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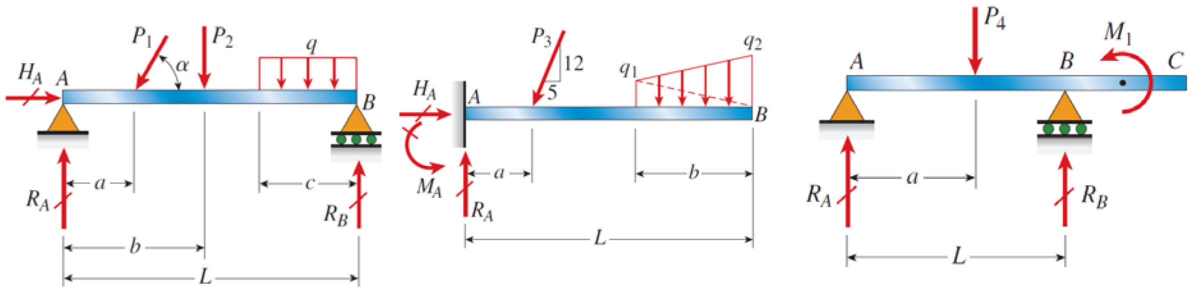
Chapter 4 Shear Forces and Bending Moments

4.2 Types of Beams, Loads, and Reactions

◉ Beams

1. Beams: structural members subjected to l_________ loads (vectors of forces and moments p__________ to the axis of the bar) cf. axially loaded bars, bars in torsion
2. Three types (defined in terms of how they are supported): (a) simply supported beam (or simple beam), (b) cantilever beam, and (c) beam with an overhang



(a) (b) (c)

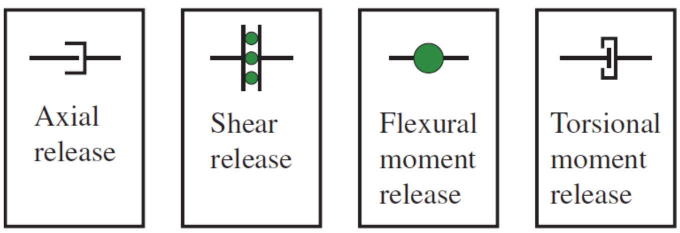
◉ Types of Loads

1. Concentrated load
2. Distributed load
   - Intensity: load per unit __________
   - Uniformly distributed load: $q =$
   - Varying load: $q = q(x)$ e.g. linearly varying load
3. Couple

◉ Reactions – Obtained from equilibrium equations defined at FBDs (See Chapter 1): **Try for the three beam types above**

◉ Releases

1. Inserted into structures to achieve desired performance
2. Add additional equilibrium equations



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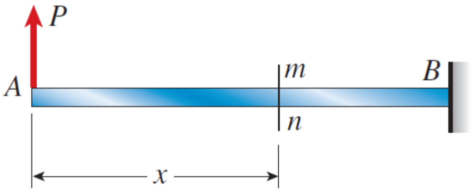
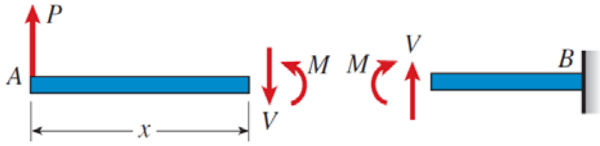
4.3 Shear Forces and Bending Moments

⊙ Stress Resultants in Beams: **Shear Force** (V) and **Bending Moment** (M)

1. Note: no axial force internally if only l_____ forces are applied.

2. Example: Cantilever beam under a concentrated lateral load





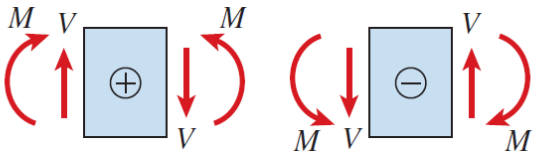
$\sum F_{\text{vert}} =$

$\sum M =$

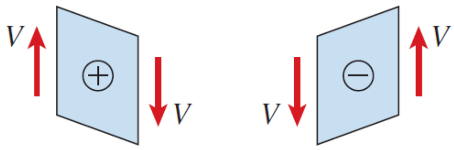
⊙ Sign Conventions

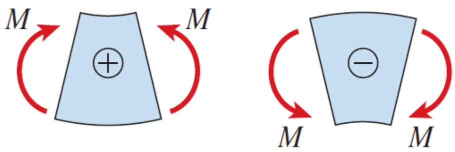
1. See above for positive signs

2. Sign conventions for force and bending moment

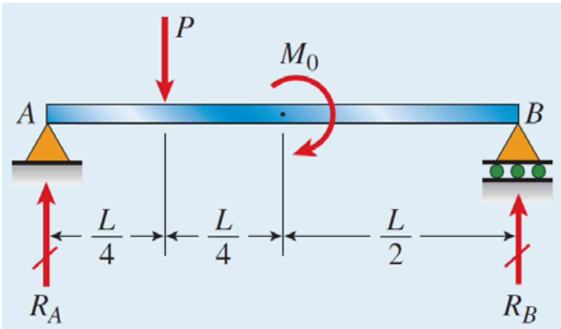


3. Sign conventions for deformation





⊙ **Example 4-1**: For the simple beam, find the shear force and bending moment at the following locations: (a) a small distance to the left of the midpoint, and (b) a small distance to the right of the midpoint



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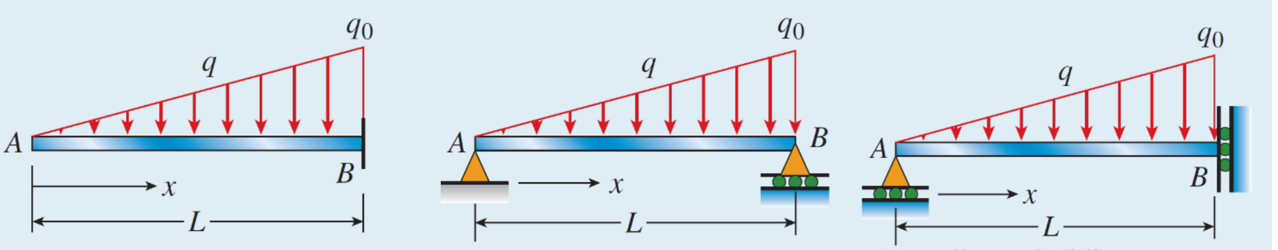
⦿ **Example 4-2**: For the following beams under linearly varying intensity $q(x) = (x/L)q_0$, find expressions for shear force $V(x)$ and bending moment $M(x)$: (a) cantilever beam, (b) simply supported beam, and (c) beam with roller support and sliding support.



(a) Cantilever (b) Simply supported (c) Roller and sliding supports

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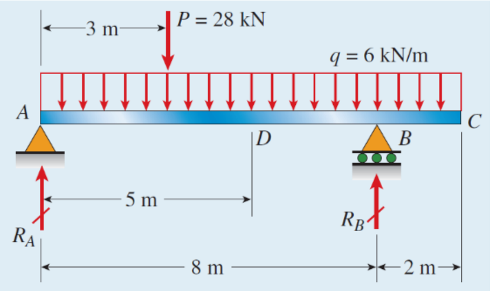
⦿ **Example 4-3**: Simple beam with overhang under uniform load $q = 200\ \text{lb/ft}$ and concentrated load $P = 14\ \text{k}$ (9 ft from the left-hand support). Span length 24 ft, length of overhang 6 ft. Shear and bending moment at cross section D located 15 ft from the left-hand support.



4.4 Relationships between Loads, Shear Forces, and Bending Moments

Note: sign conventions for loads: distributed and concentrated loads are positive when they act downward. Counterclockwise couples are positive.

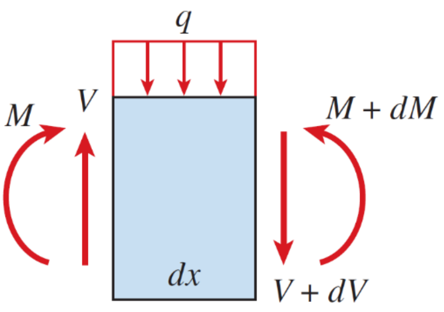
⦿ Distributed Loads

1. Shear force

$$\sum F_{\text{vert}} = 0$$

$$V - q dx - (V + dV) = 0$$

$$\boldsymbol{\frac{dV}{dx} = -q}$$



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By integrating $dV = -qdx$,

$$\int_A^B dV = -\int_A^B qdx$$

$$V_B - V_A = -\int_A^B qdx = -(\text{area of the loading diagram between } A \text{ and } B)$$

2. Bending moment

$$\sum M = 0$$

$$-M - qdx\left(\frac{dx}{2}\right) - (V + dV)dx + M + dM = 0$$

$$\boldsymbol{\frac{dM}{dx} = V}$$

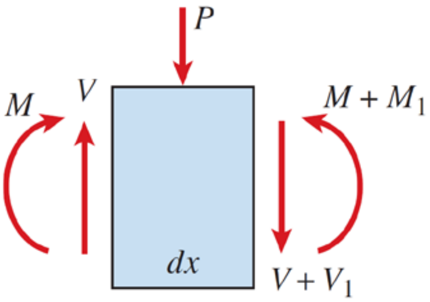
$$M_B - M_A = \int_A^B Vdx = (\text{area of the shear force diagram between } A \text{ and } B)$$

⊙ Concentrated Loads

1. Shear force

$$\sum F_{\text{vert}} = V - P - (V + V_1) = 0$$

$$\boldsymbol{V_1 = -P}$$



2. Bending moment

$$\sum M = -M - P\left(\frac{dx}{2}\right) - (V + V_1)dx + M + M_1 = 0$$

$$\boldsymbol{M_1} = P\left(\frac{dx}{2}\right) + Vdx + V_1dx \cong \boldsymbol{0}$$

Note: No change in the bending moment, but $dM/dx = V$ changes from V to $V + V_1 = V - P$

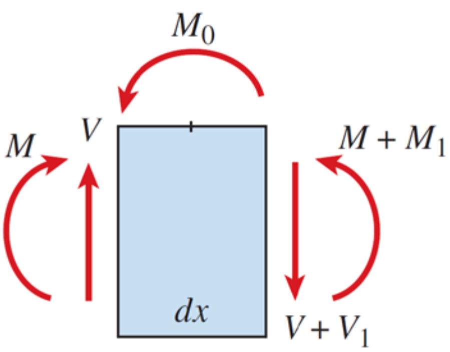
⊙ Loads in the Form of Couples

1. Shear force: $\boldsymbol{V_1 = 0}$

2. Bending moment:

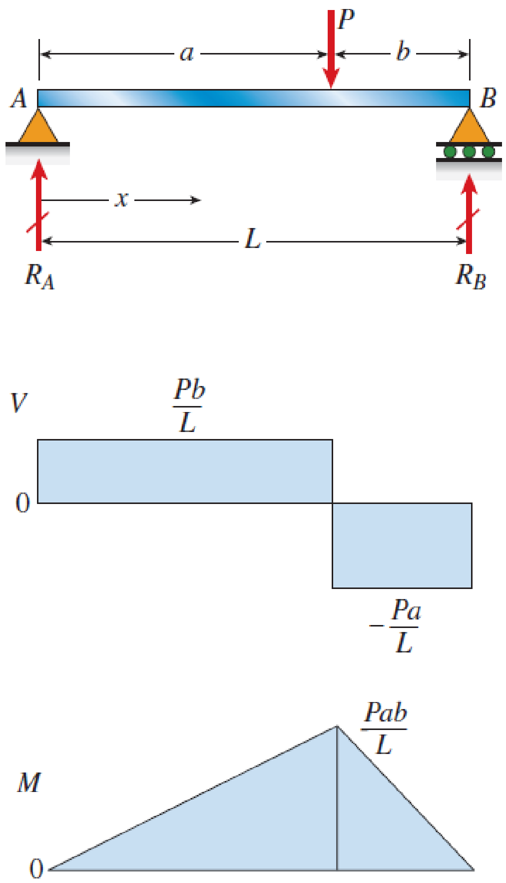
$$-M + M_0 - (V + V_1)dx + M + M_1 = 0$$

$$\boldsymbol{M_1 = -M_0}$$



4.5 Shear-Force and Bending-Moment Diagrams

◉ Concentrated Load



1. Reactions:

$$R_A = \frac{Pb}{L} \qquad R_B = \frac{Pa}{L}$$

2. Shear force and bending moment at small distance to the right of point *A*

$$V = R_A = \frac{Pb}{L} \qquad M = 0$$

3. What happens between *A* and the location of the concentrated load?

$$\frac{dV}{dx} = -q = \quad \text{and} \quad \frac{dM}{dx} = V =$$

4. Therefore, the shear-force remains c_______ and the bending moment increases l________.

5. As we pass the location of the concentrated load,

$$V_1 = -P \rightarrow V = \frac{Pb}{L} - P =$$

$$M_1 =$$

6. Between the location of the concentrated load and point *B*:

$$\frac{dV}{dx} = -q = \quad \text{and} \quad \frac{dM}{dx} = V =$$

7. Therefore, the shear-force remains c________ and the bending moment decreases l_________.

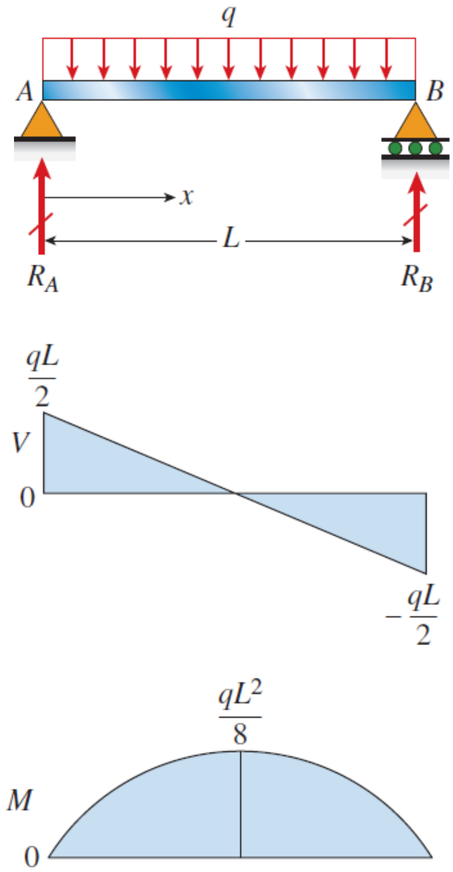
8. Maximum shear force and bending moment (in terms of absolute value):

$$V_{\max} = \qquad\qquad M_{\max} =$$

9. Location of the maximum shear force and bending moment?

10. Area of the shear-force diagram → Change in the bending-moment diagram

⦿ Uniform Load



1. Reactions:

$$R_A = \frac{qL}{2} \qquad R_B = \frac{qL}{2}$$

2. Shear force and bending moment at small distance to the right of point *A*

$$V = R_A = \frac{qL}{2} \qquad M = 0$$

3. What happens after point A?

$$\frac{dV}{dx} = -q \quad \text{and} \quad \frac{dM}{dx} = V$$

4. Therefore, the shear-force decreases l__________ and the bending moment changes as much as the area of the _____________ diagram.

5. In detail, the shear force and bending moments are derived as

$$V(x) = R_A - qx = \frac{qL}{2} - qx$$

$$M(x) = \int_0^x V(x)dx = \frac{qLx}{2} - \frac{qx^2}{2}$$

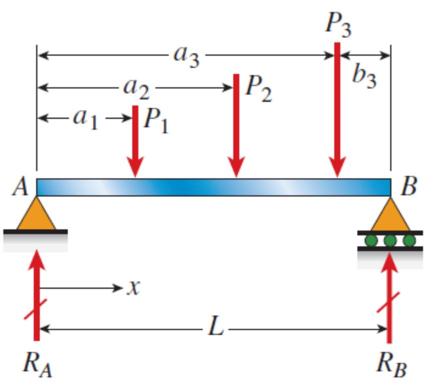
6. Maximum shear force and bending moment:

$$V_{\max} =$$

$$M_{\max} =$$

7. Locations of the maximum shear force and bending moment?

⦿ Several Concentrated Loads



1. Reactions:

$$R_B = \frac{P_1 a_1 + P_2 a_2 + P_3 a_3}{L}$$

$$R_A = P_1 + P_2 + P_3 - R_B$$

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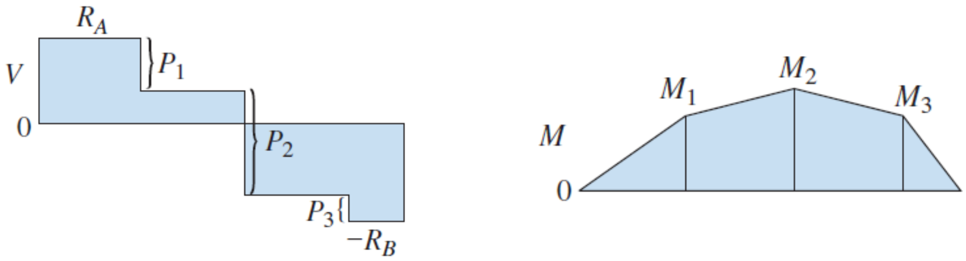
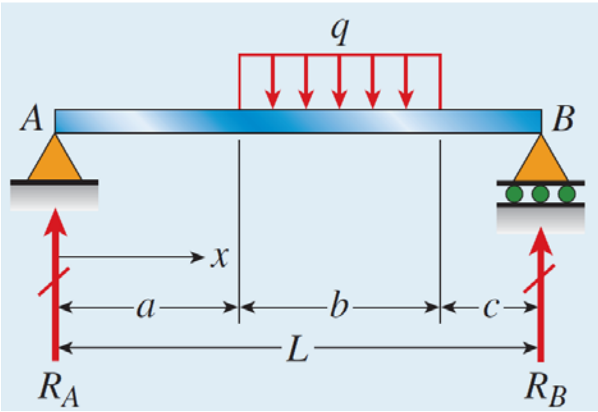
2. Shear-force and bending-moment diagrams



3. Location of maximum bending moment?

⦿ **Example 4-4**: Draw the shear-force and bending-moment diagrams for a simple beam with a uniform load of intensity q acting over part of the span. Determine the maximum bending-moment and location.



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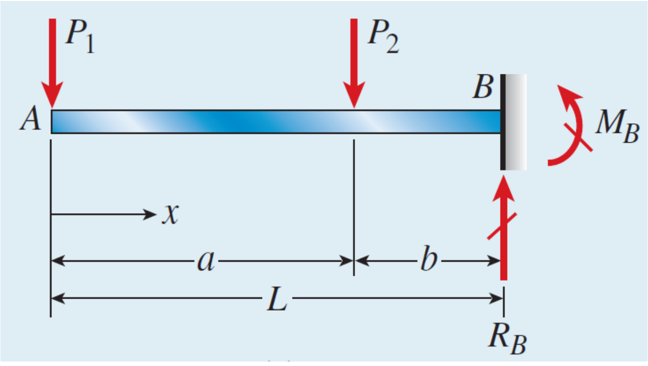
⦿ **Example 4-5**: Draw the shear-force and bending-moment diagrams for a cantilever beam with two concentrated loads. Maximum bending-moment and location?



⦿ **Example 4-7**: A beam with an overhang under uniform load $q = 1.0\ \mathrm{k/ft}$, and couple $M_0 = 12.0$ k-ft. Note $b = 4\mathrm{ft}$, and $L = 16\ \mathrm{ft}$. Shear-force and bending-moment diagrams? Maximum bending-moment?