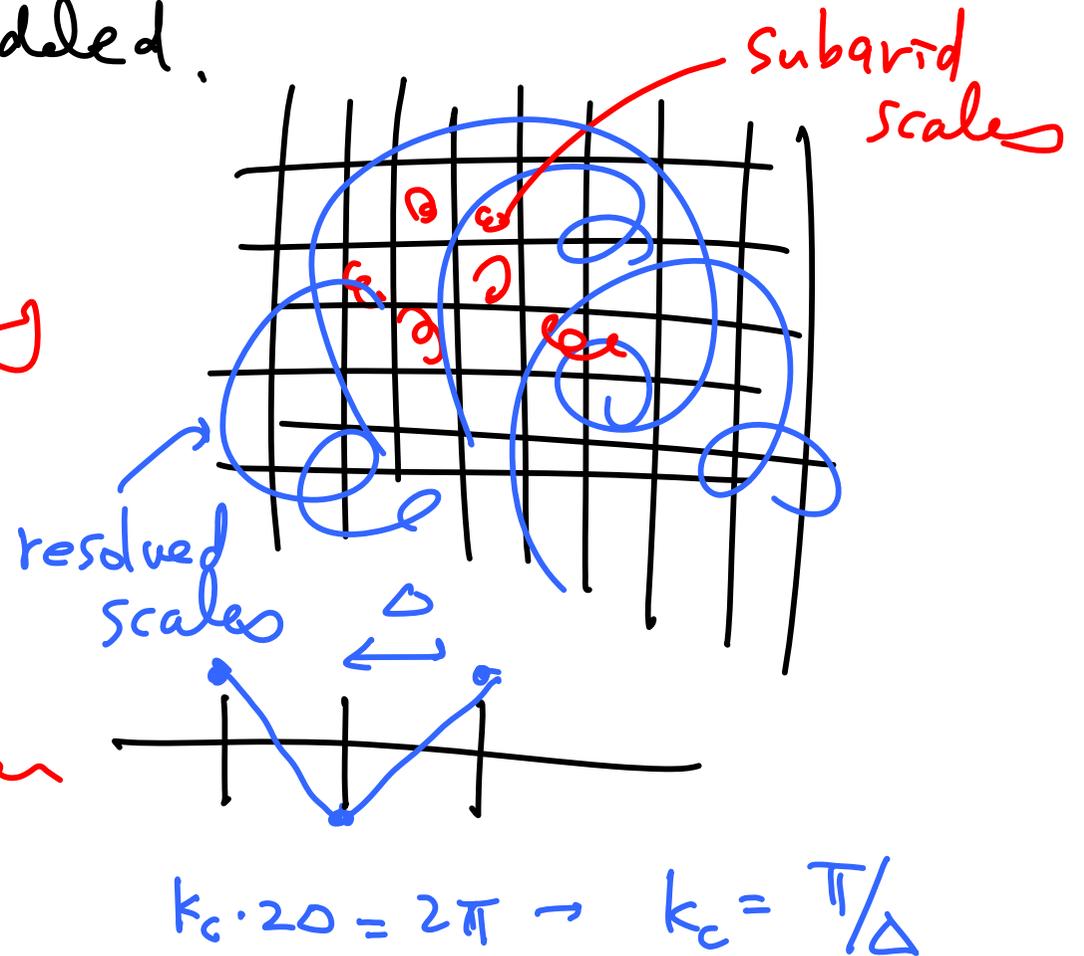
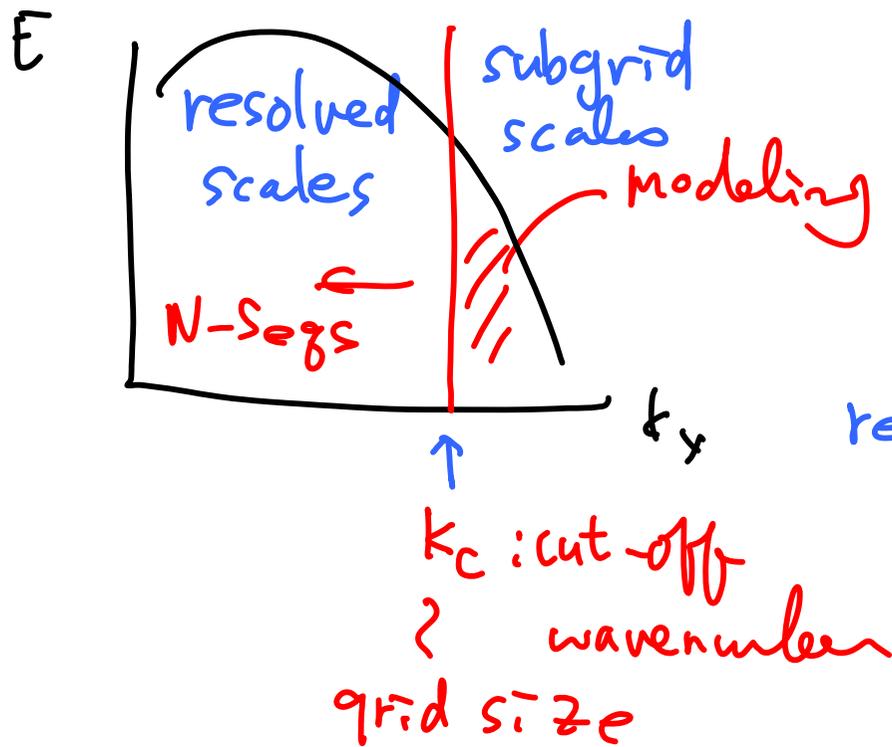
RANS, however, fails to predict massively separated flow and flows with high curvatures.

* Large eddy simulation (LES)



The premise of LES is that the motions that are resolved are the dynamically important ones and the errors introduced by modeling the small scale motions are significantly

smaller than those incurred in RANS where the entire turbulence stresses are modeled.



LES — unsteady & 3-D

turb. bdry layer flow

Choi & Moin (2012, PoF)
(Chapman (1978, AIAA J.))

DNS : $N \sim Re_L^{31/14}$

LES : $N \sim Re_L^{13/7}$

Re_L^1

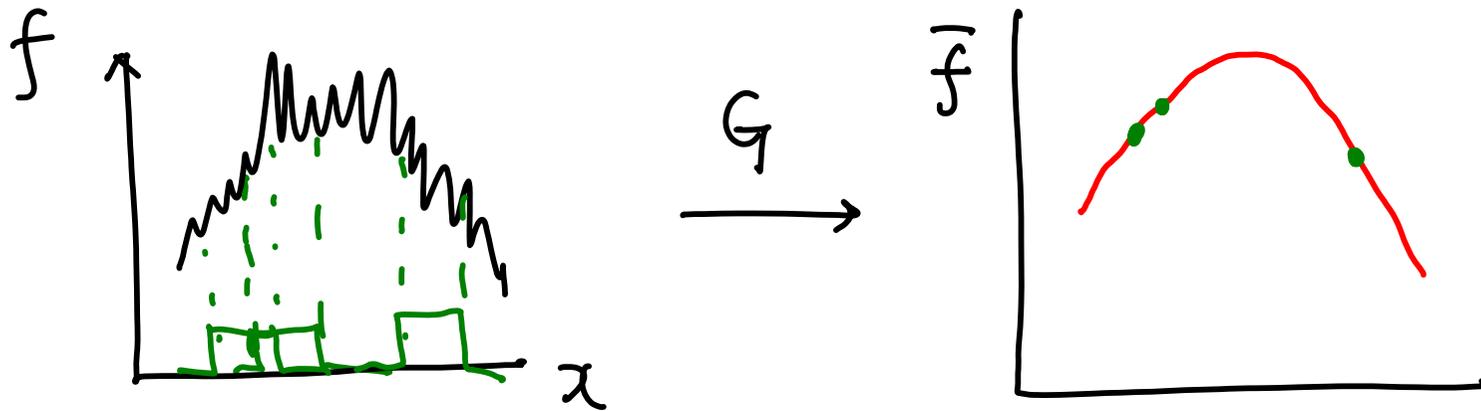
RANS : $N \sim Re_L^0$

without wall modeling

w/ " " " ← wall-modelled
LES

Filtering

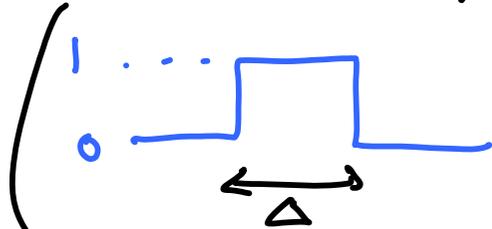
Filtering: an operation which damps out all the spatial fluctuations of flow variables smaller than a prescribed length scale (filter width).



$$\bar{f}(x) = \int f(x') G(x, x') dx' \quad ; \text{ filtered variable}$$

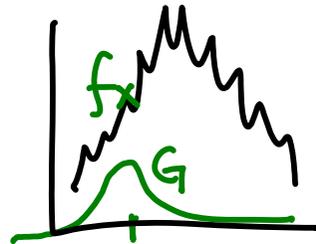
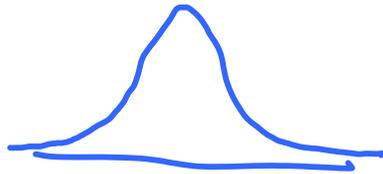
↳ convolution integral

filters: ① Box filter : $G(x, x') = \begin{cases} 1 & \text{for } x_i - \bar{\delta}/2 < x_i' < x_i + \bar{\delta}/2 \\ 0 & \text{otherwise} \end{cases}$



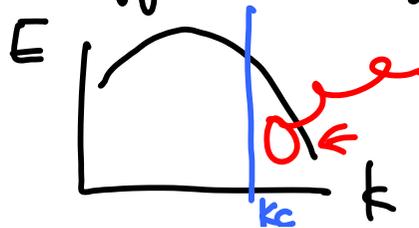
→ FVM, FDM, FEM

② Gaussian filter : $G(x, x') = \left(\frac{6}{\pi \bar{\delta}}\right)^{3/2} \exp\left[-\frac{6(x_i - x_i')^2}{\bar{\delta}}\right]$



③ sharp cut-off filter : $G(x, x') = \frac{2 \sin[\pi(x_i - x_i')/\bar{\delta}]}{\pi(x_i - x_i')}$ ←

↙ Spectral method



Governing eqs for LES

Filtered continuity and N-S eqs.

$$\overline{\frac{\partial u_i}{\partial x_i}} = 0 \rightarrow \int \frac{\partial u_i(x'_i)}{\partial x'_i} G(x_i, x'_i) dx'_i = 0 \rightarrow \boxed{\frac{\partial \bar{u}_i}{\partial x_i} = 0}$$

filtered velocity

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i$$

$$\downarrow$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_i \bar{u}_j) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \nabla^2 \bar{u}_i$$

$$= \frac{\partial}{\partial x_j} (\bar{u}_i \bar{u}_j) + \frac{\partial}{\partial x_j} (\bar{u_i u_j} - \bar{u}_i \bar{u}_j)$$

Modeling!
 $\tau_{ij} : \underline{\text{subgrid-scale stresses}}$ (CSGS)

\bar{u}_i, \bar{p}

$$\rightarrow \frac{\partial \bar{u}_i}{\partial x_i} + \frac{\partial}{\partial x_j} (\bar{u}_i \bar{u}_j) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \bar{u}_i}{\partial x_j} - \tau_{ij} \right)$$

3

- Smagorinsky eddy viscosity model (1963, SM)

$$\tau_{ij} - \frac{1}{3} \tau_{kk} \delta_{ij} = -2 \nu_T \bar{s}_{ij} \quad ; \quad \text{eddy viscosity hypothesis}$$

$$\bar{s}_{ij} = \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)$$

$$(\nu_T \sim l \times u)$$

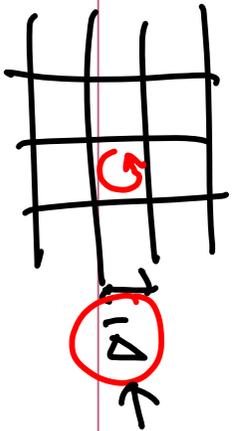
$$\nu_T = (C_S \bar{\Delta})^2 |\bar{S}|$$

$$\bar{\Delta} \times \bar{\Delta} |\bar{S}|$$

$l \times u$

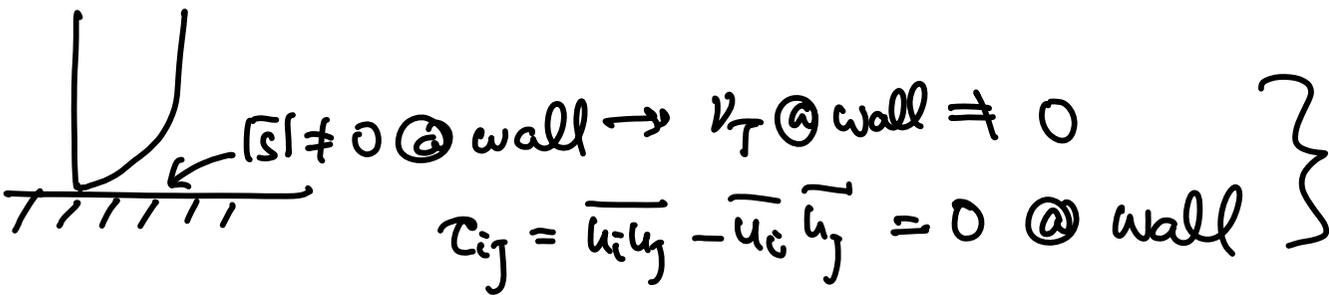
$$|\bar{S}| = \sqrt{2 \bar{s}_{ij} \bar{s}_{ij}}$$

C_S : Smagorinsky constant coefficient. $C_S = 0.1 \sim 0.3$



However, C_s is not universal and requires "damping ft" near the wall.

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j \sim 2 \nu_T \bar{S}_{ij} = 2 (C_s \bar{\Delta})^2 |\bar{S}| \bar{S}_{ij}$$

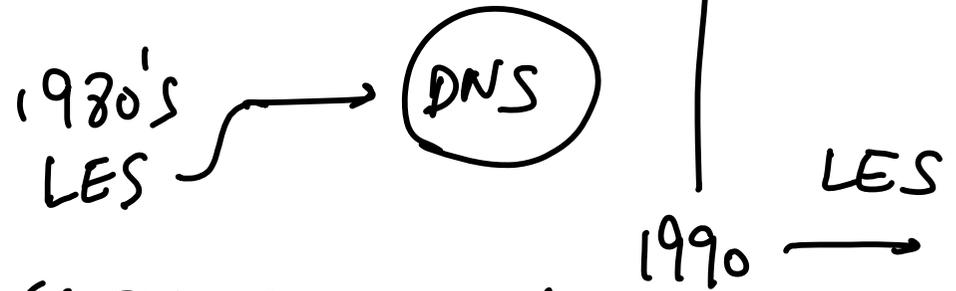
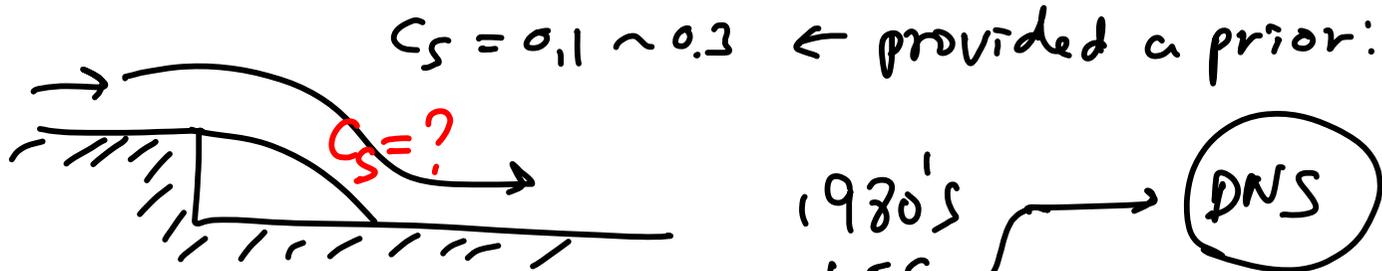
wall 

$$\left. \begin{aligned} \nu_T @ \text{wall} &= 0 \\ \tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j &= 0 @ \text{wall} \end{aligned} \right\}$$

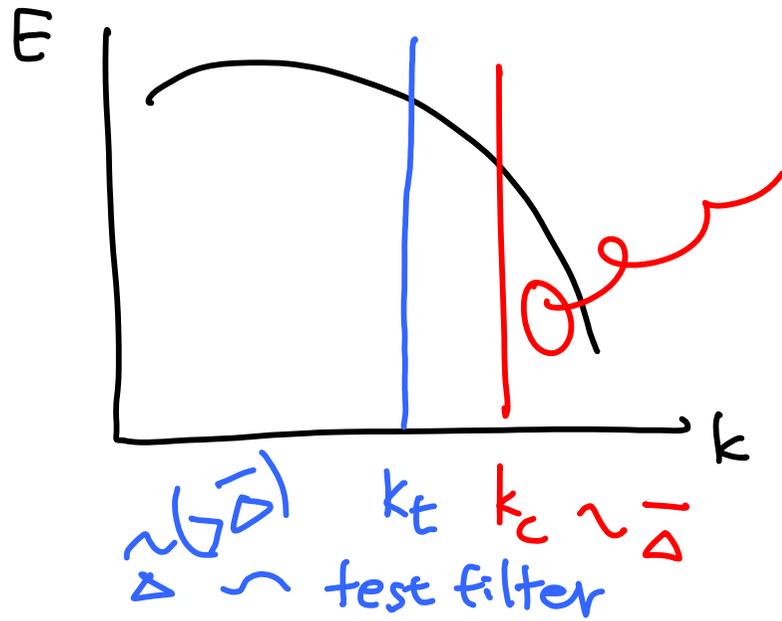
$$\Rightarrow C_s \rightarrow C_s (1 - e^{-\frac{y}{\Delta}}) \Rightarrow \nu_T \rightarrow 0 \text{ as } y \rightarrow 0$$

damping ft. ↙

1982 channel flow using LES by Kim & Moin (JFM)



• Dynamic Smagorinsky model (DSM, Germano et al. 1991, PoF)



Apply another (test) filter to N-S.:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0$$

$$\frac{\partial \bar{u}_i}{\partial x_i} + \frac{\partial \bar{u}_c \bar{u}_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \bar{u}_i}{\partial x_j} - \bar{T}_{ij} \right)$$

$$\bar{T}_{ij} \equiv \overline{u_i u_j} - \bar{u}_c \bar{u}_j$$

$$\tilde{\tau}_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$$

$$\tau_{ij} - \tilde{\tau}_{ij} = \bar{u}_i \bar{u}_j - \tilde{u}_i \tilde{u}_j \equiv L_{ij}$$

Germano identity

computable during simulation

$$\tilde{\tau}_{ij} - \frac{1}{3} \delta_{ij} \tilde{\tau}_{kk} = -2 (c_s \bar{\Delta})^2 |\bar{S}| \bar{s}_{ij}$$

$$\tau_{ij} - \frac{1}{3} \delta_{ij} \tau_{kk} = -2 (c_s \hat{\Delta})^2 |\hat{S}| \hat{s}_{ij}$$

Germano identity

$$-2 (c_s \hat{\Delta})^2 |\hat{S}| \hat{s}_{ij} + 2 (c_s \bar{\Delta})^2 |\bar{S}| \bar{s}_{ij} = L_{ij} - \frac{1}{3} \delta_{ij} L_{kk}$$

$$\tilde{\tau} - 2 (c_s \bar{\Delta})^2 \left[\left(\frac{\hat{\Delta}}{\bar{\Delta}} \right)^2 |\hat{S}| \hat{s}_{ij} - |\bar{S}| \bar{s}_{ij} \right] = L_{ij} - \frac{1}{3} \delta_{ij} L_{kk}$$

= M_{ij}

$$\Rightarrow \underline{L_{ij}} - \frac{1}{3} \delta_{ij} \underline{L_{kk}} + 2 (C_s \bar{\Delta})^2 \underline{M_{ij}} = 0 \quad \begin{array}{l} i=1,2,3 \\ j=1,2,3 \end{array} \quad \begin{array}{l} 1 C_s \\ 6 \text{ eqs.} \end{array}$$

→ least square error method (Lilly et al, 1992, PoF)

$$Q \equiv \left[L_{ij} - \frac{1}{3} \delta_{ij} L_{kk} + 2 (C_s \bar{\Delta})^2 M_{ij} \right]^2$$

$$\frac{\partial Q}{\partial C_s^2} = 0 \quad ;$$

$$(C_s \bar{\Delta})^2 = -\frac{1}{2} \frac{L_{ij} M_{ij}}{M_{ij} M_{ij}} \quad \leftarrow$$

"Dynamic" Smagorinsky model

→ $C_s \sim f(\underline{x}, t)$ and is obtained at each time step during computation.

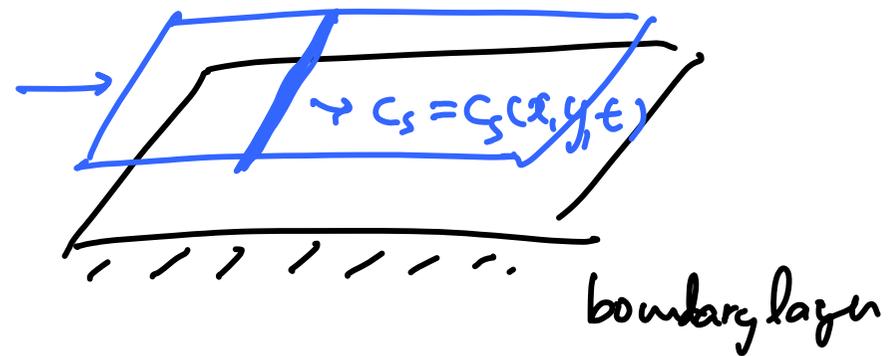
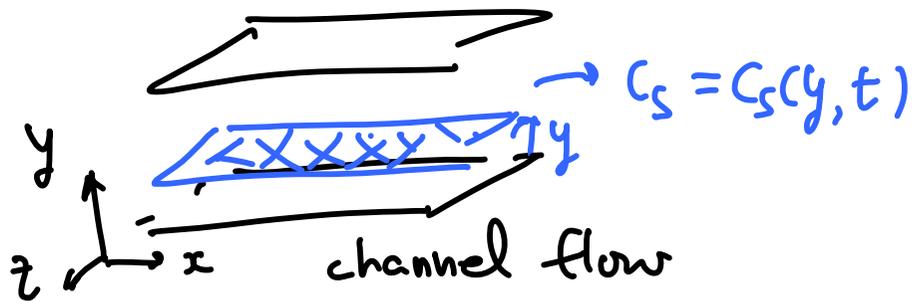
C_s^2 should be positive!

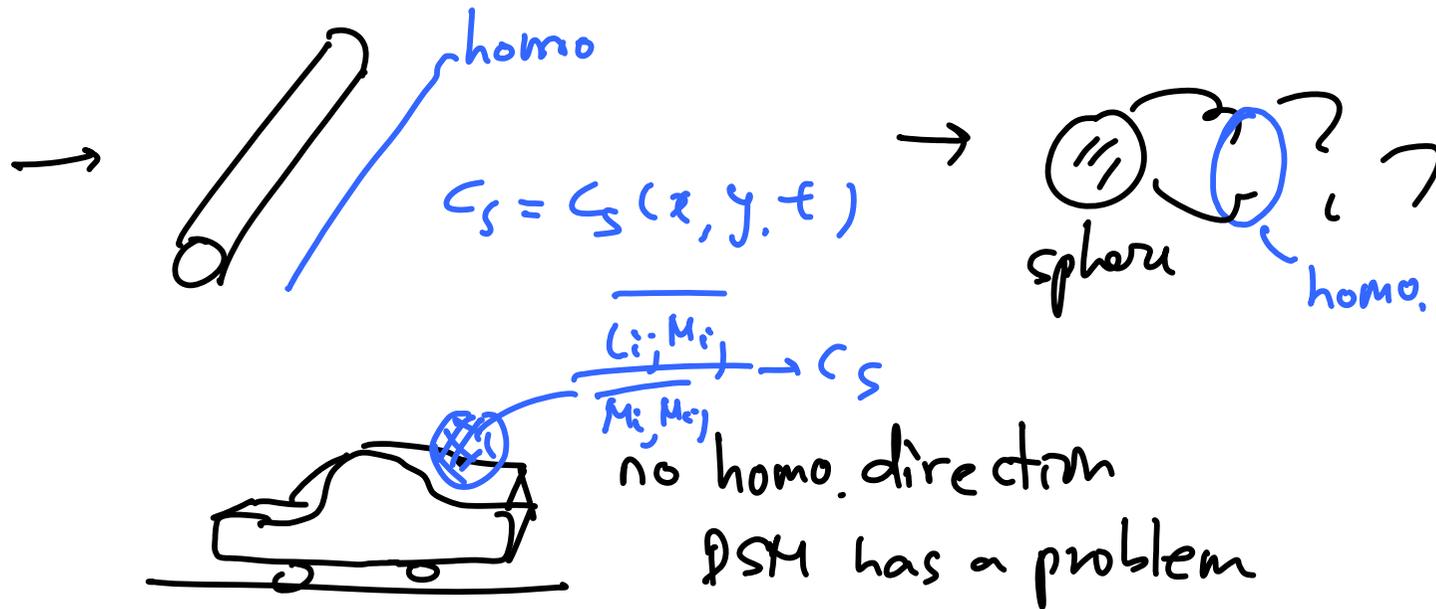
in reality, $(C_s^2) < 0$ at some grid pts. \rightarrow numerical instability

\Rightarrow requires an averaging over homogeneous directions) and/or ad hoc clipping. $\rightarrow C_s^2 \equiv 0$ if $C_s^2 < 0$.

$$(C_s^2)^2 = -\frac{1}{2} \frac{\langle L_{ij} M_{ij} \rangle_h}{\langle M_{ij} M_{ij} \rangle_h}$$

h : homogeneous directions.





• Vreman model (2004, PoF)

VM

$$\nu_T = C_\nu \sqrt{\frac{\overline{I_\beta}}{\overline{\alpha_{ij} \alpha_{ij}}}}$$

$$\overline{\alpha_{ij}} = \frac{\partial \overline{u_j}}{\partial x_i}, \quad \beta_{ij} = \sum_{n=1}^3 \overline{\Delta_n^2 \alpha_{ni} \alpha_{nj}}$$

Vreman coeff.

$$\overline{I_\beta} = \beta_{11} \beta_{22} - \beta_{12}^2 + \beta_{11} \beta_{33} - \beta_{13}^2 + \beta_{23} \beta_{33} - \beta_{33}^2$$

$C_\nu = 0.01$ for isotropic turbulence

 laminar $\rightarrow |\bar{s}_y| \neq 0 \rightarrow v_T \neq 0$ SM
wall $v_T = 0$ VM \in good!

C_v has to be provided a priori.

Park, Lee & Choi (2006, PoF) \rightarrow Dynamic global model
Lee, Choi & Park (2010, PoF) to determine C_v

C_v a function only

No clipping is required

No homo. direction is required



can be applied
to complex
geometries.