

# CH. 1

## FUNDAMENTAL PRINCIPLES OF MECHANICS

## 1.2 Generalized Procedure

### ◆ General steps to solve problems in applied mechanics

- i) Select system of interest
- ii) Postulate characteristics of system. This usually involves idealization and simplification of the real situation
- iii) Apply principles of mechanics to the idealized model. Deduce the consequences
- iv) Compare these predictions with the behavior of the actual system. This usually involves recourse to tests and experiments
- v) If satisfactory agreement is not achieved, the foregoing steps must be reconsidered. Very often progress is made by altering the assumptions regarding characteristics of the system, i.e., by constructing a different idealized model of system

→ In this course, steps i), ii), iii) are mainly treated

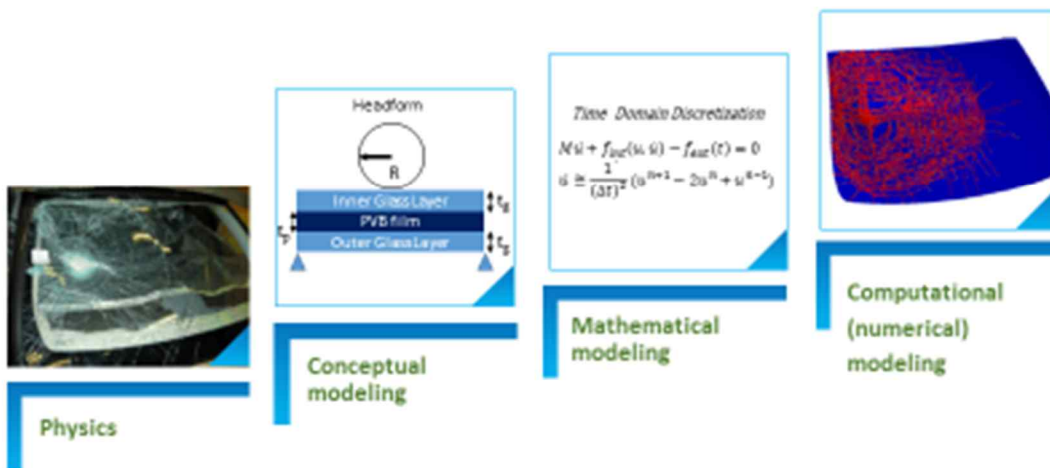
cf. In chapter 1, we apply the principles of mechanics to rigid body

## Definition of CAE



### ■ Computer-Aided Engineering (CAE)?

- The broad usage of computer software to aid in engineering analysis tasks.  
e.g., finite element analysis (FEA), computational fluid dynamics (CFD), multibody dynamics (MBD), and optimization.
- Major providers of information to help support design team in decision making.  
e.g., automotive, aero space, and shipbuilding industries.



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## 1.3 Fundamental Principles of Mechanics

### ◆ Two different types of movement which are important in the mechanics of solids

- i) Gross overall changes in position with time = motion
- ii) Local distortions of shapes = deformation
  - In solid mechanics, we shall consider situations in which there is deformation

### ◆ Analysis of a mechanical system

- i) Study of forces applied to a system
  - Focused on the equilibrium of a system
- ii) Study of motions and deformation in a system
  - Focused on the gross motion and local deformation
- iii) Application of laws relating the forces to the motion and deformation

## 1.4 Concept of Force

### ◆ Concept

- i) The development of the idea of force in mechanics has provided us with an effective means for describing a very complex physical interaction between “bodies” in terms of a simple, convenient concept.
- ii) Newton, in his third law, postulated equal and opposite effectiveness of force on the two interacting systems
 

Force interactions have two principal effects

  - 1) Alter the motion of the systems
  - 2) Deform or distort the shape of the systems
- iii) A very important property of force is that the superposition of forces satisfies the laws of vector addition
- iv) Summary
  - 1) Force is a vector interaction which can be characterized by a pair of equal and opposite vectors having the same line of action
  - 2) The magnitude of a force can be established in terms of a standardized experiment.
  - 3) When two or more forces act simultaneously, at one point, the effect is the same as if a single force equal to the vector sum of the individual forces were acting.

### ◆ Units of forces

- i) SI-unit ~ 1 Newton = 1 kgf·m/s<sup>2</sup>
- ii) United States unit ~ 1 lb·ft/s<sup>2</sup>      1 slug = 32.174 lbm =  $\frac{1 \text{ bf}}{1 \text{ f} / \text{s}^2}$

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**Prefixes for SI system**


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<i>Factor by which unit is multiplied</i>	<i>Prefix</i>	
	<i>Name</i>	<i>Symbol</i>
$10^9$	giga	G
$10^6$	mega	M
$10^3$	kilo	k
$10^{-3}$	milli	m
$10^{-6}$	micro	$\mu$
$10^{-9}$	nano	n

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**Conversion factors**


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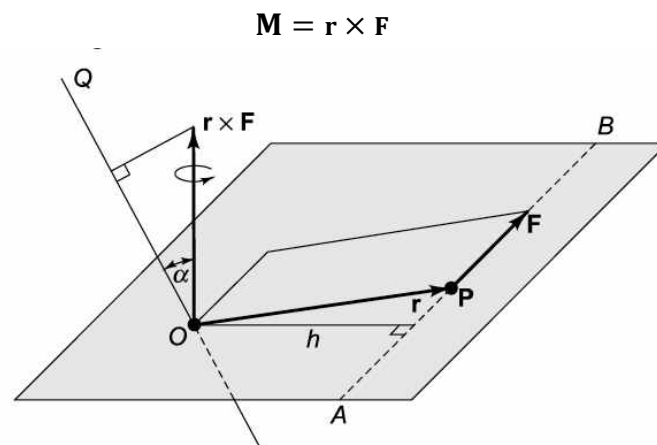
Length:	1 in. = 25.40 mm	1 m = 39.37 in.
	1 ft = 0.3048 m	1 m = 3.281 ft
Force:	1 lbf = 4.448 N	1 N = 0.2248 lb
Pressure:	1 psi = 6.895 kN/m <sup>2</sup>	1 MN/m <sup>2</sup> = 145.0 psi
	1 psf = 47.88 N/m <sup>2</sup>	1 kN/m <sup>2</sup> = 20.88 psf
Distributed load:	1 lb/ft = 14.59 N/m	1 kN/m = 68.53 lb/ft
Moment of force:	1 ft-lb = 1.356 N·m	1 N·m = 0.7376 ft-lb

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## 1.5 Moment of a force

### ◆ Definition

→ The moment or torque of  $\mathbf{F}$  about the point  $O$  is defined as the vector cross product. The resultant vector is perpendicular to the plane composed by  $\mathbf{F}$  and  $\mathbf{r}$ . The sense is determined by the right-hand rule.



**Fig. 1.5** The moment of a force  $\mathbf{F}$  about a point  $O$  is  $\mathbf{r} \times \mathbf{F}$

◆ Magnitude of the 'M'

$$|\mathbf{M}| = |\mathbf{r} \times \mathbf{F}| = F r \sin \phi$$

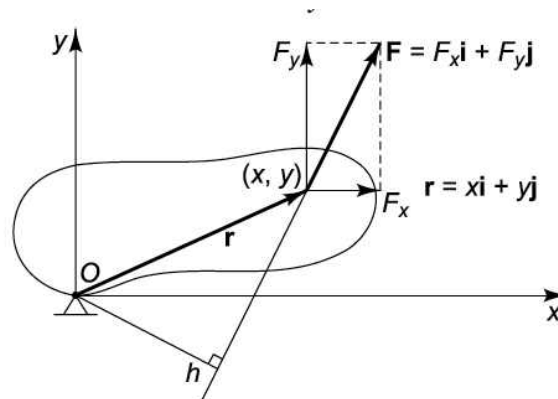
$$= h |\mathbf{F}| = hF$$

→  $|\mathbf{M}|$  is independent of the position of P along AB;

◆ Other expression of the moment (consider two-dimensional structure shown in Fig. 1.7)

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = h |\mathbf{F}| \mathbf{k}$$

$$= (x \mathbf{i} + y \mathbf{j}) \times (F_x \mathbf{i} + F_y \mathbf{j}) = (xF_y - yF_x) \mathbf{k}$$



**Fig. 1.7** Moment about O

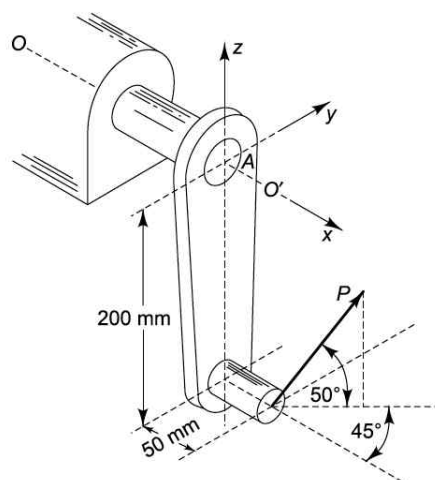
◆ The magnitude of the moment of 'F' about OQ-axis

$$(\mathbf{r} \times \mathbf{F}) \cdot \mathbf{e} = |\mathbf{r} \times \mathbf{F}| \cos \alpha = h |\mathbf{F}| \cos \alpha$$

◆ Units of the moment

→ [m·N] or [ft·lbf]

◆ **Examples 1.1** Determine the moment  $\mathbf{M}$  about the shaft axis  $OO'$  due to the force P applied



**Fig. 1.8** Example 1.1

**Sol)**

$$\mathbf{r} = 50 \mathbf{i} - 200 \mathbf{k}$$

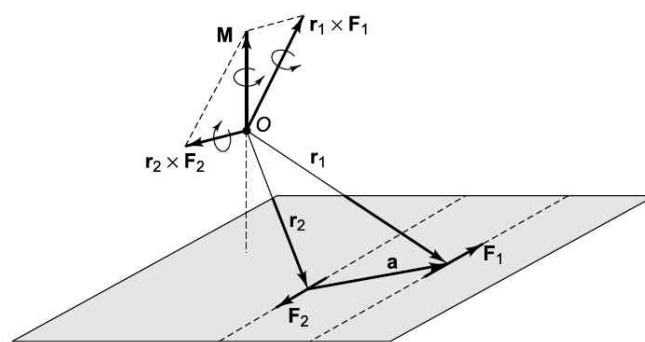
$$\mathbf{P} = P(\cos 50^\circ \cos 45^\circ \mathbf{i} + \cos 50^\circ \sin 45^\circ \mathbf{j} + \sin 50^\circ \mathbf{k})$$

$$\therefore M_{OO'} = \mathbf{i} \cdot [\mathbf{r} \times \mathbf{P}] = 200 P \cos 50^\circ \sin 45^\circ$$

### ◆ Couple

→ Two equal and parallel forces which have opposite sense

$$\begin{aligned} \mathbf{M} &= \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 \\ &= (\mathbf{r}_2 + \mathbf{a}) \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 \\ &= \mathbf{r}_2 \times (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{a} \times \mathbf{F}_1 \\ &= \mathbf{a} \times \mathbf{F}_1 \end{aligned}$$



**Fig. 1.9** The moment of a couple about the point O

∴ It is independent of the location of O

→ The moment of a couple is the same about all points in space

▷ Magnitude of the couple

$$|\mathbf{M}| = h|\mathbf{F}|$$

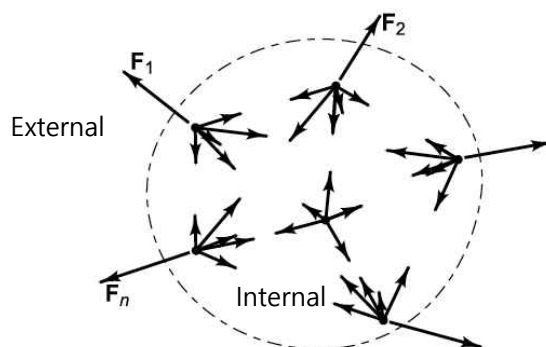
## 1.6 Conditions for equilibrium

### ◆ Equilibrium of a particle

→ According to Newton's law of motion, a particle has no acceleration if the resultant force acting on it is zero. We say that such a particle is in equilibrium.

Necessary and sufficient condition for the equilibrium

$$\mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_n = \sum_j \mathbf{F}_j = \mathbf{0}$$

◆ Equilibrium of particles

**Fig. 1.12** An isolated system of particles showing external and internal forces

- 1) Consider an isolated system of particles as indicated in Fig. 1.12. We say that such a system is in equilibrium if every one of its constituent particles is in equilibrium.
- 2) The forces acting on each particle are of two kinds, external and internal. The internal forces represent interactions with other particles in the system.
- 3) For rigid body ~ A necessary and sufficient condition for a perfectly rigid body to be in equilibrium is that the vector sum of all the external forces should be zero and that the sum of the moments of all the external forces about an arbitrary point (together with any external applied moments) should be zero.

$$\mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_n = \sum_j \mathbf{F}_j = \mathbf{0} \quad (1.5)$$

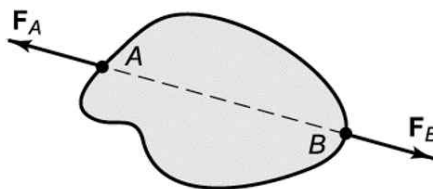
$$\mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \cdots + \mathbf{r}_n \times \mathbf{F}_n = \sum_j \mathbf{r}_j \times \mathbf{F}_j = \mathbf{0} \quad (1.6)$$

- 2) For deformable system ~ A necessary and sufficient condition for the equilibrium of a deformable system is that the sets of external forces which act on the system and on every possible subsystem isolated out of the original system should all be sets of forces which satisfy both (1.5) and (1.6).

▷ Two-force member

→ The line of action of  $\mathbf{F}_B$  must pass through A and the line of action of  $\mathbf{F}_A$  must pass through B.

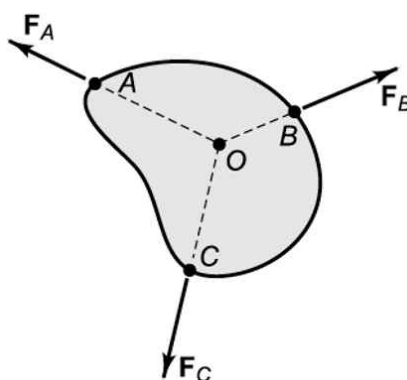
$$\rightarrow \mathbf{F}_A = -\mathbf{F}_B$$



▷ Three-force member

→ The three forces must all lie in the plane ABC and must all intersect in a common point O

∴ The total moment about the intersection of any two of the lines of action should be zero



▷ General two-dimensional equilibrium

$$\begin{aligned} \sum_j F_{jx} &= 0 \\ \sum_j F_{jy} &= 0 \end{aligned} \tag{1.7}$$

$$\sum_j \mathbf{r}_j \times \mathbf{F}_j = 0 \tag{1.8}$$

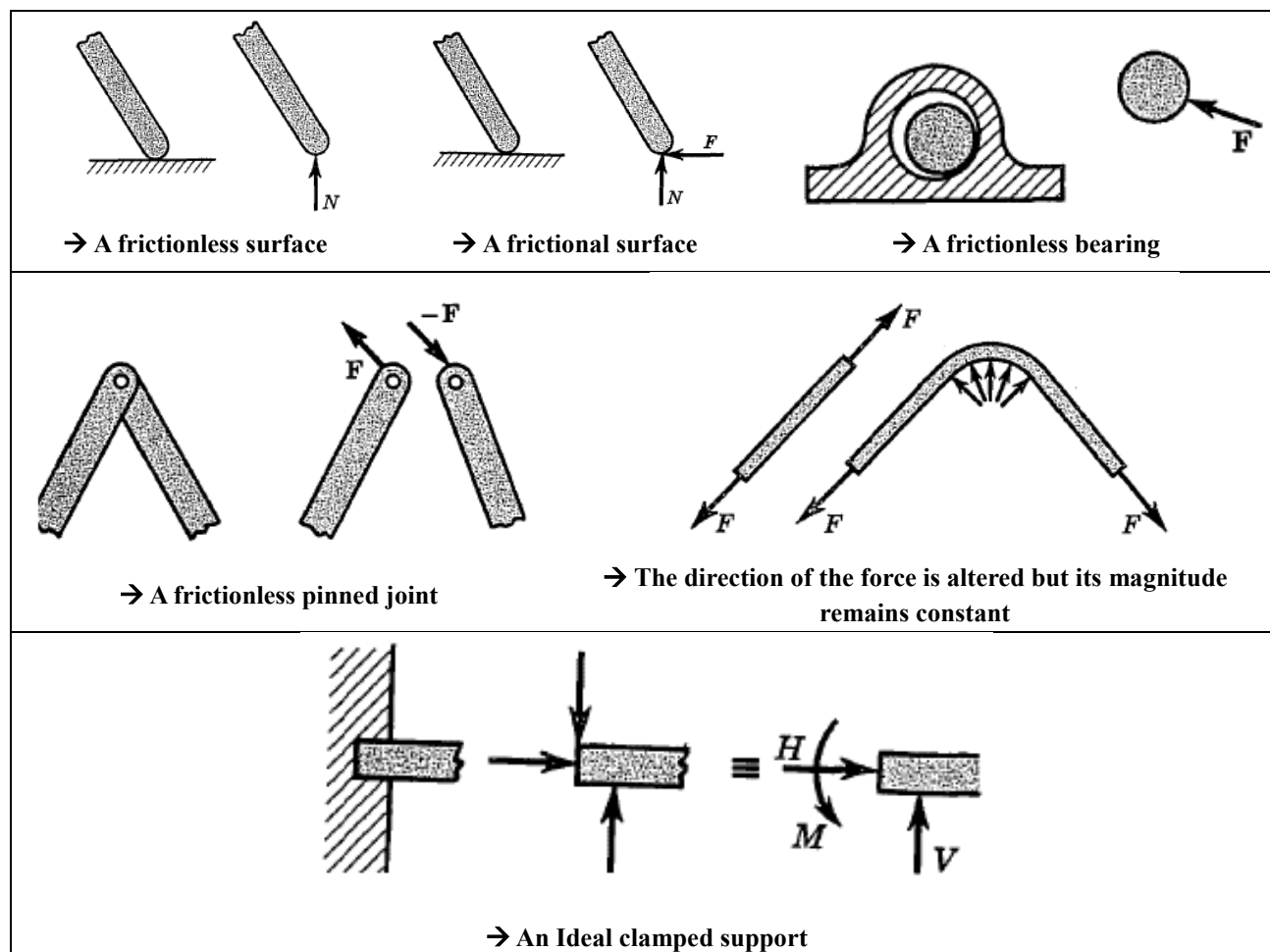
## 1.7 Engineering Applications (**Free-body diagram; Principles of mechanics; Statically (in)determinate system**)

◆ The general method of analysis that is followed throughout this book

- i) Selection of system
- ii) Idealization of system characteristics

**Table 1.2 Force-transmitting properties of some idealized mechanical elements**





iii) These are followed by an analysis based on the principles of mechanics including the following steps:

iii-1) Study of forces and equilibrium requirements  $\rightarrow$  **Static conditions** (Ch. 3)

iii-2) Study of deformation and conditions of geometric fit

iii-3) Applications of force-deformation relations

cf. **Statically determinate system**: Possible to determine all the forces involved by using only the equilibrium requirements without regard to the deformations  $\leftrightarrow$  **Statically Indeterminate system**

## 1.8 Friction

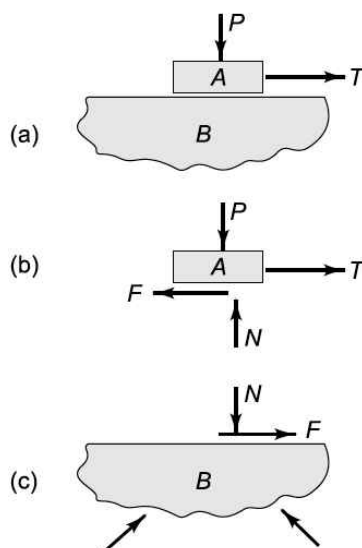
◆ Given pair of surfaces the forces  $F_s$ ,  $F_k$  are proportional to the normal force  $N$

$$F_s = f_s N$$

$$F_k = f_k N$$

where  $f_s$ : static coefficient of friction  $f_k$ : kinetic coefficient of friction.

→ These coefficients are intrinsic properties of the interface between the materials A and B, being determined by the materials A and B and by the state of lubrication or contamination at the interface

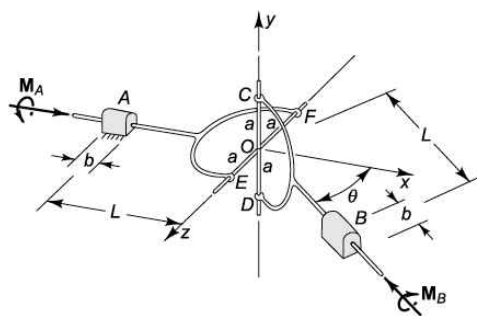


**Fig. 1.18** (a) Body A pressed against B; (b) free-body diagram of body A; (c) free-body diagram of body B

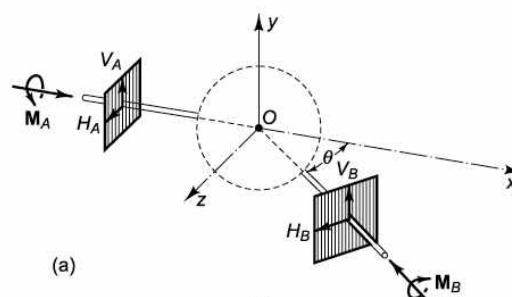
### ◆ Properties of the coefficients of friction

- i) Both coefficients of friction are nearly independent of the area of the interface. In particular, if body A in Fig. 1.18 were tipped up so that only an edge or a corner was in contact with B, we should still find approximately the same coefficients of friction. Note that under these circumstances the tangential and normal directions are determined only by the surface of B.
- ii) Both coefficients are nearly independent of the roughness of the two surfaces, although this is a conclusion which many people find hard to accept.
- iii) The static coefficient  $f_s$  is nearly independent of the time of contact of the surfaces at rest. Similarly, the kinetic coefficient  $f_k$  is nearly independent of the relative velocity of the two surfaces. Figure 1.19 shows a schematic representation of typical static-friction-time and kinetic-friction-velocity plots.

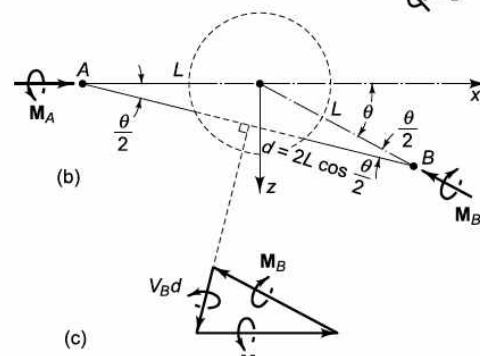
## 1.10 Hooke's Joint (Reading homework)



**Fig. 1.32** Hooke's joint mechanism



(a)



(b)

(c)

**Fig. 1.33** Isolation of entire mechanism

A Comparison an incorrect conclusion of the over-idealized model with a correct solution.

### ◆ Incorrect solution.

Considering the conditions of force balance from Fig.1.33a

$$H_A = H_B = 0$$

$$-V_A = V_B$$

From Fig. 1.33c

$$M_A = M_B$$

(a)

Alternatively, if we proceed directly with moment equilibrium about the point O, we have

$$M_A \mathbf{i} - M_B (\mathbf{i} \cos \theta + \mathbf{k} \sin \theta) - L \mathbf{i} \times V_A \mathbf{j} - L (\mathbf{i} \cos \theta + \mathbf{k} \sin \theta) \times V_A \mathbf{j} = 0$$

Working out the cross products,

$$V_A L (1 + \cos \theta) = -M_A \sin \theta$$

$$M_A + V_A L \sin \theta = M_B \cos \theta$$

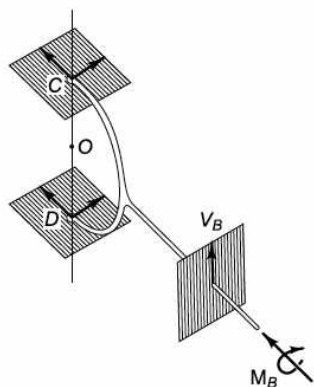
The result as above is

$$V_A L = \frac{-M_A \sin \theta}{1 + \cos \theta} \quad * \text{Incorrect solution.}$$

Suppose we were to continue our analysis further by considering the shaft B separately as in Fig. 1.34.

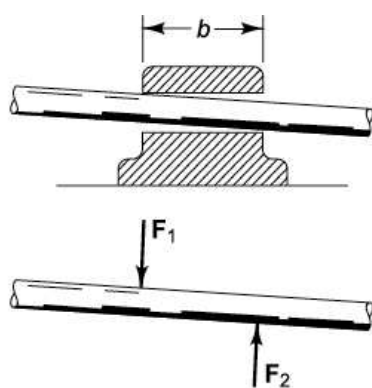
The system of Fig. 1.34. can't possibly be in equilibrium because there is nothing to balance  $V_B$  in the vertical

direction.

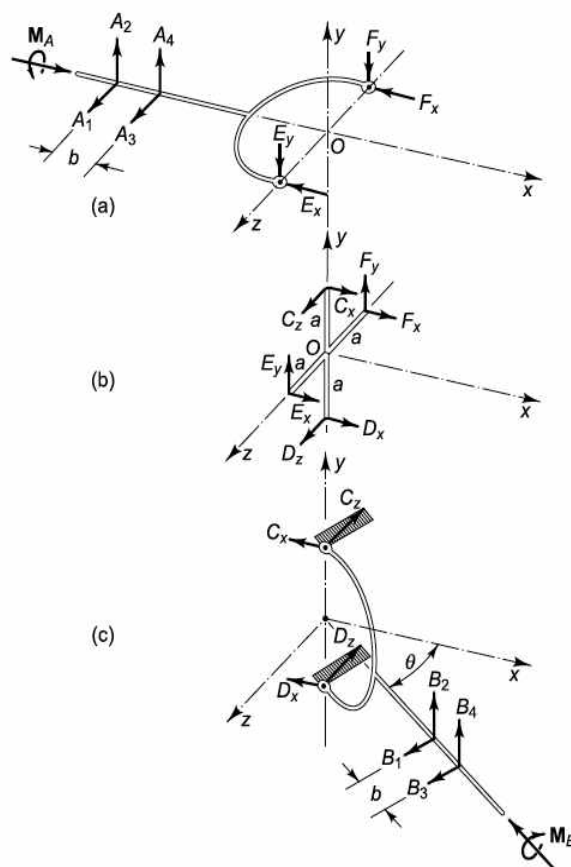


**Fig. I.34** Isolated free-body diagram of the shaft B

◆ Correct solution.



**Fig. I.35** When a shaft cocks slightly in a long bearing, there is double contact with two reactions  $F_1$  and  $F_2$



**Fig. I.36** Free-body diagrams of the three parts of the Hooke's joint

To provide for the double-contact type of reaction shown in Fig. 1.35, we have four components  $A_1$ ,  $A_2$ ,  $A_3$ , and  $A_4$  at bearing A and the four components  $B_1$ ,  $B_2$ ,  $B_3$ , and  $B_4$  at bearing B.

- From fig. 1-36(b)~

The statements of force and moment balance for the cross (Fig. 1.36b) are

$$\begin{aligned}\sum \mathbf{F} &= (C_x + D_x + E_x + F_x)\mathbf{i} + (E_y + F_y)\mathbf{j} + (C_z + D_z)\mathbf{k} = 0 \\ \sum \mathbf{M}_0 &= (aC_z - aD_z - aE_y + aF_y)\mathbf{i} + (aE_x - aF_x)\mathbf{j} + (-aC_x + aD_x)\mathbf{k} = 0\end{aligned}\quad (b)$$

Setting each component separately equal to zero,

$$\begin{aligned}C_x &= D_x = -E_x = -F_x \\ C_z &= -D_z = E_y = -F_y\end{aligned}\quad (c)$$

- From fig. 1-36 (a) ~

In Fig. 1.36a, equilibrium of forces parallel to x yields  $F_x = 0$  and equilibrium of moments about the x axis yields  $2aF_y = M_A$ .

$$\begin{aligned}C_x &= D_x = -E_x = -F_x = 0 \\ C_z &= -D_z = E_y = -F_y = -\frac{M_A}{2a}\end{aligned}\quad (d)$$

For completeness, considering the other equilibrium conditions in Fig 1.36a

$$A_1, A_2 = A_3 = A_4 = 0$$

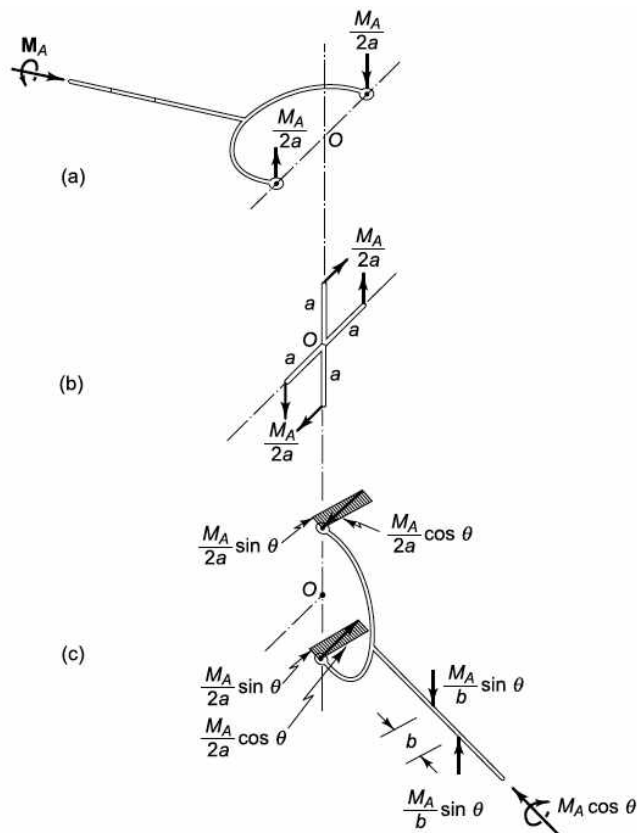
- From fig. 1-36 (c)~

Using values (d) in Fig. 1.36c

$$\begin{aligned}M_B - aD_z \cos \theta + aC_z \cos \theta &= 0 \\ M_B &= M_A \cos \theta\end{aligned}\quad (e)$$

For completeness, considering the other equilibrium conditions in Fig 1.36c

$$\begin{aligned}B_1 &= B_3 = 0 \\ B_4 &= -B_2 = \frac{M_A}{b} \sin \theta\end{aligned}$$



**Fig. 1.37** Equilibrium solution of the free-body diagrams of Fig. 1.36

To aid visualization the complete solution is shown in Fig. 1.37.

It is important to emphasize that our solution (e) is for the configuration shown in Fig. 1.32.

An exact solution for arbitrary angle of orientation  $\phi$  of shaft A measured from EF in the direction of twist of  $M_A$  (Fig. 1.32) can be found. The result is

$$M_B = \frac{\sin^2 \phi + \cos^2 \theta \cos^2 \phi}{\cos \theta} M_A \quad (f)$$

When  $\phi = 0$ , the result (f) reduces to (e).