

Chapter 3. The Second Law

Phenomena (common sense)

1. A gas expands and fills the available volume, but does not spontaneously contract into a smaller one.
2. A hot body cools to the same temperature as its surroundings, but a body does not spontaneously get hotter than its environment.
3. Heating diamonds yields graphite, but heating graphite does not give diamonds. [*Diamond by chemical vapor deposition*]
4. When a ball (the system of interest) bounces on a floor, the ball does not rise higher after each bounce, because there are inelastic losses in the materials of the ball and floor (that is, the conversion of kinetic energy of the ball's overall motion into the energy of thermal motion).

Conclusions

The direction of change leads to the greater dispersal of the total energy.

The First Law of Thermodynamics = Always Valid

A Device of 100% Efficiency ???

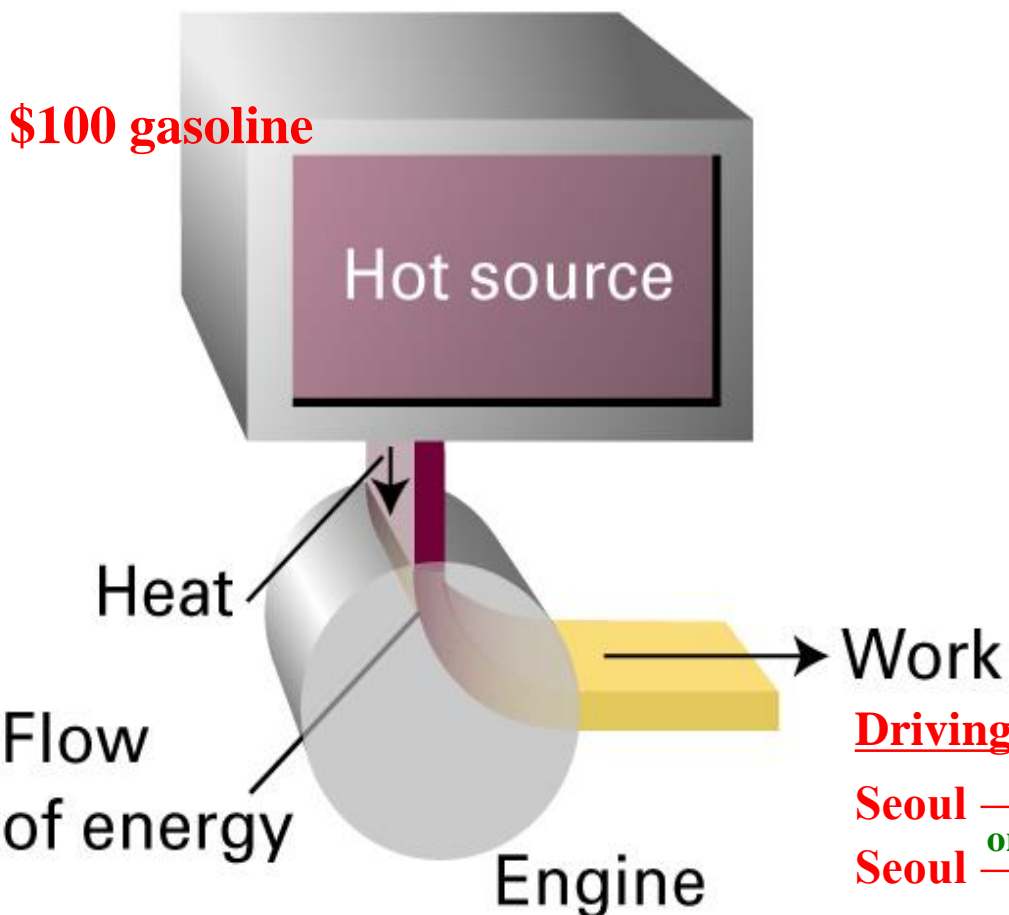


Figure 3A.3

The Kelvin statement of the Second Law **denies** the possibility of the process illustrated here, in which heat is changed completely into work, there being no other change.

The process is not in conflict with the First Law because energy is conserved.

Driving
Seoul → Finland
Seoul ^{or} → Mogolia

Irreversible.
Video (*time or -time*).

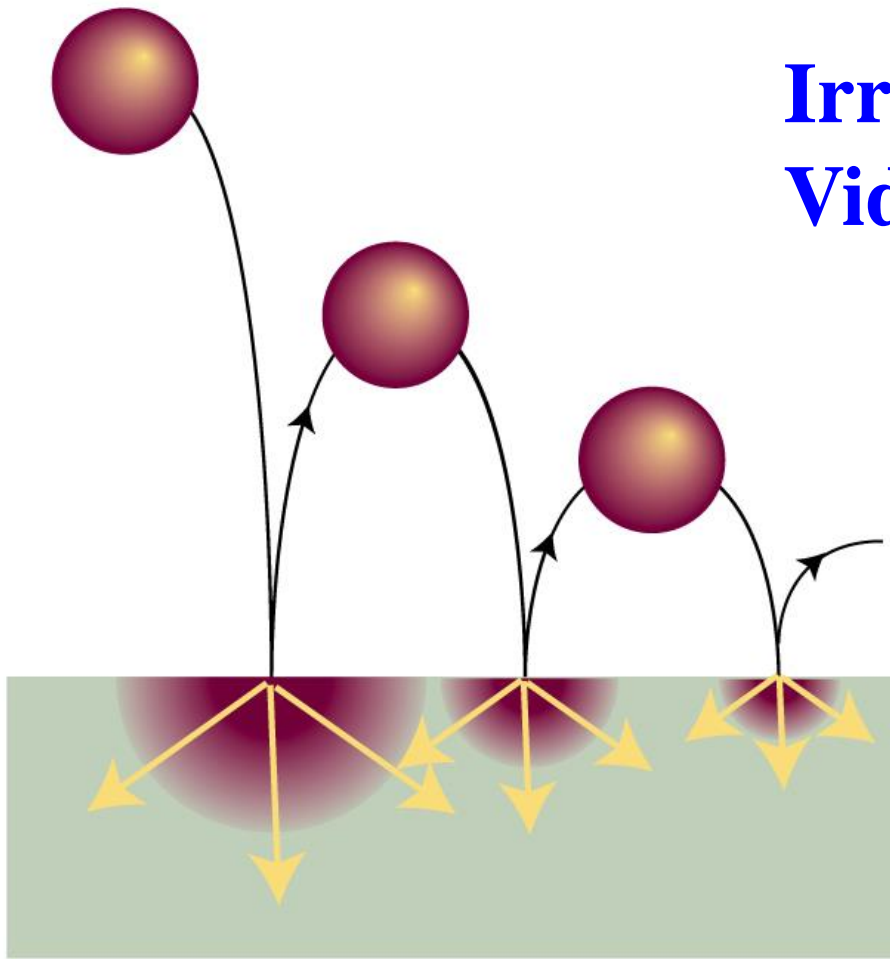


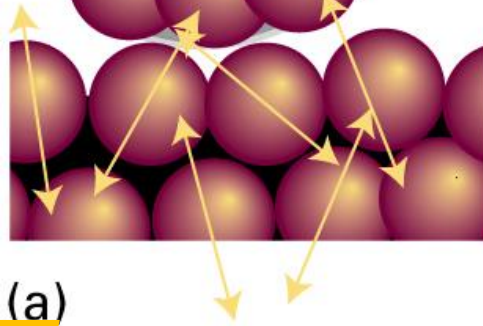
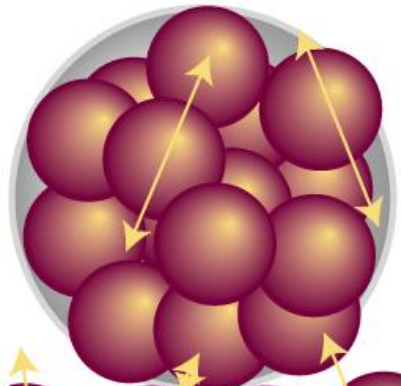
Figure 3A.1

The direction of spontaneous change for a ball bouncing on a floor. On each bounce some of its energy is degraded into the thermal motion of the atoms of the floor, and that **energy disperses**. The **reverse has never been observed** to take place on a

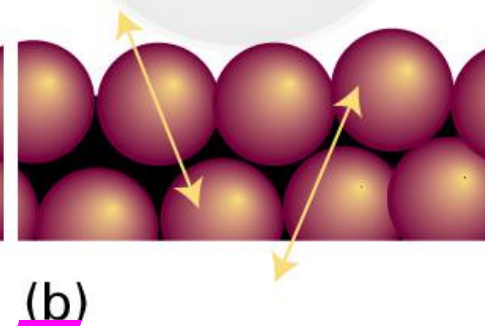
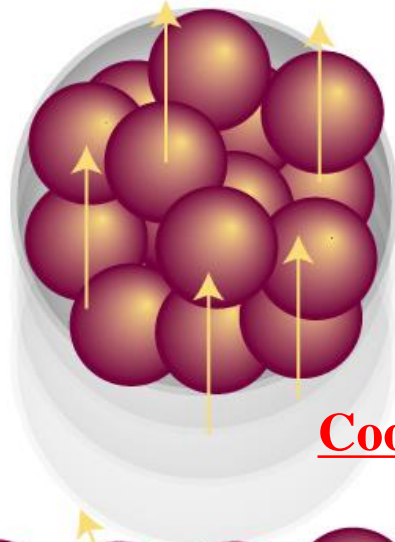
macroscopic scale.



Random Motion



(a)



(b)

Figure 3A.2

Coordinated Motion

Macroscopic World

irreversibility expressed by the Second Law.

- (a) A ball is resting on a warm surface, and the atoms are undergoing **thermal motion** (vibration, in this instance), as indicated by the arrows.
- (b) For the ball to fly upwards, **some of the random vibrational motion would have to change into coordinated, directed motion**. Such a conversion is **highly improbable**.

Measuring Dispersal: the Entropy of a System

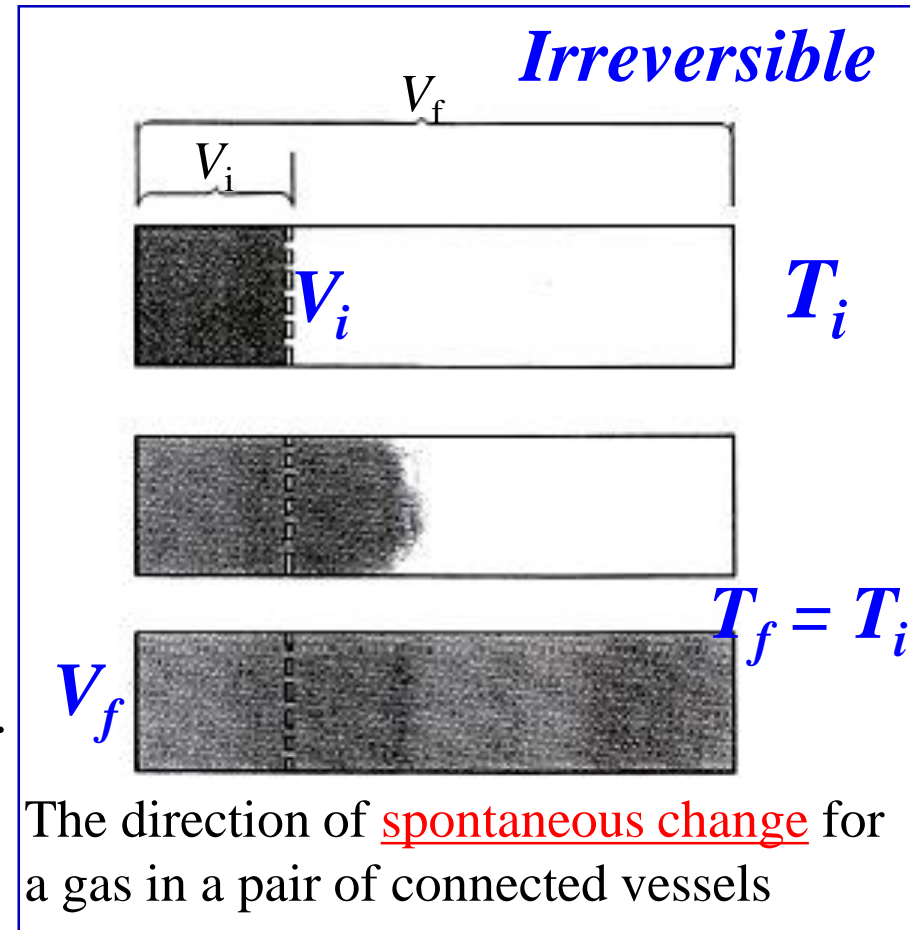
Entropy S

1. A **thermodynamic function** that measures how the dispersal of energy alters when a system changes from one state to another.
2. **A state function**

Statistical View of the Entropy

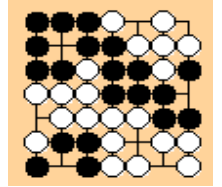
The direction of spontaneous change is:

- from a state with **low probability** of occurring
- to one of **maximum probability**.



The direction of spontaneous change for a gas in a pair of connected vessels

$$S(N, V, T) = k_B \ln \Omega(N, V, T)$$



Position + Motion

Why?

The diagram shows a rectangular container divided into two chambers by a vertical wall. The left chamber is labeled 'He' and contains a few dots representing particles. The right chamber is labeled 'Ar' and contains more dots. Above the left chamber are labels S_1 and Ω_1 . Above the right chamber are labels S_2 and Ω_2 . To the right of the diagram, the following equations are written in red ink:

$$\Omega_{\text{total}} = \Omega_1 \cdot \Omega_2$$
$$S_{\text{total}} = S_1 + S_2$$
$$S \propto \ln \Omega$$

Gedanken Experiment

Consider an Ideal Monatomic Gas (Isothermal):

Number of microstates of one atom being in V_i : $\omega(V_i) = cV_i$

Number of microstates that N atoms are in V_i : $\Omega(V_i) = [\omega(V_i)]^N = c^N V_i^N$

Number of microstates that N atoms are in V_f : $\Omega(V_f) = c^N V_f^N$

When an ideal gas expands isothermally from V_i to V_f ,

$$\begin{aligned} \Delta S &= S(V_f) - S(V_i) = k \ln(cV_f)^N - k \ln(cV_i)^N \\ &= kN(\ln cV_f - \ln cV_i) = kN \ln \frac{V_f}{V_i} \end{aligned}$$

$dU = 0$

$dw = -Nk_B T \ln V_f/V_i$
PPT 2-15

$N = nN_A$

where n : the number of moles, N_A : Avogadro's number

$\Delta S = nR \ln \frac{V_f}{V_i}$, $kN_A = R$ Ideal Gas Isothermal Expansion (3A.14)

Entropy $S \propto$ (??) Number of Microstates Ω

$S = \Omega \rightarrow$ not **extensive**

PPT 3-6

$S(N, V, T) = k \ln \Omega(N, V, T)$ where k is some constant, Boltzmann constant.

$$\Delta S = nR \ln \frac{V_f}{V_i} = Nk_B \ln \frac{V_f}{V_i}, \quad k_B N_A = R$$

**Ideal Monoatomic Gas
Isothermal**

Assumptions of this calculation

- 1) Ideal gas: neglect the interaction between particles and neglect rotational and vibrational modes.
- 2) Isothermal process: $\left(\frac{\partial U}{\partial V}\right)_T = 0$

Entropy measures the dispersal of energy, and the natural tendency of spontaneous change is towards the states of higher entropy. (3A.5)

$$S(N, V, T) = k_B \ln \Omega(N, V, T)$$

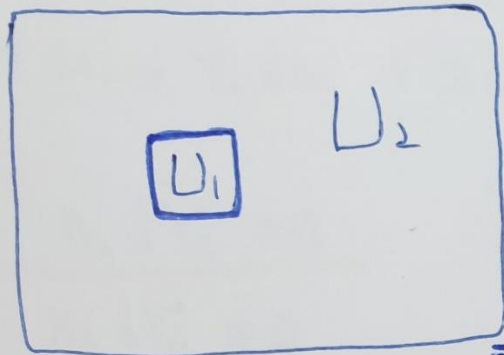
Same E
(for whatever dancing mode)

Ω = Number of Microstates

Position, Direction, Vibration, Electronic Distribution, Electronic Motion, etc.

$$dS = \frac{dq_{rev}}{T}$$

2nd Law of Thermodynamic for reversible system

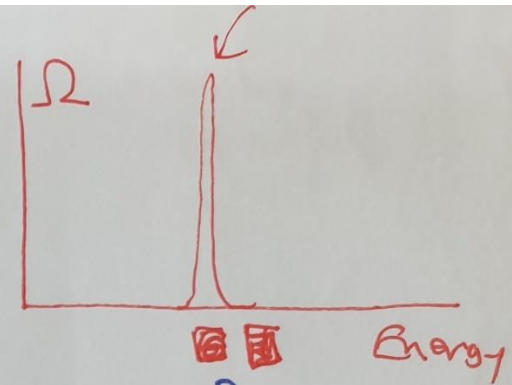


$N_1, V_1 = \text{fixed}$ $N_2, V_2 = \text{fixed}$
 $U_1 + U_2 = \text{const}$

$$\Omega_{total} = \Omega_1 \cdot \Omega_2$$

$$\frac{\partial \Omega_{total}}{\partial U_1} = 0 = \frac{\partial \Omega_1}{\partial U_1} \Omega_2 + \frac{\partial \Omega_2}{\partial U_1} \Omega_1$$

$$\left. \frac{\partial \ln \Omega_1}{\partial U_1} \right|_{N, V} = \left. \frac{\partial \ln \Omega_2}{\partial U_2} \right|_{N, V} \equiv \frac{1}{k_B T}$$



$$dU = dq - PdV$$

Thermodynamic View of Entropy

Thermal Energy → Random Motion

1. A change in entropy dS is proportional to the amount of heat added dq . **[Extensive Properties]**
2. Impact of dq on the chaos is inversely proportional to T .

For a given dq , the change in entropy is large when the temperature is low, but small if the temperature is high.

$$\underline{dS} = \frac{dq_{rev}}{T} \quad \text{The ratio of a randomizing influence (dq_{rev}) to a measure of the amount of randomness already present at T (3A.1) *Never Definition*$$

If the changes are restricted to reversible transfer of heat, then the quantity dq_{rev}/T is found to be a state function.

Ideal Gas + Isothermal

For a reversible and isothermal expansion ($V_i \rightarrow V_f$) of an ideal gas,

$$dU = dq + dw = 0 \quad dU = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV$$

$dq = -dw$

$dq_{rev} = pdV = \frac{nRT}{V} dV$

0: ideal gas
0: isothermal

On integration:

$$\Delta q_{rev} = nRT \ln \frac{V_f}{V_i}$$

$$\Delta S = \int_i^f dS = \int_i^f \frac{dq_{rev}}{T} = \frac{1}{T} \int_i^f dq_{rev} = \frac{\Delta q_{rev}}{T}$$

$$= \frac{nRT \ln(V_f / V_i)}{T} = nR \ln \frac{V_f}{V_i}$$

**2nd Law of Thermodynamics
(for reversible)**

(same as ppt 3-7)

Ideal Gas
Isothermal
(3A.14)

$$dS = \frac{dq_{rev}}{T}$$

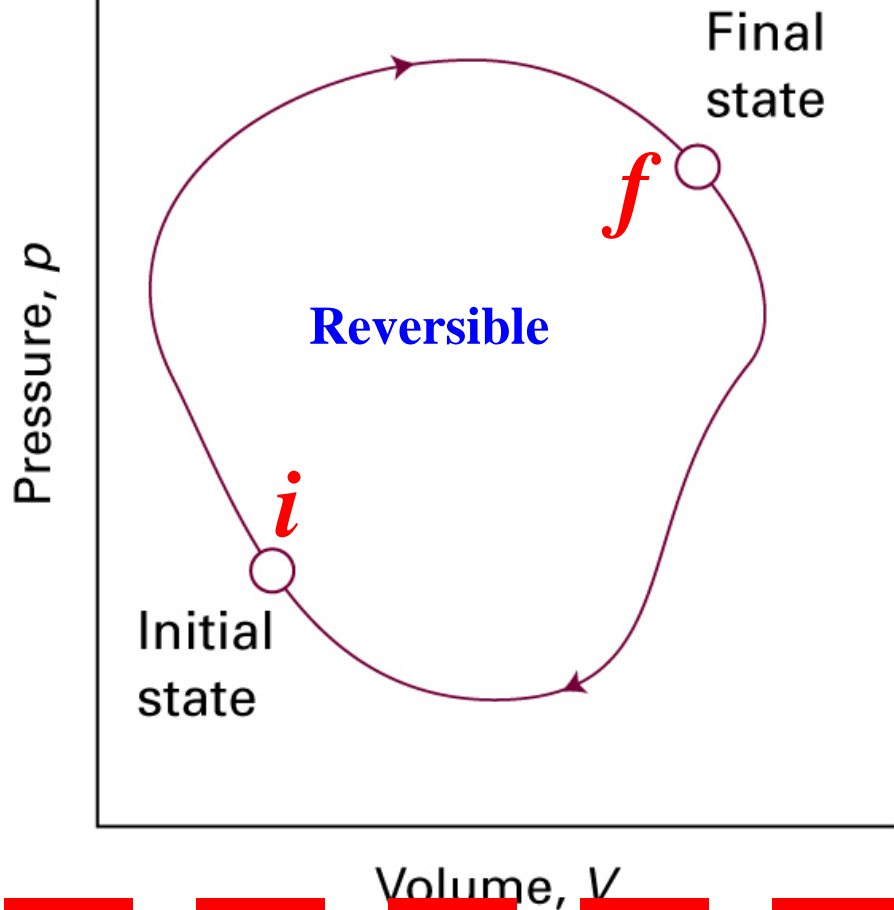


Figure 3A.6

In a thermodynamic cycle, the overall change in a state function (from the initial state to the final state and then back to the initial state again) is zero.

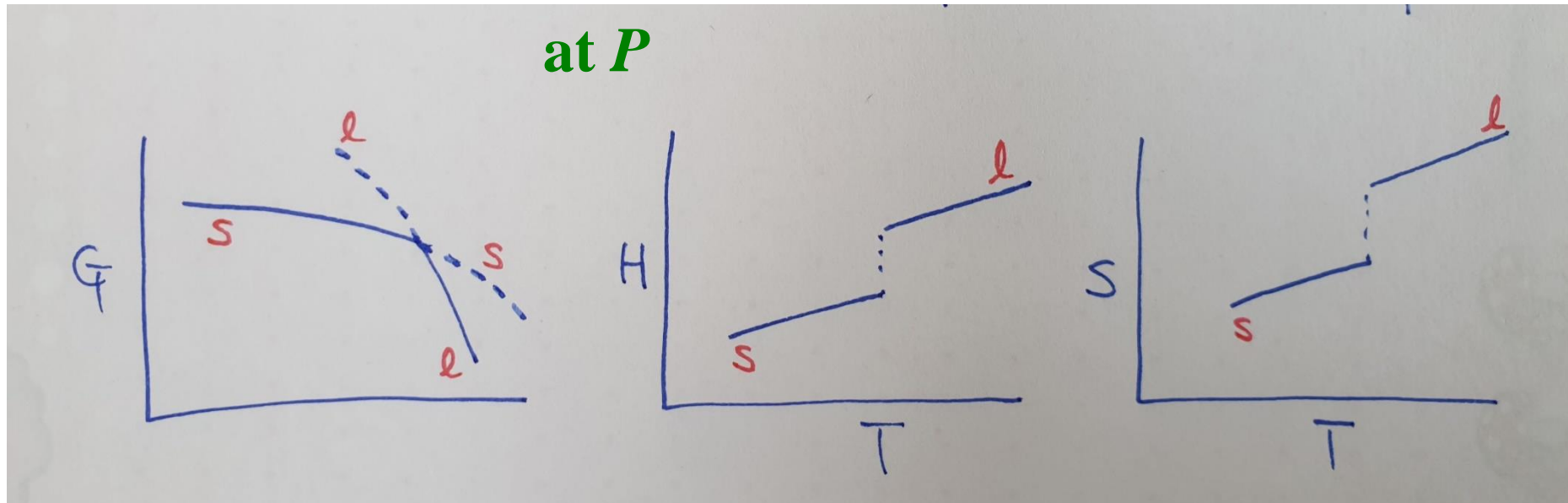
For an isolated system, no heat enters or leaves the system irreversibly, i.e. $dq = 0$.

This inequality shows that these spontaneous processes must lead to an increase in entropy of the universe.

The second law: In an isolated system, spontaneous processes occur in the direction of increasing entropy.

$$\therefore \Delta S \geq 0$$

Schematics of $G(T)$, $H(T)$ & $S(T)$



$$dH = dq + VdP$$

$$dq_{rev} = TdS$$

3A.4(b). Entropy Change Arising from a Phase Transition

Under the condition of constant pressure, the **latent heat** is an enthalpy of phase transition ΔH_t .

$$\underline{\Delta S} = \frac{\Delta H_t}{T_t} \quad T_t : \text{transition temperature}$$

~0.5 eV/atom
s-Si \leftrightarrow l-Si

Schematic G, H, & S

- Both melting and boiling are endothermic processes ($\Delta H_t > 0$). So, each is accompanied by an increase of the **system's entropy**.
- If the phase transition is exothermic ($\Delta H_t < 0$, as in freezing or condensing), then the entropy change is negative. This decrease in entropy is consistent with localization of matter and energy that accompanies the formation of a solid from a liquid or a liquid from a gas.
- If the transition is endothermic ($\Delta H_t > 0$, as in the melting and vaporization), then the entropy change is positive, which is consistent with dispersal of energy and matter in the system.

The Entropy Change of Transition:

$$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$$

$$k_B = 8.617 \times 10^{-5} \text{ eV/K}$$

A wide range of liquids gives approximately the same entropy of vaporization. \Rightarrow **Trouton's rule**

Chap. 3A.4(b)
Phase Transitions

$$\Delta S(\text{water}) = 109 \text{ JK}^{-1} \text{ mol}^{-1} \quad \text{large entropy change due to H-bonds}$$

$L \rightarrow V$

$$\Delta S \sim 10 R / \text{mol} \sim 10 k_B / \text{molecule} \quad (\text{liquid} - \text{vapor})$$

$$\Delta S \sim 1 R / \text{mol} \sim 1 k_B / \text{molecule} \quad (\text{solid} - \text{liquid})$$

Solid \rightarrow Liquid

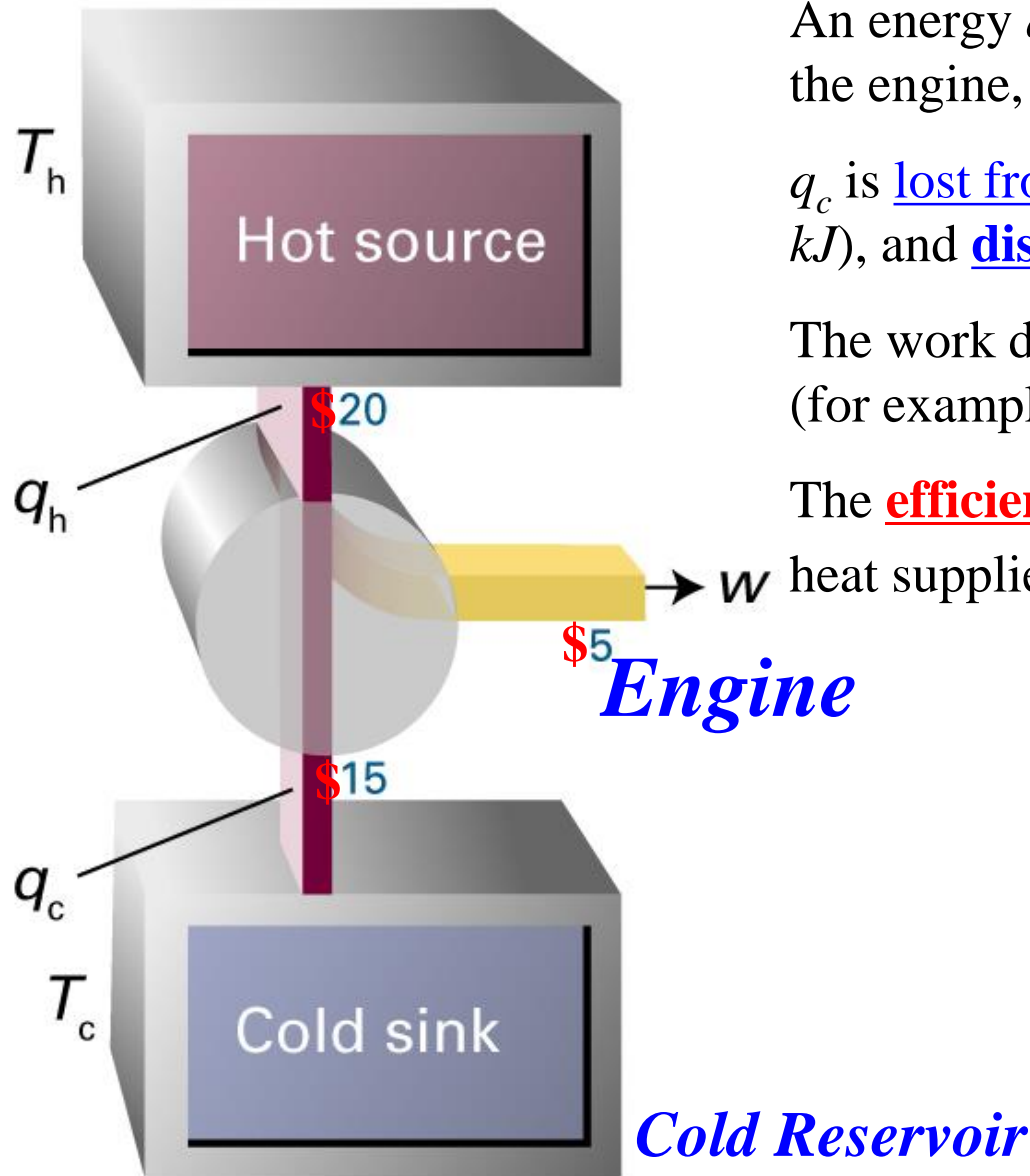
$\Delta S \approx 1 k_B$ - most metals

$\Delta S \approx 5 k_B$ - Si, Ge, Sb, etc.

$$S(N, V, T) = k_B \ln \Omega(N, V, T)$$

Microcanonical Ensemble

Engine



Suppose:

An energy q_h (for example, 20 kJ) is supplied to the engine, and

q_c is lost from the engine (for example, $q_c = -15 \text{ kJ}$), and discarded into the cold reservoir.

The work done by the engine is equal to $q_h + q_c$ (for example, $20 \text{ kJ} + (-15 \text{ kJ}) = 5 \text{ kJ}$).

The **efficiency** is the work done divided by the heat supplied from the hot source. = **25%**

Figure 3A.8

3A.3(a). Reversible Heat Engine: The Efficiency

Since the temperature of a cold reservoir is lower than that of a hot one, an overall increase of entropy can be produced even if q_h is withdrawn from the hot reservoir, and less than q_h ($= q_c$) is transferred to the cold.

$$\Delta S_{total} = \frac{q_c}{T_c} - \frac{q_h}{T_h} = 0 \quad \text{is } \underline{\text{zero}}.$$

Reversible change
by an infinitesimal modification

Therefore, we are free to use some kind of **device, an engine**, to draw off the difference $(q_h - q_c)$ as work.

The **work** that an engine may produce is:

$$w'_{\max} = q_h - q_c = q_h \left(1 - \frac{T_c}{T_h} \right)$$

Reversible Engine
= Maximum Work

The **Carnot efficiency** (ε) is the ratio of the maximum work generated to the heat absorbed.

$$\varepsilon = \frac{w'_{\max}}{q_h} = 1 - \frac{T_c}{T_h}$$

(3A.9)

Example:

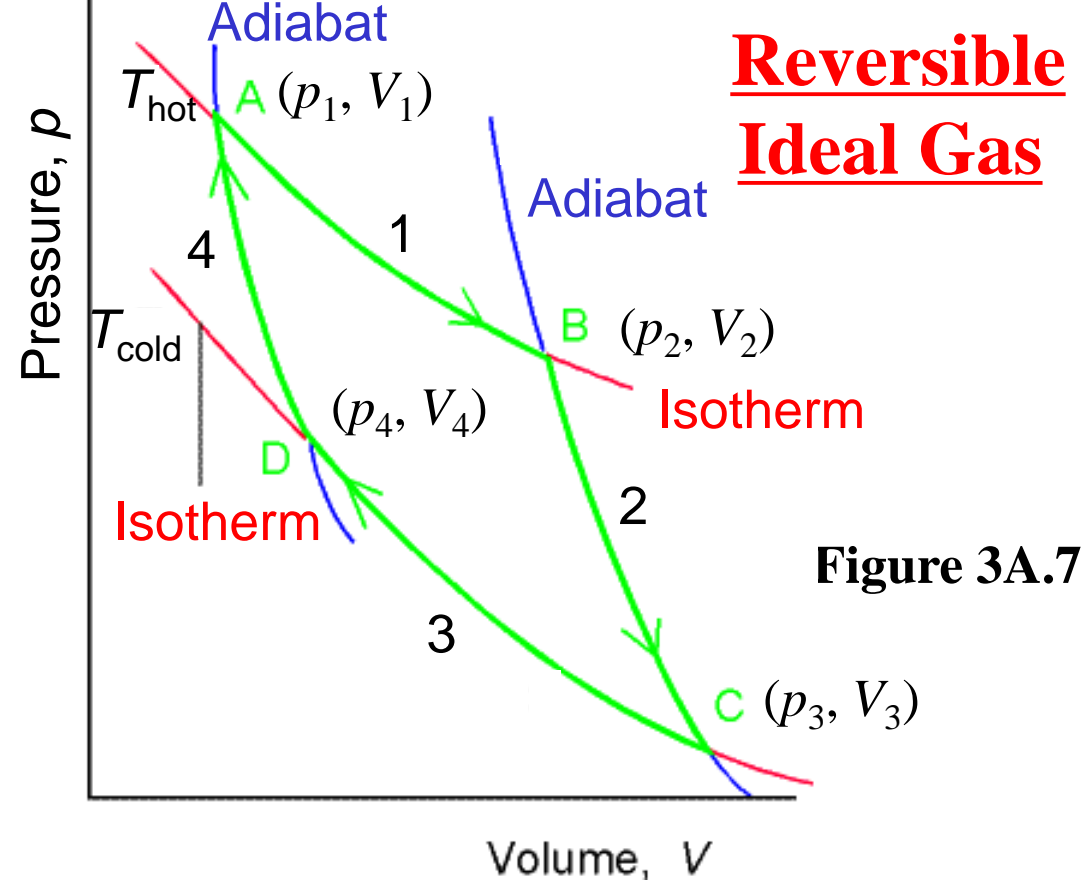
Calculate the **Carnot efficiency (reversible heat engine)** of an internal combustion engine where $T_h = 3200\text{ K}$ and $T_c = 1400\text{ K}$.

Solution:

$$\varepsilon = 1 - \frac{1400}{3200} = 0.56 = 56\%$$

Compare with the practical efficiency ($\varepsilon \approx 25\%$) of internal combustion engines.

Basic Structure of a Carnot Cycle



Step 1: Reversible isothermal expansion at temperature T_h .

Step 2: Reversible adiabatic expansion in which the temperature falls from T_h to T_c .

Step 3: Reversible isothermal compression at T_c .

Step 4: Reversible adiabatic compression from T_c to T_h .

Restores the system to its initial state.

Any Reversible Cycle

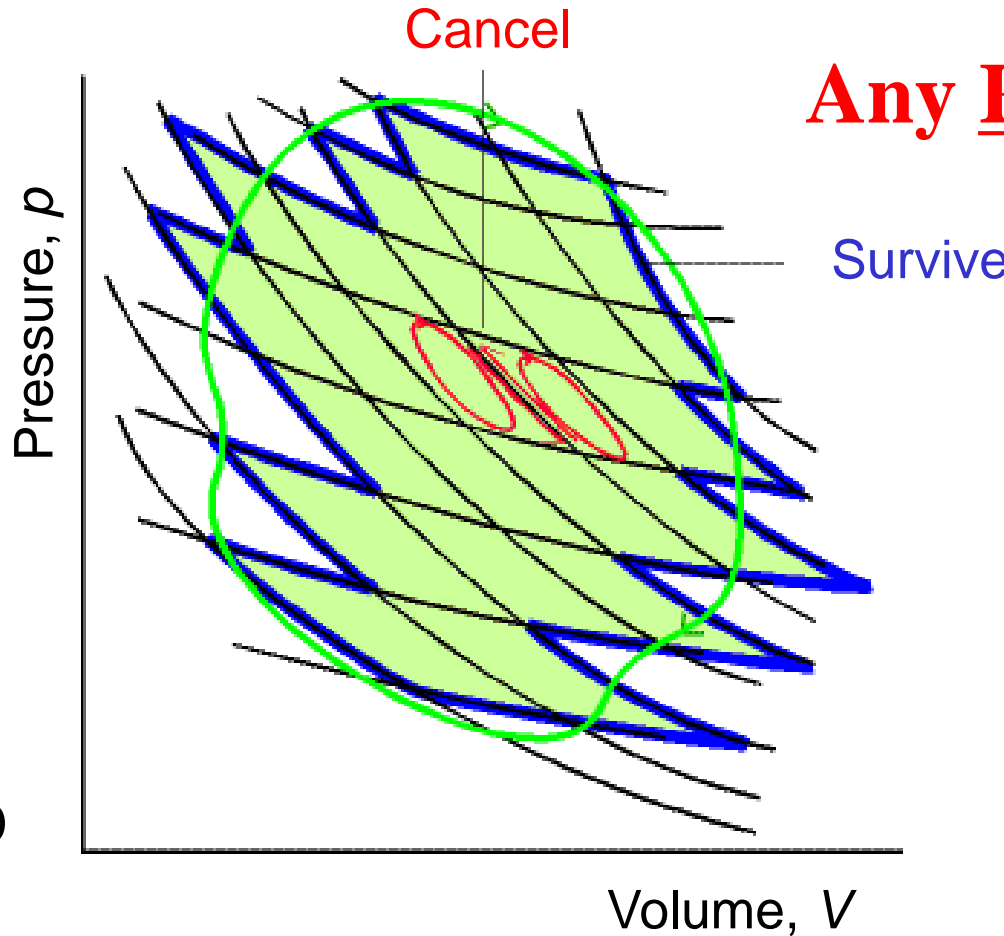


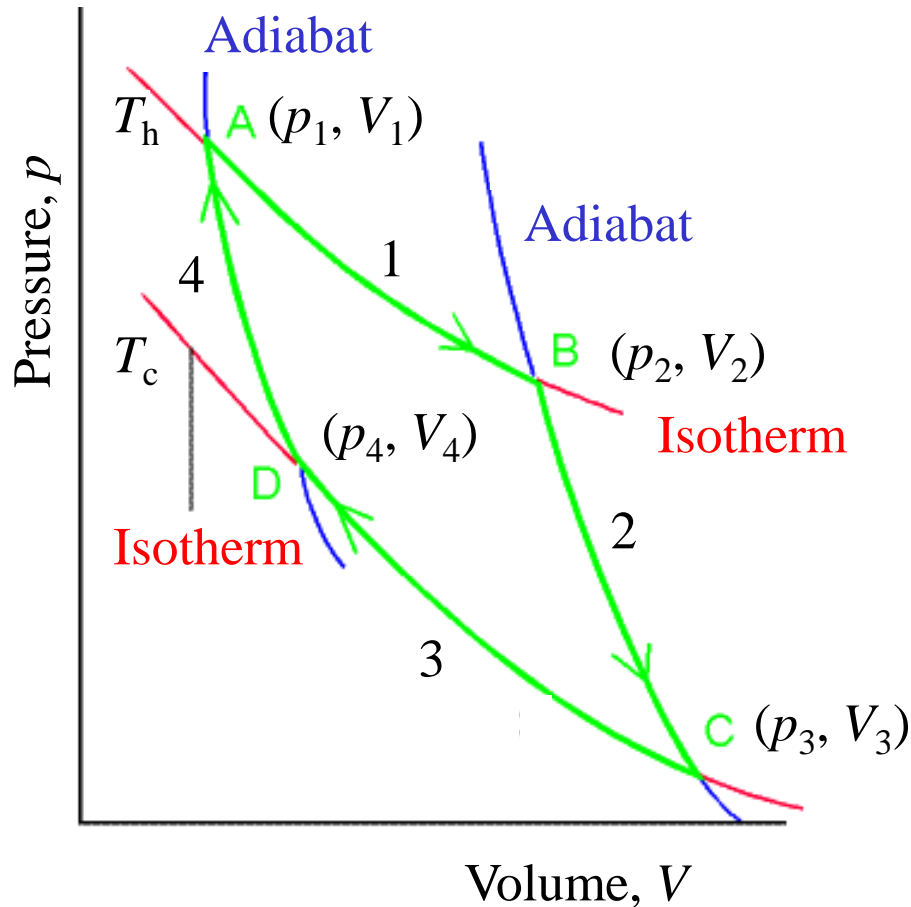
Figure 3A.10

A general cycle can be divided into small Carnot cycles. The match is exact in the limit of infinitesimally small cycles. Paths cancel in the interior of the collection, and only the perimeter, an increasingly good approximation to the true cycle as the number of cycles increases, survives.

Because the energy or entropy change around every individual cycle is zero, the integral of the energy or entropy around the perimeter is zero too.

The Carnot Cycle

All engines work on a cycle.



Step 1: [Isothermal at q_h]

The gas absorbs heat (q_h) from the high-temperature reservoir (temperature T_h) and expands isothermally and reversibly from V_1 to V_2 .

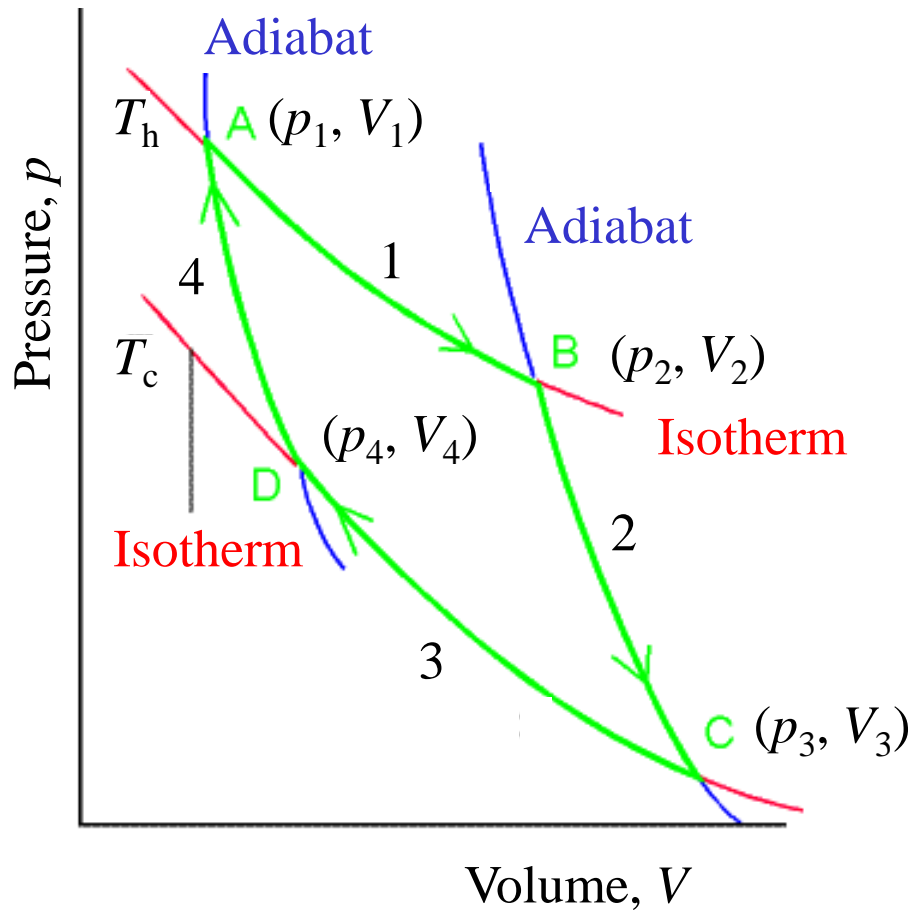
$$q_h = RT_h \ln \frac{V_2}{V_1} \quad \text{(3A.14)}$$

PPT 3-7
PPT 3-11

$$w = -RT_h \ln \frac{V_2}{V_1}$$

$$\Delta U = 0$$

(since isothermal, ideal gas)



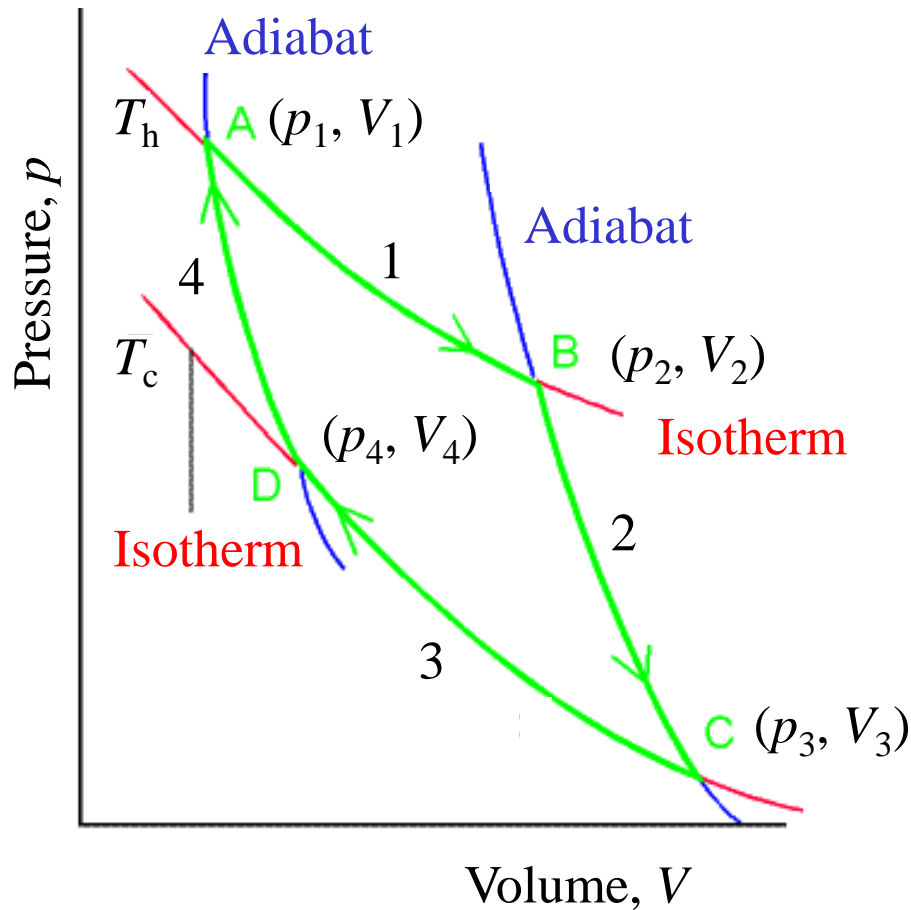
Step 2: [Adiabatic]

The gas expands adiabatically and reversibly from V_2 to V_3 ; in doing so, its temperature drops from T_h to T_c .

$$q = 0$$

$$w = \Delta U = \int_{T_h}^{T_c} C_V dT$$

Ideal gas
ppt 2-27
Chap. 2E.1



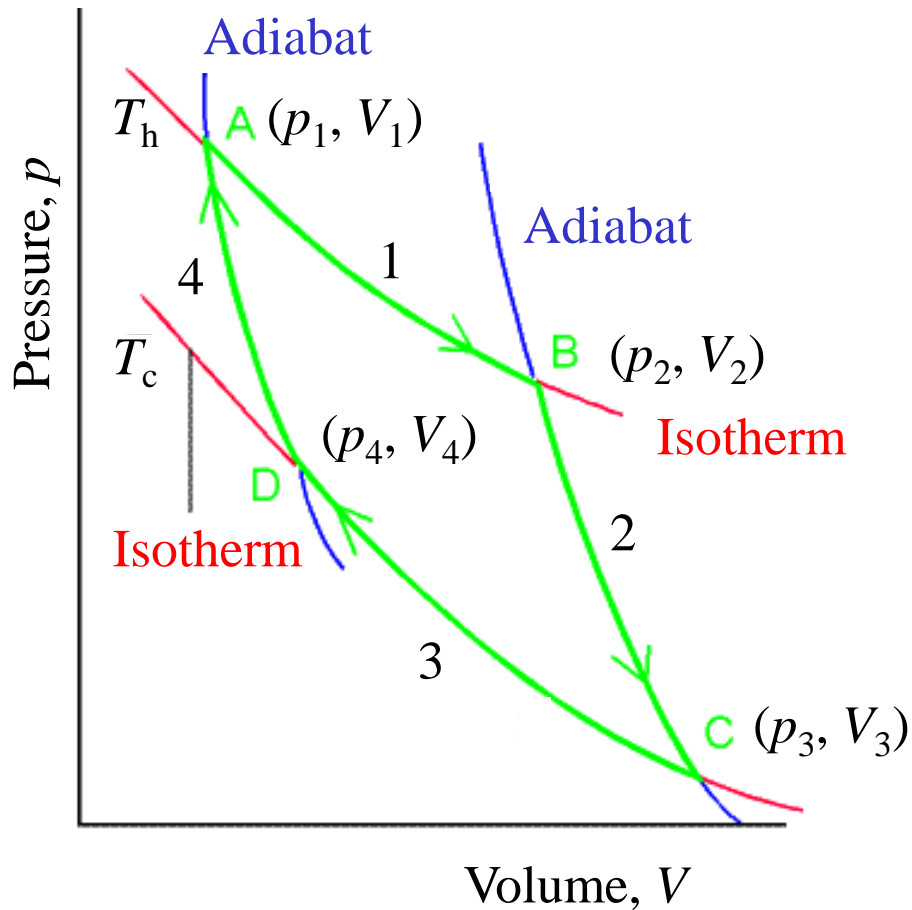
Step 3: [Isothermal at q_c]

The gas is compressed from V_3 to V_4 while in thermal contact with the low temperature reservoir (temperature T_c), isothermally and reversibly.

$$q_c = RT_c \ln \frac{V_4}{V_3}$$

$$w = -RT_c \ln \frac{V_4}{V_3}$$

$$\Delta U = 0$$



Step 4: [Adiabatic]

The gas is compressed adiabatically and reversibly from V_4 to V_1 , warming in the process from T_c to T_h .

$$q = 0$$

$$w = \Delta U = \int_{T_c}^{T_h} C_V dT$$

Adding these all together:

$$q(\text{cycle}) = q_h + q_c = \overset{\text{Step 1}}{RT_h \ln \frac{V_2}{V_1}} + \overset{\text{Step 3}}{RT_c \ln \frac{V_4}{V_3}}$$

$$w(\text{cycle}) = \overset{\text{Step 1}}{-RT_h \ln \frac{V_2}{V_1}} - \overset{\text{Step 3}}{RT_c \ln \frac{V_4}{V_3}} \quad (\text{step 2} + \text{step 4} = 0)$$

$$\Delta U(\text{cycle}) = 0 \quad \overset{\text{Step 3}}$$

For an ideal gas, and for reversible and adiabatic expansion*

$$C_{V,m} \ln \frac{T_f}{T_i} = -R \ln \frac{V_f}{V_i}$$

ppt 2-27
Chap. 2E.1

$$* dq = 0; dU = dw = -pdV \quad dU = C_V dT.$$

$$c = \frac{C_V}{nR} = \frac{C_{V,m}}{R}$$

$$\ln \left(\frac{T_f}{T_i} \right)^c = \ln \left(\frac{V_i}{V_f} \right)$$

$$w(\text{cycle}) = -RT_h \ln \frac{V_2}{V_1} - RT_c \ln \frac{V_4}{V_3}$$

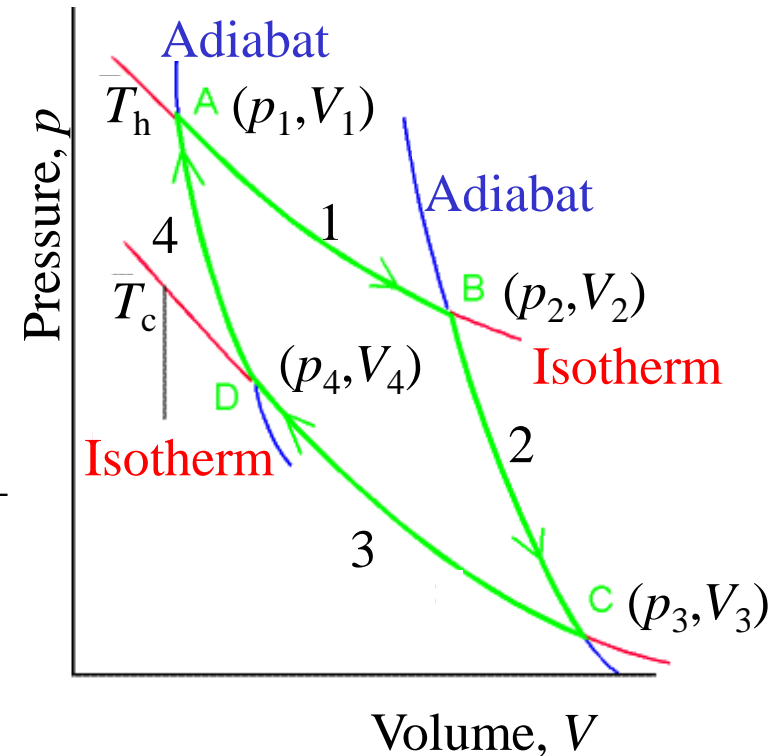
Apply the adiabatic relation in [steps 2 and 4](#):

$$C_{V,m} \ln \frac{T_h}{T_c} = -R \ln \frac{V_2}{V_3} = -R \ln \frac{V_1}{V_4}$$

$$\therefore \frac{V_2}{V_3} = \frac{V_1}{V_4} \Rightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1} \Rightarrow \ln \frac{V_2}{V_1} = -\ln \frac{V_4}{V_3}$$

$$\therefore -w = RT_h \ln \frac{V_2}{V_1} - RT_c \ln \frac{V_2}{V_1} = R(T_h - T_c) \ln \frac{V_2}{V_1}$$

$$q_h = RT_h \ln \frac{V_2}{V_1}$$



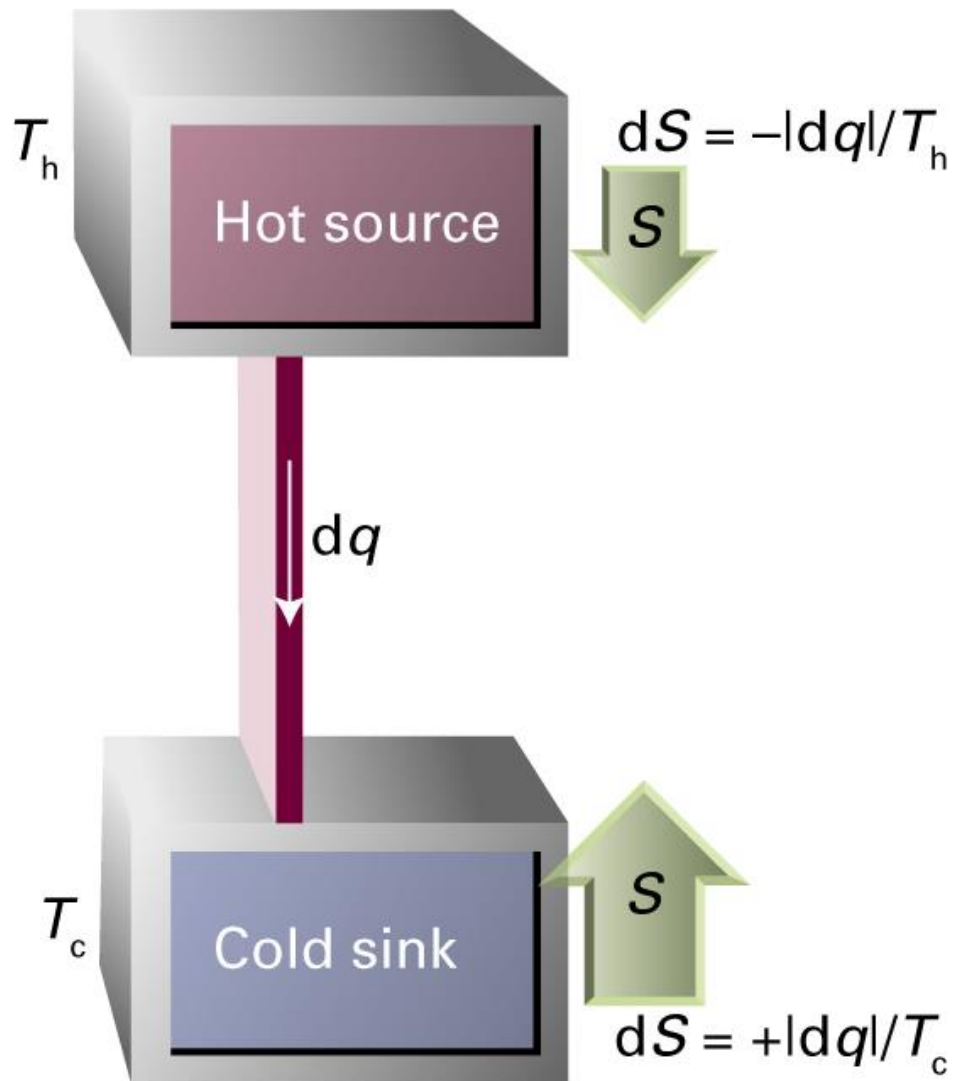
\therefore The efficiency of a Carnot engine is the ratio of the net work (-w) to the fuel burned to provide the heat q_h .

$$\text{efficiency} = \frac{-w}{q_h} = \frac{T_h - T_c}{T_h} = 1 - \left(\frac{T_c}{T_h} \right)$$

(3A.10)

(same as ppt 3-17)

Brief Illus. 3A.6

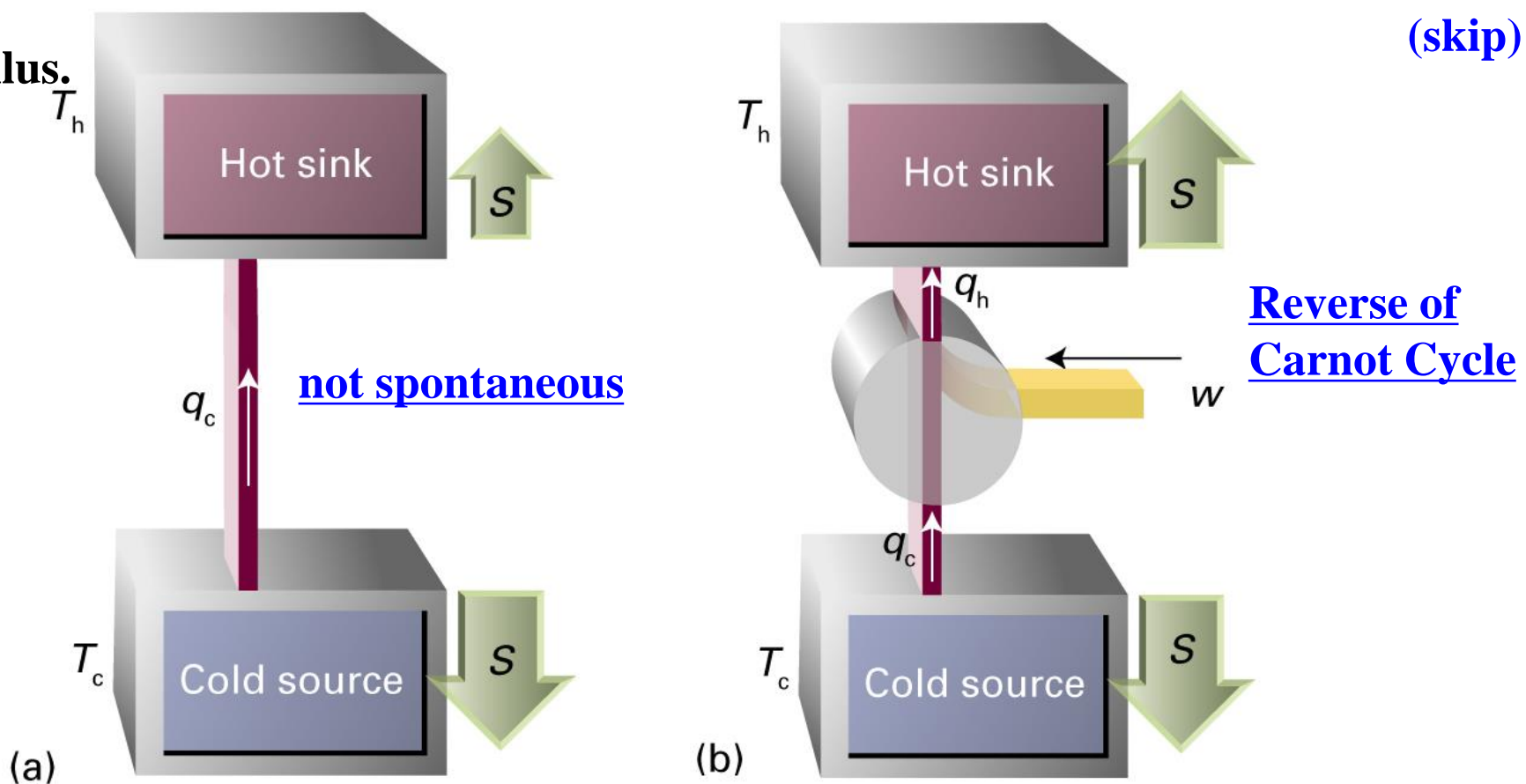


When energy leaves a hot reservoir as heat, the entropy of the reservoir decreases.

When **the same quantity of energy** enters a cooler reservoir, the entropy increases by a larger amount.

Hence, overall there is an increase in entropy and **the process is spontaneous**.

Relative changes in entropy are indicated by the sizes of the arrows.



(a) The flow of energy as heat from a cold source to a hot sink is not spontaneous. As shown here, the entropy increase of the hot sink is smaller than the entropy decrease of the cold source, so there is a net decrease of entropy.

(b) The process becomes feasible if work is provided to add to the energy stream. Then the increase of entropy of the hot sink can be made to cancel the entropy decrease of the cold source.

3C. Concentrating on the System

The criterion for **natural, spontaneous change** solely in terms of the properties of the system is:

$$dS - \frac{dq}{T} \geq 0$$

i) Constant volume: $(dq)_V = dU$

$$dS - \frac{dU}{T} \geq 0 \quad TdS \geq dU \quad \text{for spontaneous}$$

ii) Constant pressure: $(dq)_p = dH$

$$dS - \frac{dH}{T} \geq 0 \quad TdS \geq dH \quad \text{for spontaneous}$$

Now we can **define** new thermodynamic functions for the criterion of spontaneous process:

Helmholz Free Energy: $\underline{A \equiv U - TS}$



Gibbs Free Energy: $\underline{G \equiv H - TS}$

Why $\Delta G \leq 0$ at the given T and P ?

Why $\Delta F \leq 0$ at the given T and V ?

$$G \equiv H - TS$$

$$dG = \delta q + VdP - Tds - SdT$$

at $T \triangleright P$

$$dG = \delta q - Tds \leq 0$$

irreversible
equilibrium

$$A = F \equiv U - TS$$

$$dF = \delta q - PdV - Tds - SdT$$

at $T \triangleright V$

$$dF = \delta q - Tds \leq 0$$

irreversible
equilibrium

$$dS - \frac{dq}{T} \geq 0$$

3B. The Variation of Entropy with T

$$\therefore C_p = \left(\frac{\partial q}{\partial T} \right)_p = \left(\frac{\partial H}{\partial T} \right)_p$$

$$S(T_f) = S(T_i) + \int_i^f \frac{dq_{rev}}{T}, \quad dq_{rev} = C_p dT$$

constant P

$$\underline{S(T_f) = S(T_i) + \int_i^f \frac{C_p dT}{T}}$$

**After Measuring C_p Experimentally,
We Can Identify $S(T)$, $H(T)$ & $G(T)$.**

If C_p is independent of temperature in the temperature range of interest, we obtain:

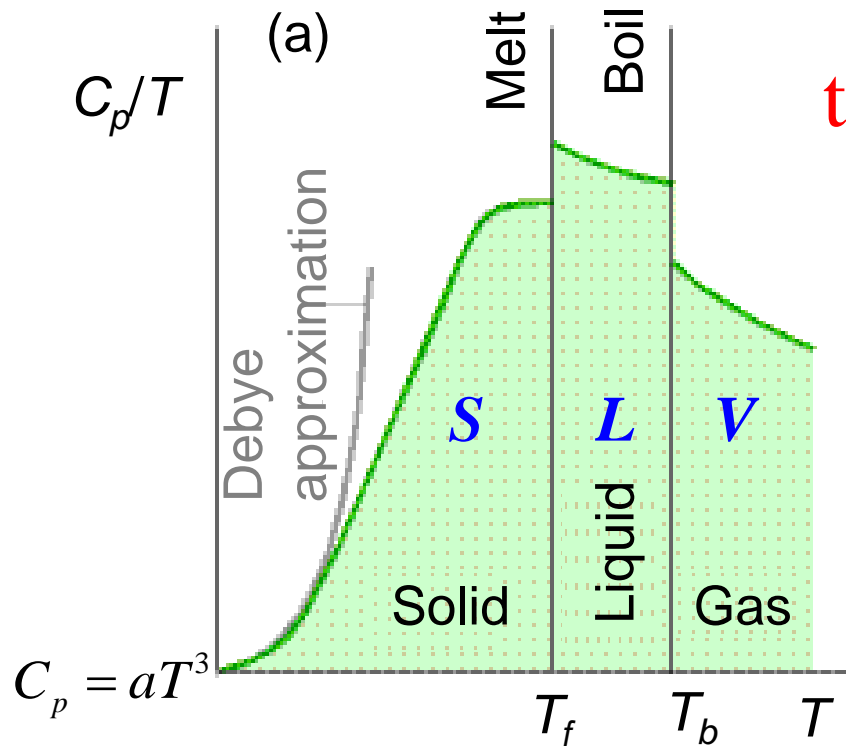
$$S(T_f) = S(T_i) + C_p \int_i^f \frac{dT}{T} = S(T_i) + C_p \ln \left(\frac{T_f}{T_i} \right)$$

At some temperatures between 0 and T , the materials may change its phase and absorb heat in the process:

$$S(T) = S(0) + \int_0^{T_f} \frac{\underline{S} C_p^{solid}}{T} dT + \frac{\underline{S \rightarrow L} \Delta H_{melt}}{T_f} + \int_{T_f}^{T_b} \frac{\underline{L} C_p^{liq}}{T} dT + \frac{\underline{L \rightarrow V} \Delta H_{evap}}{T_b} + \int_{T_b}^T \frac{\underline{V} C_p^{gas}}{T} dT$$

In the vicinity of absolute zero: $C_p = aT^3$

$C_p \approx C_v \approx 3k_B/\text{atom}$
in solid



temperature axis: not linear

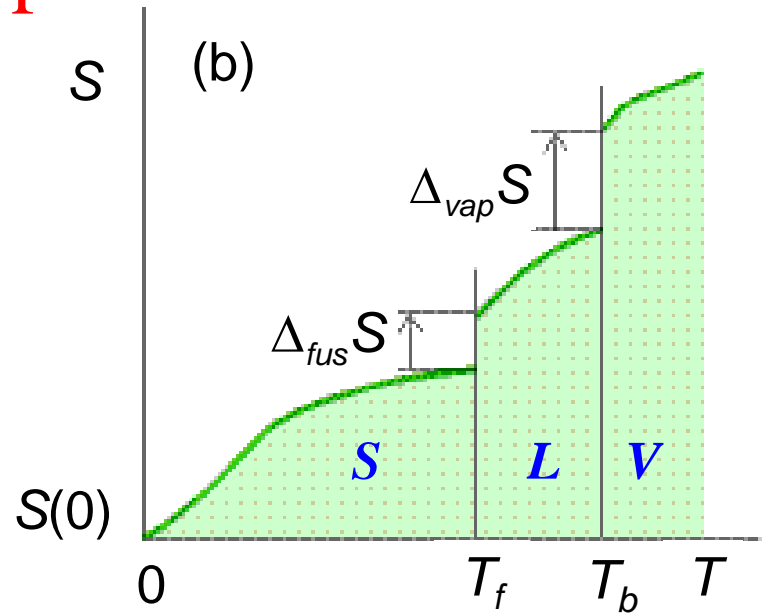


Figure 3B.1

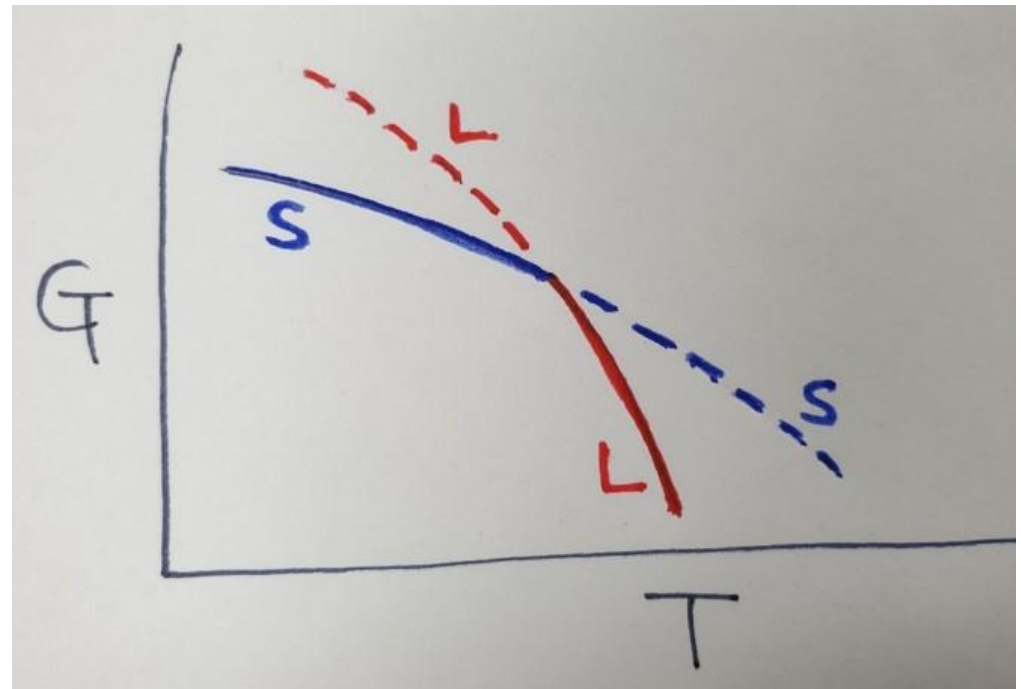
The determination of entropy from heat capacity data.

Chapter 3B.2.

At absolute zero, all quenchable energy has been quenched. In the case of a perfect crystal at absolute zero, all the atoms are in a regular, uniform array, and the absence of disorder and thermal chaos suggests that the entropy is the same in every case.

The Third Law of Thermodynamics:

All perfect crystalline (equilibrium) compounds have zero entropies at $T = 0$ K.



Chapter 3D. Combining the First and Second Laws

$$G = G(p, T, n) \quad n: \text{composition}$$

Combining the first law and second law

The first law says

$$dU = dq + dw \quad (1)$$

For **reversible** processes in the absence of any kind of work other than pV -work,

$$dw_{rev} = -pdV \quad (2)$$

$$dq_{rev} = TdS \quad (3)$$

$$dU = TdS - pdV \quad (4)$$

└───┬───> Master equation or fundamental equation

$$\underline{dU = TdS - pdV} \quad \text{equilibrium}$$

$$U = U(S, V)$$

$$\underline{dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV} \quad (5)$$

Comparing eqs (4) and (5), we obtain relationships. $dU = TdS - pdV$

$$\underline{\left(\frac{\partial U}{\partial S}\right)_V = T} \quad (6)$$

$$\underline{-\left(\frac{\partial U}{\partial V}\right)_S = p} \quad (7)$$

Equation (6) enables a **temperature** to be expressed solely in terms of extensive thermodynamic quantities. If the volume is constant, the relation states that the ratio of the change in energy (a First Law concept) to the corresponding change in entropy (a Second Law concept) is equal to the temperature of the system, whatever its nature or composition.

3.8(a). Various Maxwell Relationships

$$dU = TdS - pdV$$

U : exact differential

$$(4) \quad \begin{aligned} \left(\frac{\partial U}{\partial S}\right)_V &= T \\ -\left(\frac{\partial U}{\partial V}\right)_S &= p \end{aligned}$$

By using the relation No. 4,

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V \quad \text{Maxwell relation from } U \quad (8)$$

$$\underline{G = H - TS}$$

$$dG = dU + p dV + V dp - T dS - S dT$$

$$\therefore dG = V dp - S dT$$

$$\because dU = TdS - pdV$$

$$(9)$$

G : exact differential

$$\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T \quad \text{Maxwell relation from } G \quad (10)$$

Note: Relations between partial derivatives

If f is a function of x and y , then when x and y change by dx and dy , f changes by

$$df = \left(\frac{\partial f}{\partial x}\right)_y dx + \left(\frac{\partial f}{\partial y}\right)_x dy$$

Partial derivatives may be taken in any order:

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

In the following, z is a variable on which x and y depend (for example, x , y , and z might correspond to p , V , and T).

Relation No. 1. When x is changed at constant z :

$$\left(\frac{\partial f}{\partial x}\right)_z = \left(\frac{\partial f}{\partial x}\right)_y + \left(\frac{\partial f}{\partial y}\right)_x \left(\frac{\partial y}{\partial x}\right)_z$$

Relation No. 2 (the Inverter).

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}$$

Relation No. 3 (the Permuter).

$$\left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x$$

By combining this and Relation No. 2 we obtain Euler's chain relation:

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

Relation No. 4. This relation establishes whether or not df is an exact differential.

$$df = g dx + h dy \text{ is exact if } \left(\frac{\partial g}{\partial y}\right)_x = \left(\frac{\partial h}{\partial x}\right)_y$$

If df is exact, its integral between specified limits is independent of the path.

$$\underline{A = U - TS}$$

$$dA = dU - TdS - SdT \quad (11)$$

From equations (4) and (11)

$$dA = -pdV - SdT$$

A : exact differential

$$\left(\frac{\partial S}{\partial V} \right)_T = \left(\frac{\partial p}{\partial T} \right)_V \quad \text{Maxwell relation from } A = F \quad (12)$$

Maxwell relation

$$\underline{H = U + PV} \quad (13)$$
$$dH = dU + pdV + Vdp$$

From equations (4) and (13), $dU = TdS - pdV$
 $dH = TdV + Vdp$

H : exact differential

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p \quad \text{Maxwell relation from } H \quad (14)$$

Maxwell relation

Maxwell's Relations

equilibrium

$$T = \left. \frac{\partial U}{\partial S} \right|_V = \left. \frac{\partial H}{\partial S} \right|_P$$

$$P = - \left. \frac{\partial U}{\partial V} \right|_S = - \left. \frac{\partial A}{\partial V} \right|_T$$

$$V = \left. \frac{\partial H}{\partial P} \right|_S = \left. \frac{\partial G}{\partial P} \right|_T$$

$$S = - \left. \frac{\partial A}{\partial T} \right|_V = - \left. \frac{\partial G}{\partial T} \right|_P$$

~~Maxwell's Equations~~
Relations

equilibrium

$\frac{dy}{dx}$



Maxwell's Equations

$$\nabla \cdot \vec{D} = 4\pi P$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{E} = - \frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\vec{B} = \mu \vec{H} \quad \vec{D} = \epsilon \vec{E}$$

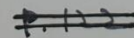
Maxwell's Equations

from U $\left. \frac{\partial T}{\partial V} \right|_S = - \left. \frac{\partial P}{\partial S} \right|_V$

from H $\left. \frac{\partial T}{\partial P} \right|_S = \left. \frac{\partial V}{\partial S} \right|_P$

from A $\left. \frac{\partial S}{\partial V} \right|_T = \left. \frac{\partial P}{\partial T} \right|_V$

from G $\left. \frac{\partial S}{\partial P} \right|_T = - \left. \frac{\partial V}{\partial T} \right|_P$



3D.1(b). The Variation of Internal Energy with Volume

$$\underline{\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p} \quad \text{(Sec. 2D.2)}$$

Derivation

Homework

$$dU(S, V) = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV \quad (5)$$

Dividing equation (5) by dV and imposing constant temperature,

$$\left(\frac{\partial U}{\partial V}\right)_T = \left(\frac{\partial U}{\partial S}\right)_V \left(\frac{\partial S}{\partial V}\right)_T + \left(\frac{\partial U}{\partial V}\right)_S \quad (15)$$

Combining with Eqs. (6) and (7), one can obtain

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p \quad (16)$$

Substitution of Eq. (12) into equation (16)

Maxwell's relation
Eq. (12)

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$\underline{\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p} \quad \text{(3D.6)}$$

Example 3D.2

Show thermodynamically that $\left(\frac{\partial U}{\partial V}\right)_T$ is zero for an ideal gas, and compute its value for a van der Waals gas.

Solution:

For an ideal gas, $pV=nRT$.

$$\left(\frac{\partial p}{\partial T}\right)_T = \frac{nR}{V} \quad \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

Definition of internal pressure

Combining two equations,

$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{nRT}{V} - p = 0$$

The equation of state of a van der Waals gas $p = \frac{nRT}{V-nb} - \frac{n^2a}{V^2}$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{nR}{V-nb}$$

$$\therefore \left(\frac{\partial U}{\partial V}\right)_T = \frac{nRT}{V-nb} - p = \frac{nRT}{V-nb} - \left(\frac{nRT}{V-nb} - \frac{n^2a}{V^2}\right) = \frac{n^2a}{V^2}$$

ppt 2-61

3D.2. Properties of Gibbs Free Energy

$$G = H - TS$$

$$H = U + pV$$

$$\begin{aligned} dG &= dH - TdS - SdT \\ &= dU + pdV + Vdp - TdS - SdT \end{aligned} \quad (17)$$

Combining equation (4) and (17),

$$\begin{aligned} \therefore \underline{dG} &= TdS - pdV + pdV + Vdp - TdS - SdT \\ &= \underline{Vdp - SdT} \quad \text{reversible} \end{aligned} \quad (18)$$

$$G = G(p, T)$$

$$dG = \left(\frac{\partial G}{\partial p} \right)_T dp + \left(\frac{\partial G}{\partial T} \right)_p dT \quad (19)$$

Combining eqs (18) and (19), one obtains

$$\underline{\left(\frac{\partial G}{\partial p} \right)_T} = V, \quad \underline{\left(\frac{\partial G}{\partial T} \right)_p} = -S \quad (20) \quad (3d.8)$$

$$G(T,P)$$

$$dG = -SdT + Vdp + \sum_i \mu_i dn_i$$

reversible (Chap. 5)

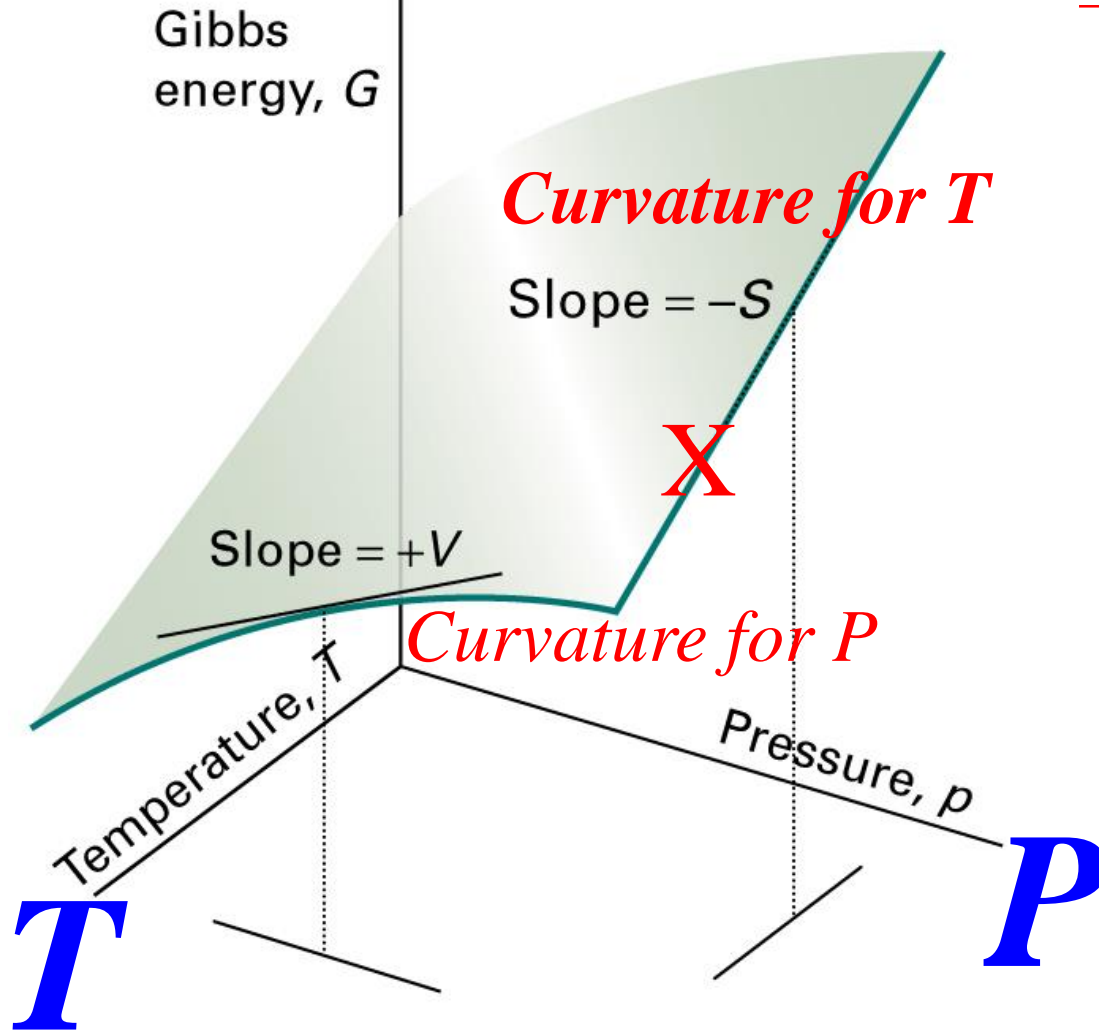


Figure 3D.1

The variation of the Gibbs energy of a system:

- with temperature at constant P
- with pressure at constant T

The slope of the former is equal to the negative of the entropy of the system and that of the latter is equal to the volume.

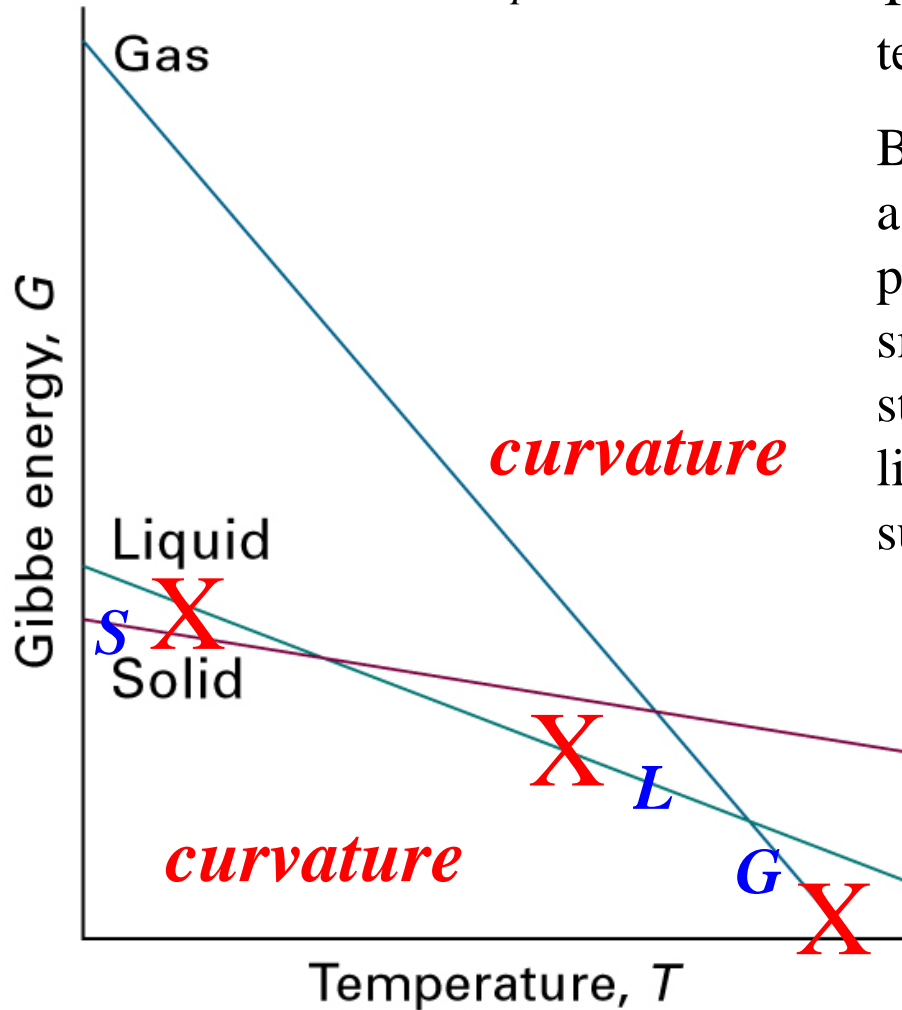
$$G \equiv H - TS$$

reversible
or
irreversible

$$dG = -SdT + Vdp + \sum_i \mu_i dn_i$$

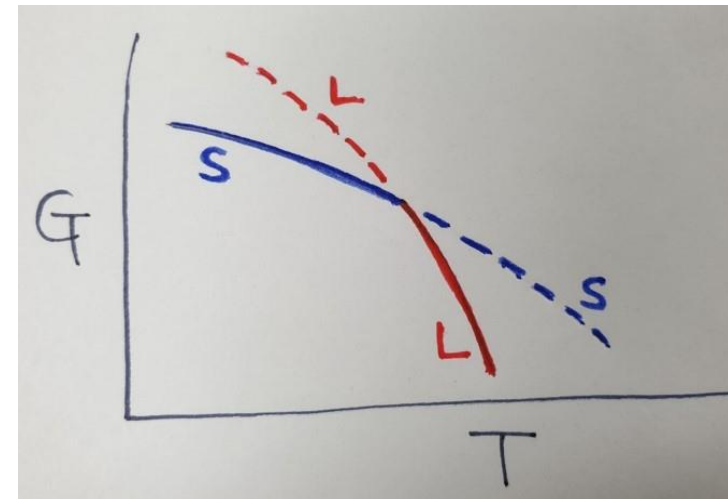
Figure 3D.2

$$\left(\frac{\partial G}{\partial T} \right)_p = -S$$



The variation of Gibbs energy with the temperature is determined by the **entropy**.

Because the entropy of the gaseous phase of a substance is greater than that of the liquid phase, and the entropy of the solid phase is smallest, the Gibbs energy changes most steeply for the gas phase, followed by the liquid phase, and then the solid phase of the substance.



3D.2(b). The Change of Gibbs Energy with Temperature

Equation (20) implies that, as S is always a positive quantity, G must decrease when the temperature is raised at constant pressure (see Figure 3D.1).

$$G = H - TS \qquad S = \frac{H - G}{T} \qquad (21)$$

Combining Eqs. (20) and (21),

$$\left(\frac{\partial G}{\partial T}\right)_p = \frac{G - H}{T} \qquad (22) \qquad \left(\frac{\partial G}{\partial T}\right)_p = -S$$

$$\begin{aligned} \left(\frac{\partial(G/T)}{\partial T}\right)_p &= \frac{1}{T} \left(\frac{\partial G}{\partial T}\right)_p + G \left(\frac{\partial}{\partial T} \left(\frac{1}{T}\right)\right)_p \\ &= \frac{1}{T} \left\{ \left(\frac{\partial G}{\partial T}\right)_p - \frac{G}{T} \right\} \end{aligned} \qquad (23)$$

Combining Eqs. (22) and (23), one obtains

$$\underline{\left(\frac{\partial(G/T)}{\partial T}\right)_p = -\frac{H}{T^2}} \qquad (24) \qquad G(T) \rightarrow H(T) \qquad (3D.10)$$

Gibbs-Helmholtz equation

$$\left(\frac{\partial(G/T)}{\partial T} \right)_p = -\frac{H}{T^2}$$

Gibbs-Helmholtz equation

For a chemical reaction,

initial state (reactants) → final state (products)

$$\Delta G = G_f - G_i$$

$$\left(\frac{\partial(\Delta G/T)}{\partial T} \right)_p = -\left(\frac{H_f}{T^2} - \frac{H_i}{T^2} \right)$$

$$\left(\frac{\partial(\Delta G/T)}{\partial T} \right)_p = -\frac{\Delta H}{T^2} \quad (25) \quad (3D.11)$$

3D.2(c). The Change of Gibbs Free Energy with Pressure

$$\left(\frac{\partial G}{\partial p}\right)_T = V$$

$$dG = -SdT + Vdp + \sum_i \mu_i dn_i$$

For a reaction in which G_i changes to G_f ,

$$\left(\frac{\partial \Delta G}{\partial p}\right)_T = \Delta V, \quad \text{where } \Delta V = V_f - V_i.$$

Integration of the above equation results in:

$$G(p_f) = G(p_i) + \int_{p_i}^{p_f} V(p) dp \quad (*) \quad (3D.12b)$$

G is virtually independent of pressure for solid or liquid.

For **liquid or solid**, volume depends only very weakly on the pressure:

~1 atm.

$$G_m(p_f) \cong G_m(p_i) + (p_f - p_i)V_m$$

Except at very high pressure, $(p_f - p_i)V_m$ is **very small**,
and virtually no error is introduced.

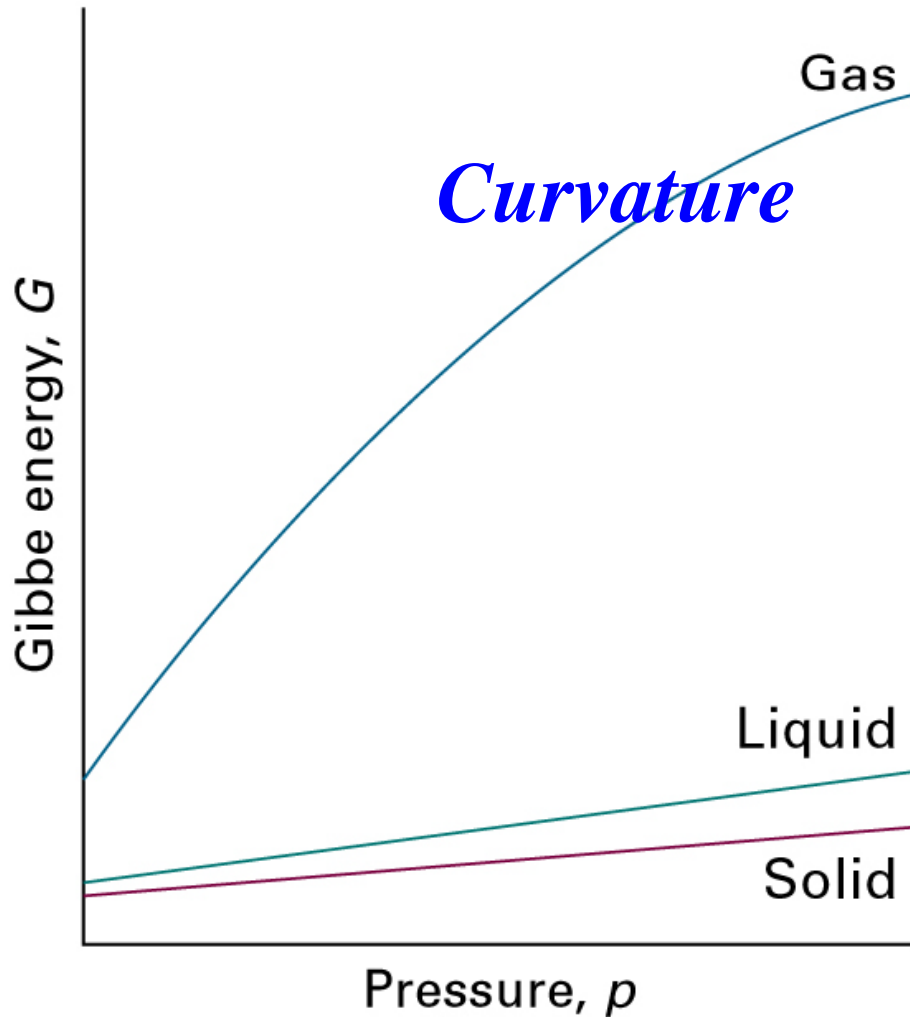
(≠ Earth Science)

∴ For solids and liquids, $G_m(p_f) \approx G_m(p_i)$.

$$\left(\frac{\partial G}{\partial p}\right)_T = V$$

Figure 3D.3

The variation of the Gibbs energy with the pressure is determined by the volume of the sample.



Because the volume of the gaseous phase of a substance is greater than that of the same amount of liquid phase, and the volume of the solid phase is smallest (for most substances), the Gibbs energy changes most steeply for the gas phase, followed by the liquid phase, and then the solid phase of the substance.

Because the volumes of the solid and liquid phases of a substance are similar, they vary by similar amounts as the pressure is changed.

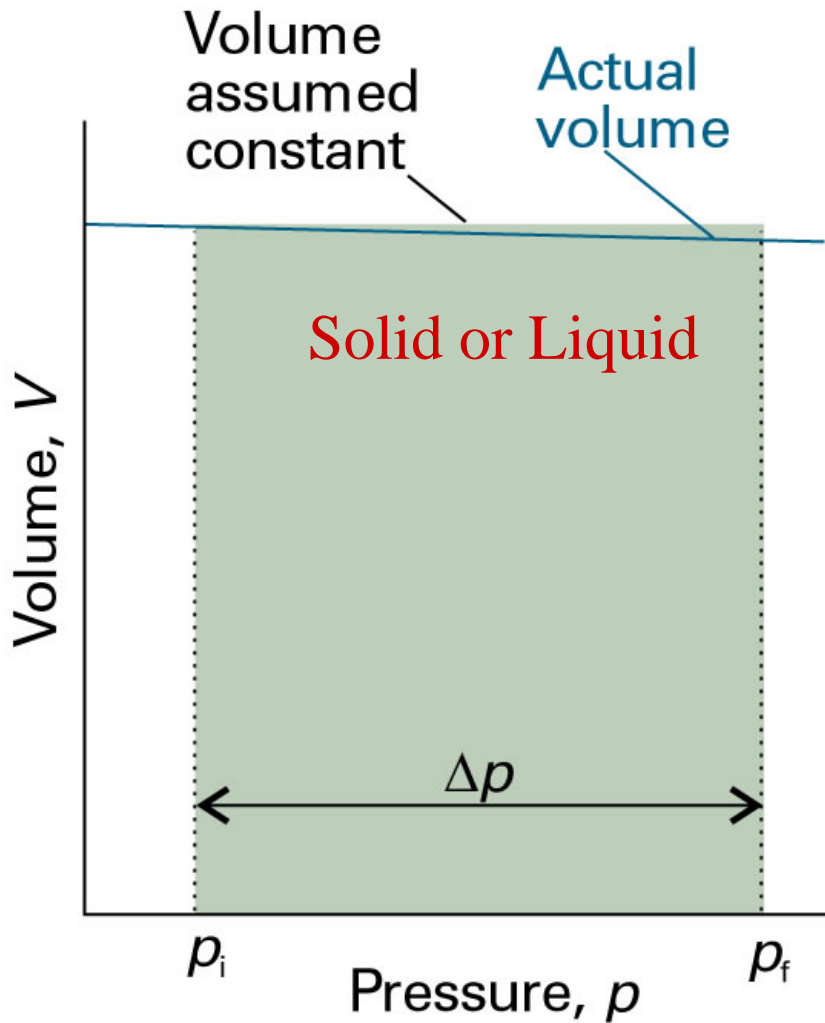
$\sim 5 \times 10^{22}$ atoms / cm³

for typical solid or liquid

-Mon/Apr/04/22

Lithiated or Delithiated Electrode

$$dG = -SdT + Vdp + \sum_i \mu_i dn_i \quad (\text{Chap. 5})$$



$$G(p_f) = G(p_i) + \int_{p_i}^{p_f} V(p) dp$$

$$G_m(p_f) \cong G_m(p_i) + (p_f - p_i) V_m$$

$$\therefore \Delta G_m = V_m \Delta p$$

Figure 3D.4

The difference in Gibbs energy of a **solid or liquid** at two pressures is equal to the rectangular area shown.

We have assumed that the variation of volume with pressure is negligible.

Ideal Gas + Constant T

For an ideal gas, $V = \frac{nRT}{p}$

$$G(p_f) = G(p_i) + \int_{p_i}^{p_f} V(p) dp \quad (*) \quad (3D.12b)$$

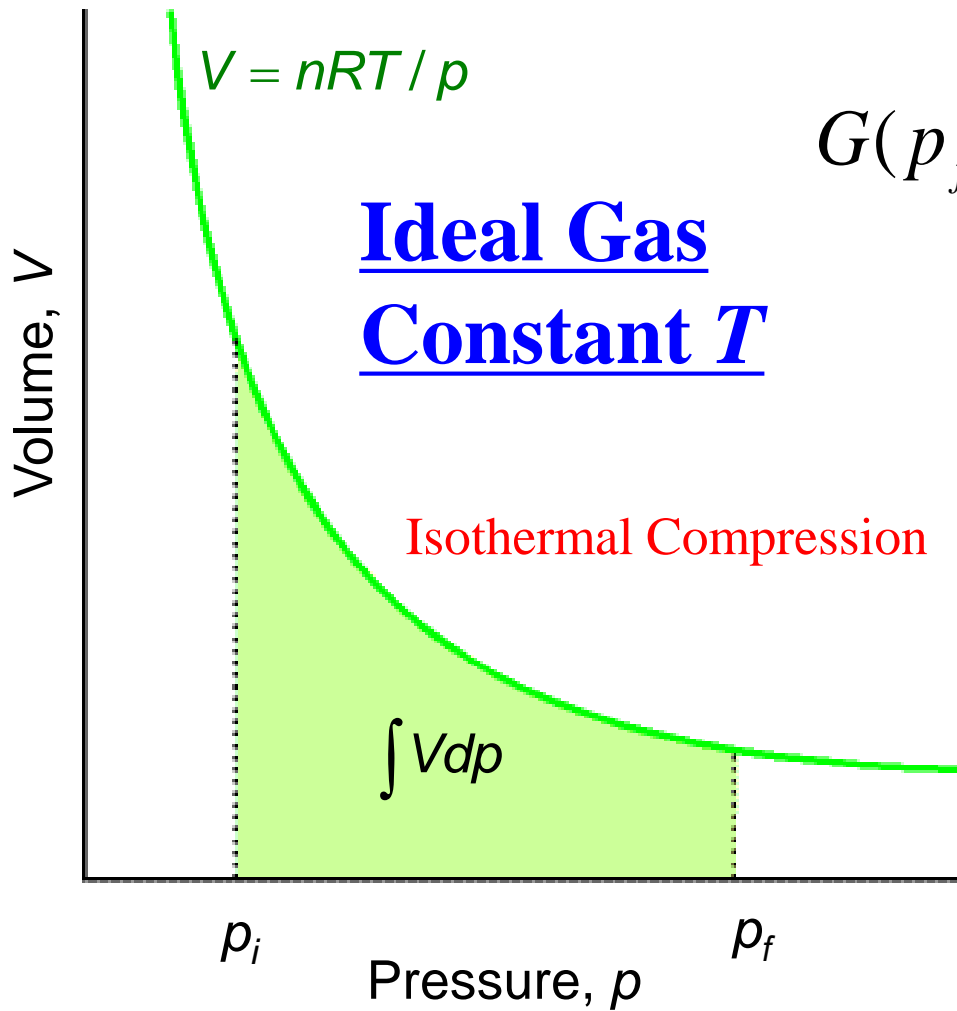
Inserting the above equation into equation (*) yields

$$G(p_f) = G(p_i) + \int_{p_i}^{p_f} \frac{nRT}{p} dp$$

$$\therefore \underline{G(p_f) = G(p_i) + nRT \ln \frac{p_f}{p_i}} \quad \text{Ideal Gas} \quad (3D.15)$$

Constant T

Single Component



$$G(p_f) = G(p_i) + nRT \ln \frac{p_f}{p_i}$$

Figure 3D.5

The difference in Gibbs energy for a perfect gas at two pressures is equal to the area shown below the perfect-gas isotherm.

3D.2(c). Chemical Potential of an Ideal Gas

Single Component

Standard state of an ideal gas = 1 atm

Gibbs function at 1 atm = G^0

$$G(p_f) = G(p_i) + nRT \ln \frac{p_f}{p_i}$$

At any other pressure p ,

$$G(p) = G^0 + nRT \ln(p / \text{atm})$$

For one mole of material,

$$G_m(p) = G_m^0 + RT \ln(p / \text{atm})$$

Molar Gibbs Free Energy

We write $\mu = G_m(p)$:

Chemical Potential

$$\underline{\mu(p, T) = \mu^0(T) + RT \ln p} \quad \text{Ideal Gas} \quad (3D.15)$$

Chemical potential = G^{total} / mole (= G^{total} / atom) Single Component

(Chap. 5)

$$\underline{\mu_j(p, T, p_1, p_2, p_3, \dots) = \mu_j^0(p, T) + RT \ln p_j} \quad \text{Ideal Gas}$$

Multi Components

Single Component + Ideal Gas

$$\mu(p, T) = G_m(p, T) = \mu^0(T) + RT \ln p$$

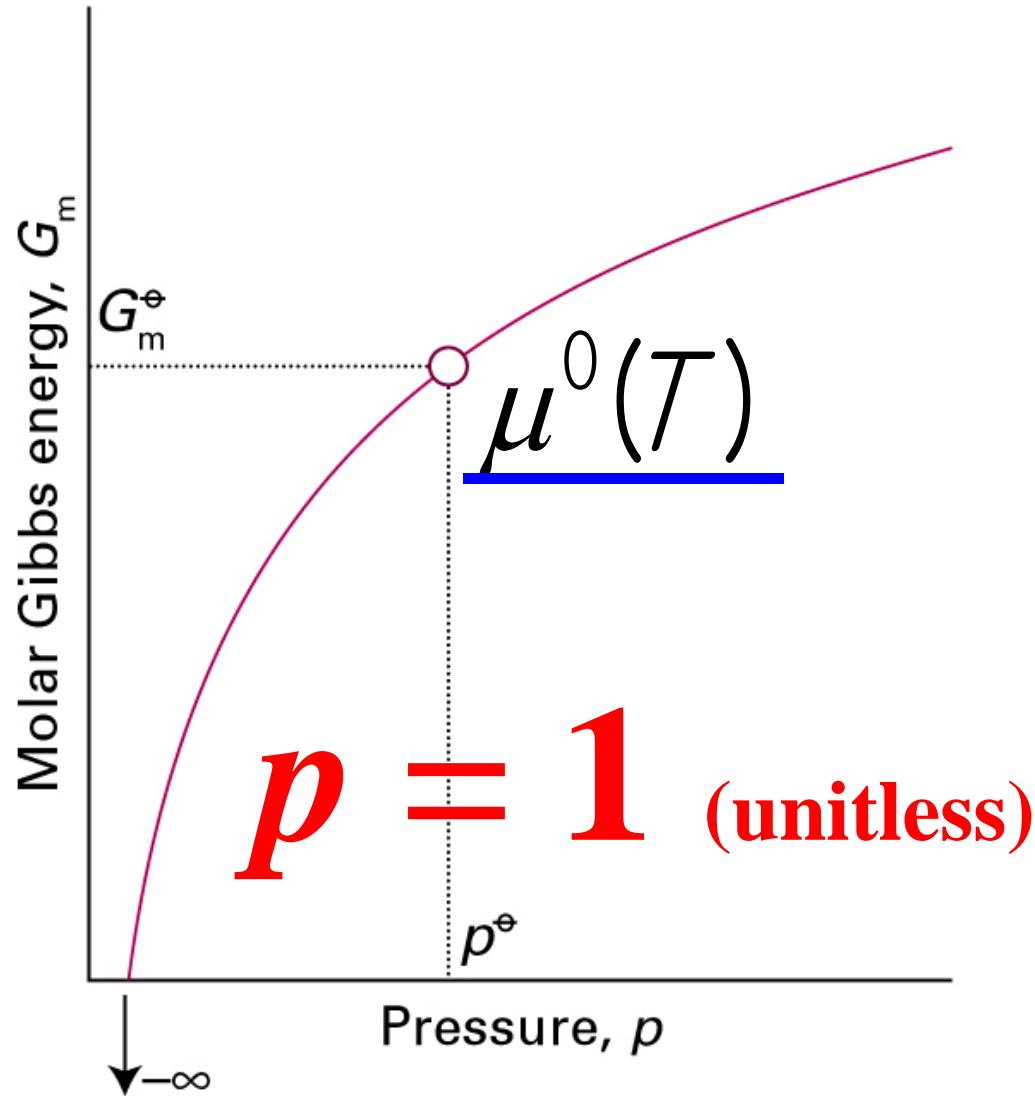


Figure 3D.6

The molar Gibbs free energy of a perfect gas is proportional to $\ln p$.

Real Gases: the Fugacity

$$G_m(p) = G_m^0 + RT \ln(p / atm) \quad : \text{applicable to ideal gases}$$

For real gases, we are to determine the pressure dependence of the volume of a sample of real gas and to calculate $G(p)$ from $\int V(p)dp$ by numerical integration.

$$G_m = G_m^0 + RT \ln(f / atm) \Rightarrow \mu = \mu^0 + RT \ln(f / atm)$$

The quantity f plays the role of the pressure, but it has a value which ensures that the chemical potential is given by the last equation whatever the pressure. Here f is called as **the fugacity** of gas.

The following equation is true for all substances.

$$G(p_f) = G(p_i) + \int_{p_i}^{p_f} V(p)dp; \quad G_m(p_f) = G_m(p_i) + \int_{p_i}^{p_f} V_m(p)dp$$

Hence, for a real gas,

$$\int_{p_i}^{p_f} V_m^{real} dp = \mu^{real}(p_f, T) - \mu^{real}(p_i, T) = RT \ln \frac{f_f}{f_i}$$

For an ideal gas,

$$\int_{p_i}^{p_f} V_m^{real} dp = \mu^{real}(p_f, T) - \mu^{real}(p_i, T) = RT \ln \frac{f_f}{f_i}$$

The difference of the two equations

$$\int_{p_i}^{p_f} \{V_m^{real}(p) - V_m^{ideal}(p)\} dp = RT \ln \frac{f_f}{f_i} - RT \ln \ln \frac{p_f}{p_i}$$

$$\ln \left\{ \frac{(f_f / p_f)}{(f_i / p_i)} \right\} = \frac{1}{RT} \int_{p_i}^{p_f} \{V_m^{real}(p) - V_m^{ideal}(p)\} dp$$

$$p_i \rightarrow 0 \quad \frac{f_i}{p_i} \rightarrow 1$$

$$\ln \left(\frac{f}{p} \right) = \frac{1}{RT} \int_0^p \{V_m^{real}(p) - V_m^{ideal}(p)\} dp$$

$$V_m^{ideal} = \frac{RT}{p}$$

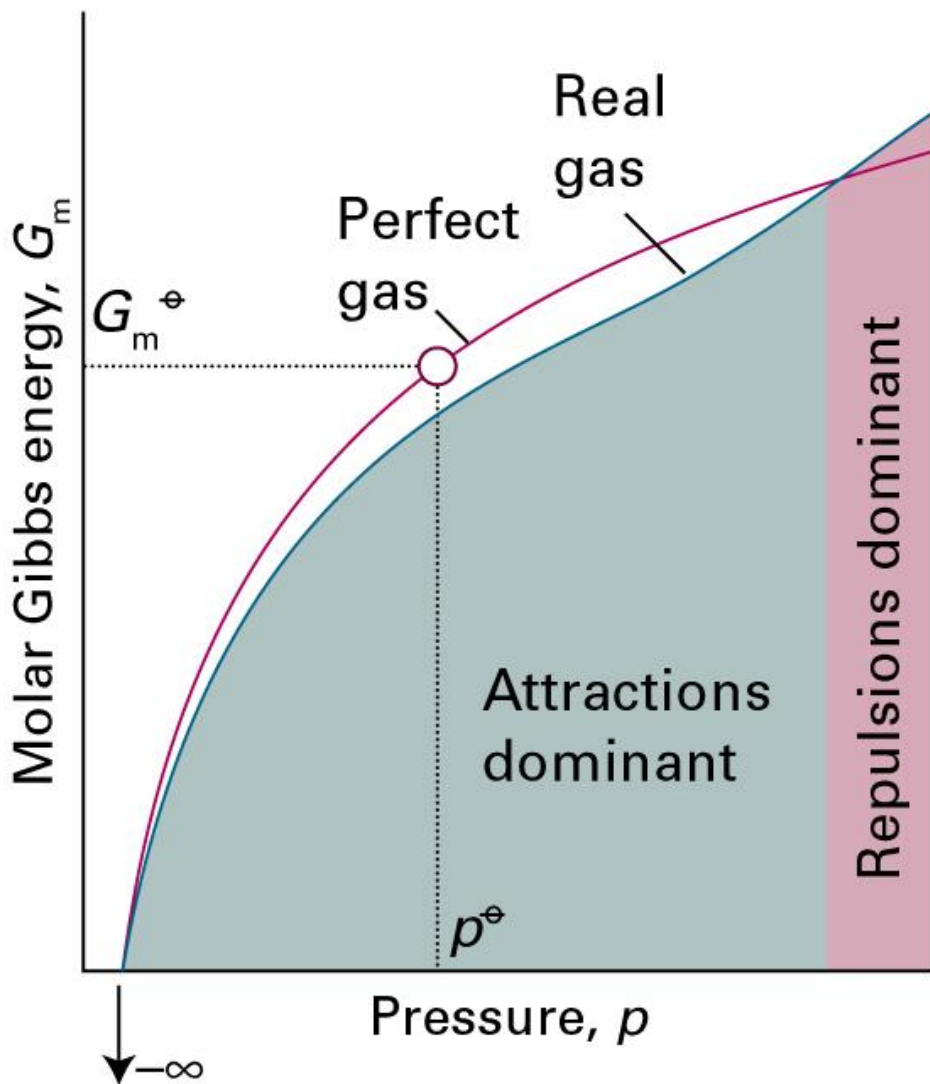


Figure 3.25

The molar Gibbs free energy of a real gas. As $p \rightarrow 0$, the molar Gibbs free energy coincides with the value for a perfect gas. When attractive forces are dominant (at intermediate pressures), the molar Gibbs free energy is less than that of a perfect gas and the molecules have a lower ‘escaping tendency’. At high pressures, when repulsive forces are dominant, the molar Gibbs free energy of a real gas is greater than that of a perfect gas. Then the ‘escaping tendency’ is increased.

The Real Volume-Pressure Dependence

Compression factor

$$z = \frac{pV_m^{real}}{RT} \Rightarrow V_m^{real} = \frac{zRT}{p}$$

For a real gas, $z = z(p, T)$

$$\ln\left(\frac{f}{p}\right) = \frac{1}{RT} \int_0^p \{V_m^{real}(p) - V_m^{ideal}(p)\} dp$$

$$\begin{aligned} \ln\left(\frac{f}{p}\right) &= \frac{1}{RT} \int_0^p \left(\frac{zRT}{p} - \frac{RT}{p}\right) dp = \frac{RT}{RT} \int_0^p \left(\frac{z(p, T) - 1}{p}\right) dp \\ &= \int_0^p \frac{z(p, T) - 1}{p} dp \end{aligned}$$

$$\therefore f = p \exp \int_0^p \left(\frac{z(p, T) - 1}{p}\right) dp$$

Standard States of Real Gases

For an ideal gas, $p = 1 \text{ atm}$: standard state

For a real gas, $f = 1 \text{ atm}$: standard state

$$f = \gamma p$$

γ : fugacity coefficient

$$\mu(p) = \mu^0 + RT \ln(f / \text{atm})$$

$$\mu(p) = \mu^0 + RT \ln(p / \text{atm}) + RT \ln \gamma$$

The chemical potential of
an ideal gas

Deviation from ideality

Open System and Changes of Composition

$$dG = -SdT + Vdp \quad \text{Single Component}$$

When composition change, Gibbs function changes:

$$\therefore G(p, T) \Rightarrow G(p, T, n_1, n_2, n_3, \dots)$$

$$G = \sum_i \mu_i n_i$$

Multi Components

Derivation in
Chap. 5

$$dG = -SdT + Vdp + \sum_i \mu_i dn_i$$

$$\left(\frac{\partial G}{\partial p} \right)_{T, n_1, n_2, \dots} = V \quad \left(\frac{\partial G}{\partial T} \right)_{p, n_1, n_2, \dots} = -S$$

Suppose now that substance 1 is the only substance present.

(Chap. 5)

$G \propto$ the amount of material present

$G = n_1 G_{1m} = n_1 \mu_1$ At constant temperature and pressure,


$$dG = \mu_1 dn_1$$
$$\left(\frac{\partial G}{\partial n_1} \right)_{p,T} = \mu_1$$

Therefore, the chemical potential of species 1 is the measure of how the Gibbs function of species 1 changes when the amount present is varied.

More than One Component:

$$\mu_i(T, p, n_j) \equiv \left(\frac{\partial G}{\partial n_i} \right)_{T, p, n_j} \quad (4) \quad \text{in Chap. 5}$$

(a definition)



Partial Molar Gibbs Free Energy for atom i

= Chemical Potential for atom i

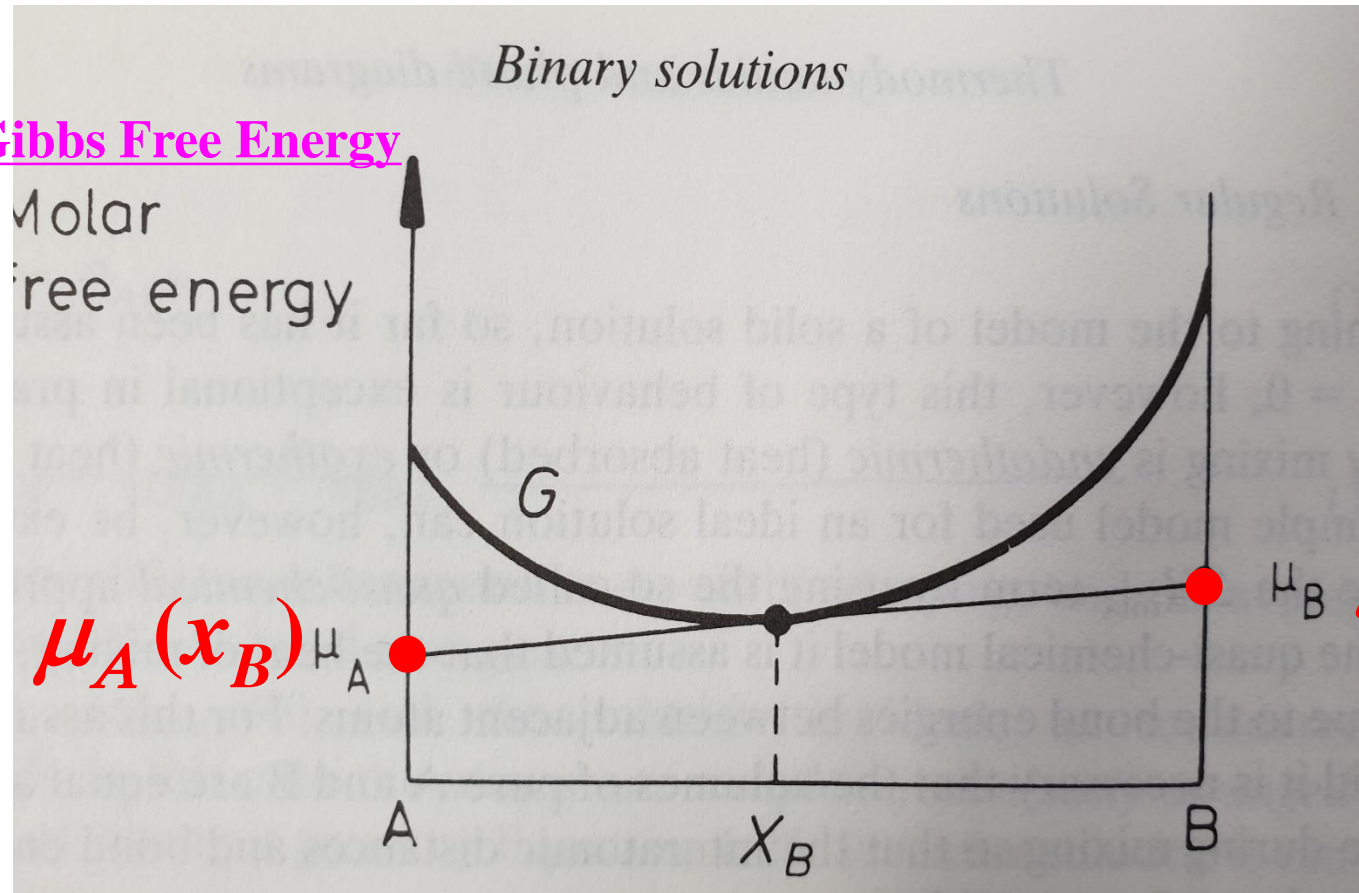
Real Materials + Reversible

$$dG(T, p, n_j) = -SdT + Vdp + \sum_i \mu_i dn_i$$

Derivation in
Chap. 5

the master equation of chemical thermodynamics

Binary Alloy – Location of μ_A and μ_B



$\mu_A(x_B)$

$\mu_B(x_B)$

100% A Atomic % (Mole %) 100% B

Partial Molar Gibbs Free Energy
= Chemical Potential

- Entropy (Boltzmann Formula) *****

- 2nd Law of Thermodynamics *****

Binary Alloy – Location of μ_A and μ_B : (Chap. 5) *****

Problems from Chap. 3

3A.2

3A.1(b) 3A.2(b) 3A.3(b) 3A.5(b) 3A.7(b)

3A.2

3B.8

3D.2

3D.1(b) 3D.2(b) 3D.3(b)

3.2