

201 12 28 6½ 2½ ← Dynamic Analysis and
Response of Linear systems.

Modal Analysis (Mode superposition Method)

7 Linear Structural Dynamics

$$\underline{m} \dot{\underline{y}} + \underline{c} \dot{\underline{y}} + \underline{k} \underline{y} = \underline{p} \text{ (f)}$$

$$\text{model: } \left\{ \begin{array}{l} y = \underline{\underline{y}}_1, \quad \dot{y} = \underline{\underline{\dot{y}}}_1, \quad \ddot{y} = \underline{\underline{\ddot{y}}}_1 \\ \text{expansion} \end{array} \right.$$

$$y = \sum_{r=1}^N \phi_r \times g_r$$

$$\phi_T \times [m \pm \frac{\epsilon}{2} + \frac{c}{2}, m \pm \frac{\epsilon}{2}] = P(t)$$

$$M_m \ddot{\theta}_m + C_m \dot{\theta}_m + K_m \theta_m = P_m(t) \quad (m=1, 2, \dots, N)$$

Orthogonal
damping $\gamma t^{2/3}$

$$\ddot{\delta}_m + 2\zeta_m \omega_m \dot{\delta}_m + \omega_m^2 = \frac{P_m(t)}{M_m} = \frac{\Phi_m^T P(t)}{\Phi_m^T M \Phi} \quad (m=1, 2, \dots, N)$$

To write decoupled equations \Rightarrow

(i) Displacement response (at any time)

$$\underline{y}(t) = \sum_{m=1}^N \underline{\varphi}_m(t) = \sum_{m=1}^N \underline{\phi}_m \times \underline{\varepsilon}_m(t)$$

Superpose!

(iii) Element for as

$$\text{iii) Element forces} \quad \downarrow$$

$\left[\begin{array}{l} \text{element stiffness matrix} \\ \text{① } F_m(t) = k_e \cdot u_m(t) = k_e \Phi_m \varphi_m(t), \quad F(t) = \sum_m F_m(t) \end{array} \right]$

② "Equivalent static force approach"

$$f_m(\theta) = \frac{h}{m} U_m(\theta) = \omega_m^2 m \times \Phi_m Q_m(\theta)$$

$$= \omega_m^2 \underline{m} \oplus_m \times^g b_m(t)$$

221?

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01 2123 vector on cylinder 23 3 8 11 12 13 14 15 16 17 18 19

modal analysis : summary

matrix or
finite element
formulation

1. Define the structural properties: \underline{M} , $\underline{\underline{E}}$, \underline{S}_m

2. Determine ω_n and ϕ_m ← Eigenvalue analysis

3. Compute the response in each mode: $\bar{q}_m(t)$

4. Combine the contributions of all the modes

to determine the total responses, $\bar{y}(t)$ and $\bar{r}(t)$.
 ↳ 12.3 ~ 12.6 232
 (easy to follow)

Modal Response Contributions

Chopra 23.9
unique & \underline{S} (not
really?)

12.8 Modal expansion of excitation vector $\underline{P}(t) = \underline{S} \times p(t)$

12.8

(extra)

$$\underline{P}(t) = \underline{S} \times p(t) \quad \text{(constant vector)}$$

$$\left(\begin{array}{l} \underline{P}(t) = -\underline{M} \{1\} \ddot{u}_g(t) \\ \underline{S} = -\underline{M} \{1\}, \quad p(t) = \ddot{u}_g(t) \end{array} \right) \quad \text{support motion } \ddot{u}_g(t)$$

Note:

$$\ddot{u}_m = \underline{\Phi}_m \ddot{\varphi}_m(t)$$

$$(f_I)_m = -\underline{M} \ddot{u}_m(t) = -\underline{M} \underline{\Phi}_m \ddot{\varphi}_m(t)$$

* Expand the vector \underline{S} as

$$\underline{S} = \sum_{r=1}^N S_r = \sum_{r=1}^N \Gamma_r \underline{m} \underline{\Phi}_r \quad \text{--- (*)}$$

which scalar?

$$\begin{aligned} \underline{\Phi}_m^T \cdot \underline{S} &= \underline{\Phi}_m^T \cdot \left(\sum_{r=1}^N \Gamma_r \underline{m} \underline{\Phi}_r \right) \\ &= \Gamma_m (\underline{\Phi}_m^T \underline{m} \underline{\Phi}_m) = \Gamma_m \times M_m \end{aligned}$$

Orthogonality
of $\underline{\Phi}_m$

$$\therefore \underline{\Phi}_m^T \underline{m} \underline{\Phi}_m = \Gamma_m \times M_m$$

Example)
p. 483
Fig. 12.8.1 ~ 12.8.3

$$\therefore \Gamma_m = \frac{\underline{\Phi}_m^T \underline{S}}{M_m} = \frac{\underline{\Phi}_m^T \underline{S}}{\underline{\Phi}_m^T \underline{m} \underline{\Phi}_m}$$

$$\therefore \underline{S}_m = \Gamma_m \times \underline{m} \underline{\Phi}_m \quad \text{the contribution of the } n^{\text{th}} \text{ mode to } \underline{S}$$

$$(**) \text{ 1401 } \frac{\partial \mathbf{u}}{\partial t} = \frac{1}{2} \mathbf{u}^T \mathbf{M}^{-1} \mathbf{u} - \frac{1}{8} \mathbf{u}^T \mathbf{M} \quad \leftarrow \underline{\underline{S}} = \sum \underline{\underline{S_r}} \\ = \sum \underline{\underline{F_r}} \underline{\underline{m}} \underline{\underline{\Phi}}$$

(3)

$$\underline{\underline{P}}(t) = \underline{\underline{S}} \times \underline{\underline{p}}(t)$$

$$= [\underline{\underline{S_1}} + \underline{\underline{S_2}} + \dots + \underline{\underline{S_m}} + \dots + \underline{\underline{S_N}}] \times \underline{\underline{p}}(t)$$

$$= [\underbrace{\underline{\underline{S_1}} \times \underline{\underline{p}}(t)}_{\underline{\underline{P_1}}(t)} + \dots + \underbrace{\underline{\underline{S_m}} \times \underline{\underline{p}}(t)}_{\underline{\underline{P_m}}(t)} + \dots + \underbrace{\underline{\underline{S_N}} \times \underline{\underline{p}}(t)}_{\underline{\underline{P_N}}(t)}]$$

" $\underline{\underline{S_m}} \times \underline{\underline{p}}(t) = \underline{\underline{P_m}}(t)$ 는 대체로 Σ 를 대체하는 원리"

$$\therefore \underline{\underline{m}} \ddot{\underline{\underline{y}}} + \underline{\underline{c}} \dot{\underline{\underline{y}}} + \underline{\underline{k}} \underline{\underline{y}} = \underline{\underline{S}} \underline{\underline{p}}(t)$$

$$= [\underbrace{\underline{\underline{F_1}} \underline{\underline{m}} \underline{\underline{\Phi_1}}}_{\underline{\underline{S_1}}} + \dots + \underbrace{\underline{\underline{F_N}} \underline{\underline{m}} \underline{\underline{\Phi_N}}}_{\underline{\underline{S_N}}}] \times \underline{\underline{p}}(t)$$

$$\underline{\underline{\Phi}}_m^T [\underline{\underline{m}} \ddot{\underline{\underline{y}}} + \underline{\underline{c}} \dot{\underline{\underline{y}}} + \underline{\underline{k}} \underline{\underline{y}}] = \{ \underline{\underline{\Phi}}_m^T \underline{\underline{F_m}} \underline{\underline{m}} \underline{\underline{\Phi_m}} \} \times \underline{\underline{p}}(t) = \underline{\underline{F_m}} \underline{\underline{M_m}} \underline{\underline{p}}(t)$$

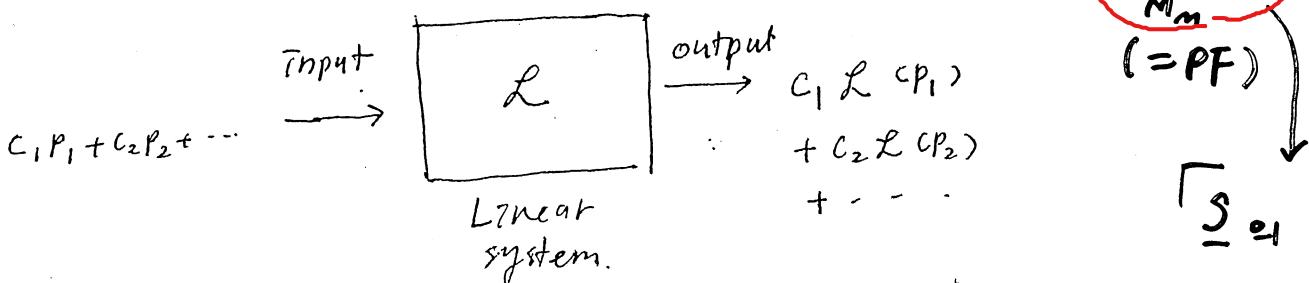
$$\underline{\underline{M_m}} \ddot{\underline{\underline{\delta_m}}}(t) + \underline{\underline{C_m}} \dot{\underline{\underline{\delta_m}}}(t) + \underline{\underline{K_m}} \underline{\underline{\delta_m}}(t) = \underline{\underline{\Phi}}_m^T \underline{\underline{S}} \underline{\underline{m}} \times \underline{\underline{p}}(t) = \underline{\underline{F_m}} \underline{\underline{M_m}} \underline{\underline{p}}(t) \quad \dots (**)$$

$\ddot{\underline{\underline{\delta_m}}}(t) + 2\underline{\underline{\zeta_m}} \dot{\underline{\underline{\delta_m}}}(t) + \underline{\underline{\omega_m^2}} \underline{\underline{\delta_m}}(t) = \underline{\underline{\Phi}}_m^T \underline{\underline{S}} \underline{\underline{m}} \times \underline{\underline{p}}(t)$

force vector $\underline{\underline{S}} \underline{\underline{m}} \times \underline{\underline{p}}(t)$ on $\underline{\underline{\delta_m}}$

particular solution $\ddot{\underline{\underline{\delta_m}}}_p(t) = \underline{\underline{\Phi}}_m^T \underline{\underline{S}} \underline{\underline{m}} \times \underline{\underline{p}}(t)$

$\ddot{\underline{\underline{\delta_m}}}_p(t) = \underline{\underline{\Phi}}_m^T \underline{\underline{S}} \underline{\underline{m}} \times \underline{\underline{p}}(t)$



12-9 Modal analysis for $\underline{\underline{P}}(t) = \underline{\underline{S}} \underline{\underline{p}}(t)$

(**) 의 원칙에 M_m 을 대체하는 원칙,

$\underline{\underline{\Phi}}_m$ on m
projection
 c_1 of Φ_1
 \vdots

$$\ddot{\gamma}_m + 2\zeta_m \omega_m \dot{\gamma}_m + \omega_m^2 \gamma_m = \Gamma_m P(f) \quad \text{--- (12.9.1)}$$

~~\neq~~ modal participation factor ($\frac{\phi_m}{\sqrt{\lambda_m}}$ PF)

$$= \frac{\phi_m^T S}{\phi_m^T M \phi_m}$$

↑
 \$\phi_m\$ မှ ဒုက္ခန်းရဲ့ ပုံစံများ
 အတွက် ပုံစံ
 ပုံစံများ အတွက်

$\frac{1}{\sqrt{\lambda_m}}$ " "

To utilize many available

results for soft systems,

$$\ddot{D}_m + 2\zeta_m \omega_m \dot{D}_m + \omega_m^2 D_m = P(t) \quad \text{--- (12.4.4)}$$

"Unit" excitation.

$$f_m(t) = \sum D_m(t) \quad \dots \quad (12.9.3)$$

\dagger of $\frac{1}{2}$ as scale factor

0386 (0101 Gen. 500 ft \pm mm σ^2_{ss})

$$\underline{y}_m = \underline{\phi}_m \times \Gamma_m D_m(t) \quad \leftarrow \text{the } n^{\text{th}} \text{ mode contribution to } \underline{y}(t)$$

The equivalent ^{method} static forces,

$$f_m(t) = k \underline{y}_m(t) = \frac{\cancel{k}}{\cancel{t}} \underline{w}_m^2 \underline{m} \underline{\phi}_m \Gamma_m D_m(t) = \cancel{\underline{w}_m^2 \underline{m} \underline{\phi}_m} \Gamma_m D_m(t)$$

\underline{S}_m

$$= S_m \times (\omega_m^2 D_m(t)) \quad \dots (12.9.5)$$

$\cancel{t} \underline{A}_m(t)$ if EQ input.

\rightarrow Amplitude + $\text{ca} \cdot \text{p} \dots$
 The contribution $r_m(t)$ to any response

The m^{th} mode contribution $r_m(t)$ to any response quantity $r(t)$ is determined by static analysis of the structure subjected to forces $f_m(t)$.

↳ 见书 p. 488 例 3 , $r_m(t) = (r_m^{st} \times w_m^2 d_m(t))$

(5)

$$r(t) = \sum_{n=1}^N r_m q_n = \sum_{n=1}^N r_m^{st} [\omega_n^2 b_m(t)] \quad \text{--- (12.9.7)}$$

Summary : (1) The modal analysis procedure presented has the advantage of providing a basis for identifying and understanding the factors that influence the relative modal contributions to the response.

(2) Modal analysis requires static analysis of the structure for N sets of external forces, \underline{s}_m , $m=1, 2, \dots, N$, and dynamic analysis of N different SDF systems.

Combining the modal responses gives the dynamic response of the structure.



12.10

Modal contribution factors, \bar{F}_m

$$\bar{F}_m = \frac{\underline{r}_m \cdot \underline{s}_m}{\|\underline{r}_m\| \|\underline{s}_m\|} \quad \text{unit vector}$$

$$\underline{P}(t) = \underline{s} \times \underline{p}(t) = \underline{s} \quad \leftarrow \begin{array}{l} \text{unit loading all} \\ \text{eigenvectors} \end{array}$$

Loading : $\underline{s} = s_1 + s_2 + \dots + s_N$ $\Rightarrow \therefore \bar{F}_m = \frac{\underline{r}_m^{st}}{\underline{r}^{st}}$

Response : $\underline{r}^{st} = r_1^{st} + r_2^{st} + \dots + r_N^{st}$

- ① $\underline{r}_1^{st} \cdot \underline{s}_1 = 1.0$ (210%)
- ② $\underline{r}_2^{st} \cdot \underline{s}_2 = 0.0$ (210%)
- ③ $\sum \bar{F}_m = 1.0$ (210%)

Ch. 12 21/211 : ~~P.12.1 (d) (ii)~~ modal analysis

12.2

Premultiplying both sides of Eq. (12.8.2) by ϕ_n^T and utilizing the orthogonality property of modes gives

$$\underline{\phi}^T \underline{s} = \underline{\phi}_n^T \sum \Gamma_r \underline{m} \underline{\phi}_r = \Gamma_n \times M_n; \quad \Gamma_n = \left(\frac{\phi_n^T s}{M_n} \right) \text{ mode participation factor} \quad (12.8.3)$$

The contribution of the n th mode to s is

$$s_n = \Gamma_n m \phi_n \quad (12.8.4)$$

which is independent of how the modes are normalized. This should be clear from the structure of Eqs. (12.8.3) and (12.8.4).

(12.8.2) $\underline{\phi}_n^T \underline{s}$ in Eq. 12.8.3 Equation (12.8.2) may be viewed as an expansion of the distribution s of applied forces in terms of inertia force distributions s_n associated with natural modes. This interpretation becomes apparent by considering the structure vibrating in its n th mode with accelerations $\ddot{\mathbf{u}}_n(t) = \ddot{q}_n(t) \phi_n$. The associated inertia forces are

$$(\mathbf{f}_I)_n = -\mathbf{m}\ddot{\mathbf{u}}_n(t) = -\mathbf{m} \phi_n \ddot{q}_n(t) \quad \begin{array}{l} \text{m가 BEEZ 진동식이} \\ \text{계수} \end{array}$$

and their spatial distribution, given by the vector $\mathbf{m}\phi_n$, is the same as that of s_n .

Eqs. (12.8.2) $\underline{\phi}_n^T \underline{s}$ in Eq. 12.8.3 The expansion of Eq. (12.8.2) has two useful properties: (1) the force vector $s_n p(t)$ produces response only in the n th mode but no response in any other mode; and (2) the dynamic response in the n th mode is due entirely to the partial force vector $s_n p(t)$ (see Derivation 12.1).

P.485 To study the modal expansion of the force vector $sp(t)$ further, we consider the structure of Fig. 12.8.1: a five-story shear building (i.e., flexurally rigid floor beams and slabs) with lumped mass m at each floor, and same story stiffness k for all stories.

Floor Mass Story Stiffness

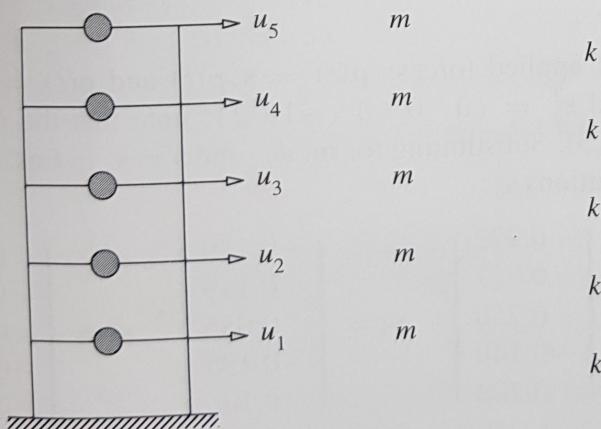


Figure 12.8.1 Uniform five-story shear building.

The mass and stiffness matrices of the structure are

$$\mathbf{m} = m \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix} \quad \mathbf{k} = k \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{bmatrix}$$

Determined by solving the eigenvalue problem, the natural frequencies are

From Eigen solution →

$$\omega_n = \alpha_n \left(\frac{k}{m} \right)^{1/2}$$

where $\alpha_1 = 0.285$, $\alpha_2 = 0.831$, $\alpha_3 = 1.310$, $\alpha_4 = 1.682$, and $\alpha_5 = 1.919$. For a structure with $m = 100$ kips/g, the natural vibration modes, which have been normalized to obtain $M_n = 1$, are (Fig. 12.8.2)

$$\phi_1 = \begin{Bmatrix} 0.334 \\ 0.641 \\ 0.895 \\ 1.078 \\ 1.173 \end{Bmatrix} \quad \phi_2 = \begin{Bmatrix} -0.895 \\ -1.173 \\ -0.641 \\ 0.334 \\ 1.078 \end{Bmatrix} \quad \phi_3 = \begin{Bmatrix} 1.173 \\ 0.334 \\ -1.078 \\ -0.641 \\ 0.895 \end{Bmatrix} \quad \phi_4 = \begin{Bmatrix} -1.078 \\ 0.895 \\ 0.334 \\ -1.173 \\ 0.641 \end{Bmatrix} \quad \phi_5 = \begin{Bmatrix} 0.641 \\ -1.078 \\ 1.173 \\ -0.895 \\ 0.334 \end{Bmatrix}$$

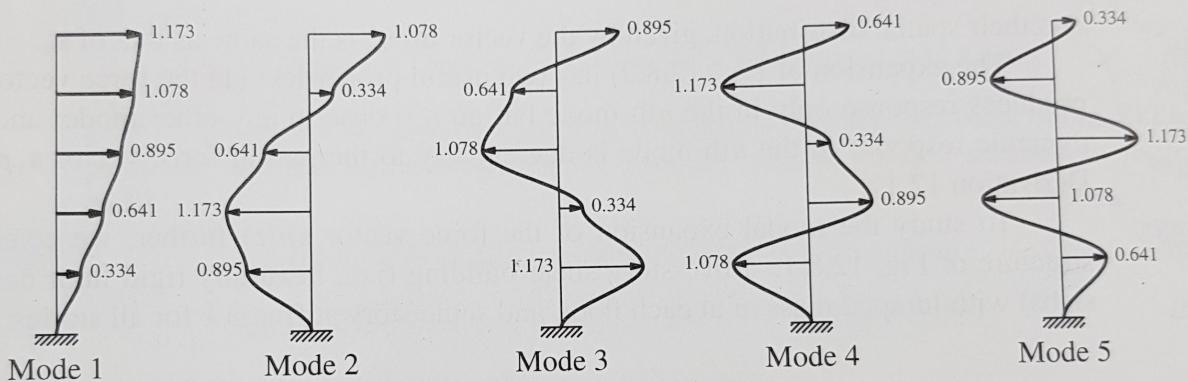


Figure 12.8.2 Natural modes of vibration of uniform five-story shear building.

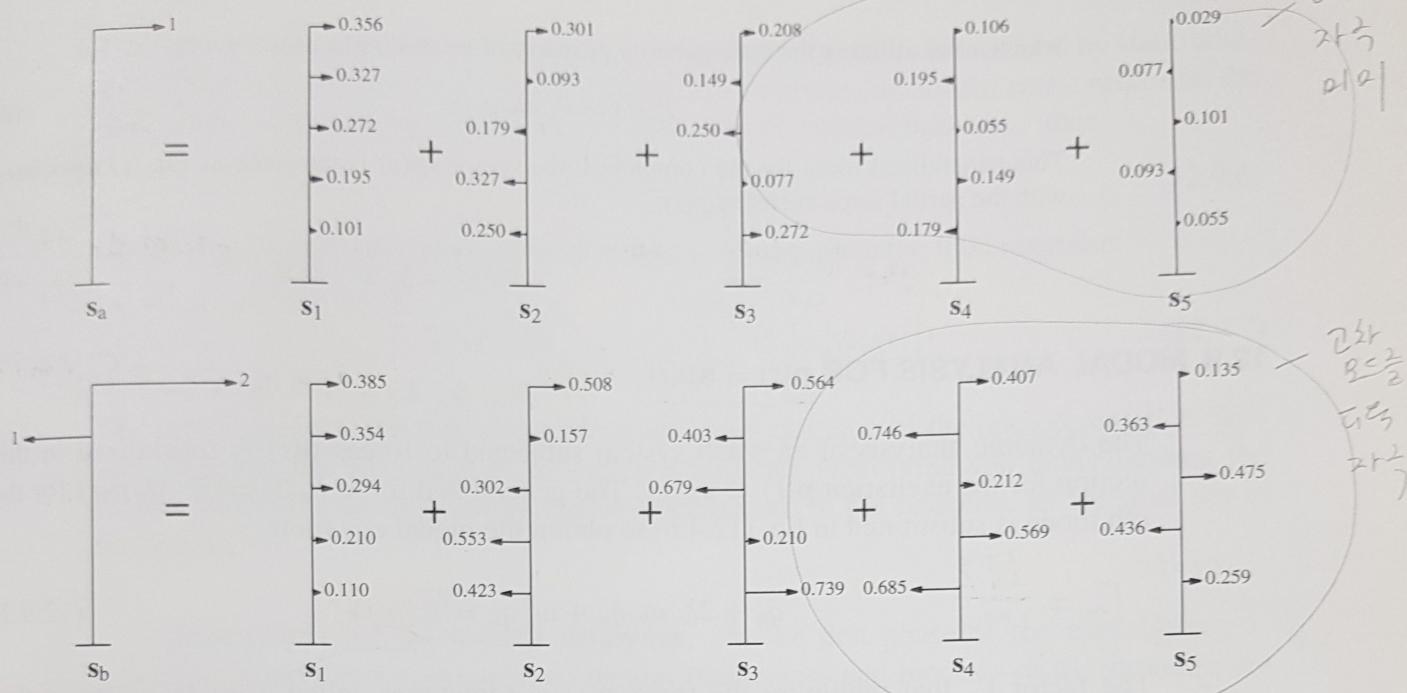
12V8E
 Consider two different sets of applied forces: $\mathbf{p}(t) = \mathbf{s}_a p(t)$ and $\mathbf{p}(t) = \mathbf{s}_b p(t)$, where $\mathbf{s}_a^T = \langle 0 \ 0 \ 0 \ 0 \ 1 \rangle$ and $\mathbf{s}_b^T = \langle 0 \ 0 \ 0 \ -1 \ 2 \rangle$; note that the resultant force is unity in both cases (Fig. 12.8.3). Substituting for \mathbf{m} , ϕ_n , and $\mathbf{s} = \mathbf{s}_a$ in Eqs. (12.8.4) and (12.8.3) gives the modal contributions \mathbf{s}_n :

$$\Sigma_a = \begin{bmatrix} 0 \\ 6 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{s}_1 = \begin{Bmatrix} 0.101 \\ 0.195 \\ 0.272 \\ 0.327 \\ 0.356 \end{Bmatrix} \quad \mathbf{s}_2 = \begin{Bmatrix} -0.250 \\ -0.327 \\ -0.179 \\ 0.093 \\ 0.301 \end{Bmatrix} \quad \mathbf{s}_3 = \begin{Bmatrix} 0.272 \\ 0.077 \\ -0.250 \\ -0.149 \\ 0.208 \end{Bmatrix} \quad \mathbf{s}_4 = \begin{Bmatrix} -0.179 \\ 0.149 \\ 0.055 \\ -0.195 \\ 0.106 \end{Bmatrix} \quad \mathbf{s}_5 = \begin{Bmatrix} 0.055 \\ -0.093 \\ 0.101 \\ -0.077 \\ 0.029 \end{Bmatrix}$$

Similarly, for $\mathbf{s} = \mathbf{s}_b$, the \mathbf{s}_n vectors are

$$\Sigma_b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 2 \end{bmatrix} \quad \mathbf{s}_1 = \begin{Bmatrix} 0.110 \\ 0.210 \\ 0.294 \\ 0.354 \\ 0.385 \end{Bmatrix} \quad \mathbf{s}_2 = \begin{Bmatrix} -0.423 \\ -0.553 \\ -0.302 \\ 0.157 \\ 0.508 \end{Bmatrix} \quad \mathbf{s}_3 = \begin{Bmatrix} 0.739 \\ 0.210 \\ -0.679 \\ -0.403 \\ 0.564 \end{Bmatrix} \quad \mathbf{s}_4 = \begin{Bmatrix} -0.685 \\ 0.569 \\ 0.212 \\ -0.746 \\ 0.407 \end{Bmatrix} \quad \mathbf{s}_5 = \begin{Bmatrix} 0.259 \\ -0.436 \\ 0.475 \\ -0.363 \\ 0.135 \end{Bmatrix}$$

23v8E

Figure 12.8.3 Modal expansion of excitation vectors s_a and s_b .

Both sets of vectors are displayed in Fig. 12.8.3. The contributions of the higher modes to s are larger for s_b than for s_a , suggesting that these modes may contribute more to the response if the force distribution is s_b than if it is s_a . We will return to this observation in Section 12.11.

Derivation 12.1

The first property can be demonstrated from the generalized force for the r th mode:

$$P_r(t) = \phi_r^T s_n p(t) = \Gamma_n (\phi_r^T \mathbf{m} \phi_n) p(t) \quad (a)$$

Because of Eq. (10.4.1b), the orthogonality property of modes,

$$P_r(t) = 0 \quad r \neq n \quad (b)$$

indicating that the excitation vector $s_n p(t)$ produces no generalized force and hence no response in the r th mode, $r \neq n$. Equation (a) for $r = n$ is

$$P_n(t) = \Gamma_n M_n p(t) \quad (c)$$

which is nonzero, implying that $s_n p(t)$ produces a response only in the n th mode.

The second property becomes obvious by examining the generalized force for the n th mode associated with the total force vector:

$$P_n(t) = \phi_n^T s p(t)$$

Substituting Eq. (12.8.2) for s gives

$$P_n(t) = \sum_{r=1}^N \Gamma_r (\phi_n^T \mathbf{m} \phi_r) p(t)$$

$$\underline{M} \ddot{\underline{Y}} + \underline{C} \dot{\underline{Y}} + \underline{K} \underline{Y} = \underline{s} p(t) = \left(\sum_{r=1}^N \Gamma_r \underline{M} \underline{\Phi}_r \right) \times p(t)$$

$$= [\Gamma_1 \underline{M} \underline{\Phi}_1 + \Gamma_2 \underline{M} \underline{\Phi}_2 + \dots + \Gamma_N \underline{M} \underline{\Phi}_N] \times p(t)$$

$$C_1 P_1 + C_2 P_2 + \dots + C_N P_N \rightarrow \boxed{\underline{L}} \rightarrow C_1 \underline{L} (P_1) + C_2 \underline{L} (P_2) + \dots + C_N \underline{L} (P_N)$$

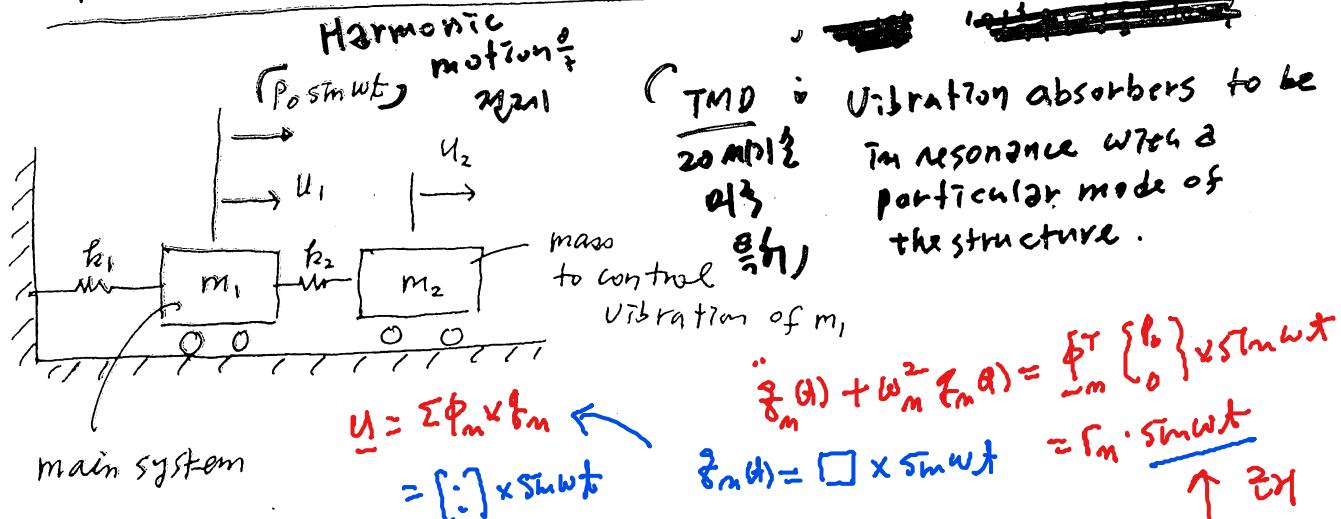
linear system

$\left\langle \frac{f_1}{1225} \frac{f_2}{2} \frac{f_3}{2} \right\rangle$

'Passive'

(1) ~~Diagram~~

Two - DOF systems without damping : Tuned Mass Dumper



$$\text{Eq. of motion : } \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} P_0 \\ 0 \end{bmatrix} \sin \omega t \quad \dots (1)$$

Because the system is undamped, the steady-state solution can be assumed as

$$\begin{bmatrix} u_1(\omega) \\ u_2(\omega) \end{bmatrix} = \begin{bmatrix} u_{10} \\ u_{20} \end{bmatrix} \times \sin \omega t \quad \dots (2)$$

Substituting this into Eq. (1),

$$\begin{bmatrix} k_1 + k_2 - m_1 \omega^2 & -k_2 \\ -k_2 & k_2 - m_2 \omega^2 \end{bmatrix} \begin{bmatrix} u_{10} \\ u_{20} \end{bmatrix} = \begin{bmatrix} P_0 \\ 0 \end{bmatrix} \quad \dots (3)$$

$$\text{or } \begin{bmatrix} k - \omega^2 m \\ k - \omega^2 m \end{bmatrix} \begin{bmatrix} u_{10} \\ u_{20} \end{bmatrix} = \begin{bmatrix} P_0 \\ 0 \end{bmatrix} \quad \dots (4)$$

$$\begin{bmatrix} u_{10} \\ u_{20} \end{bmatrix} = \frac{\text{adj} \begin{bmatrix} k - \omega^2 m \\ k - \omega^2 m \end{bmatrix}}{\det \begin{bmatrix} k - \omega^2 m \\ k - \omega^2 m \end{bmatrix}} \times \begin{bmatrix} P_0 \\ 0 \end{bmatrix} \quad \dots (5)$$

or

$$u_{10} = \frac{P_0 (k_2 - m_2 \omega^2)}{m_1 m_2 (\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)} ; u_{20} = \frac{P_0 k_2}{(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)}$$

$\omega_1 \approx \omega_2 \approx \det [k - \omega^2 m] = 0$ \Rightarrow ω_1, ω_2 are 2 natural frequencies.

2-DOF TMD
1st & 2nd frequency

~~EE223 mode superposition method~~

~~P.68, Eq (3.1.1)
18/1~~

~~Damping ω_d
Based on $\frac{k_2}{m_2}$~~

~~H.W.H.
t₀ = 0~~

(6)

(2)

Example : $m_1 = 2m$, $m_2 = m$, $k_1 = 2k$, $k_2 = k$ or $\omega_1 = \sqrt{k/m}$

$\omega_1 = \sqrt{k/2m}$, $\omega_2 = \sqrt{2k/m}$ of $\frac{\omega_2}{\omega_1} = \sqrt{2}$ or $\frac{1}{2}$ (6) \Rightarrow system is very stiff

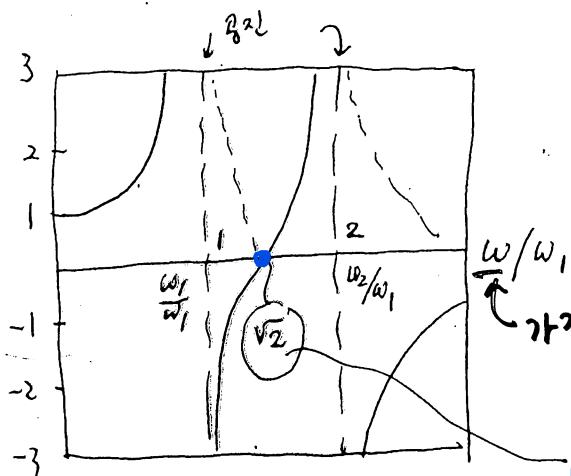
\Rightarrow Not good! (too heavy)

$$\frac{u_{10}}{(u_{1st})_0} = \frac{1 - \frac{1}{2}(\omega/\omega_1)^2}{[1 - (\omega/\omega_1)^2][1 - (\omega/\omega_2)^2]} \quad \left. \right\} \dots (7)$$

$$\frac{u_{20}}{(u_{2st})_0} = \frac{1}{[1 - (\omega/\omega_1)^2][1 - (\omega/\omega_2)^2]}$$

where $(u_{1st})_0 = P_0/2k$; $(u_{2st})_0 = P_0/2k$

Main system response $\frac{u_{10}}{(u_{1st})_0}$



Depend on frequency ratios (ω/ω_1) and (ω/ω_2) , not separately on ω_1 , ω_1 and ω_2 .

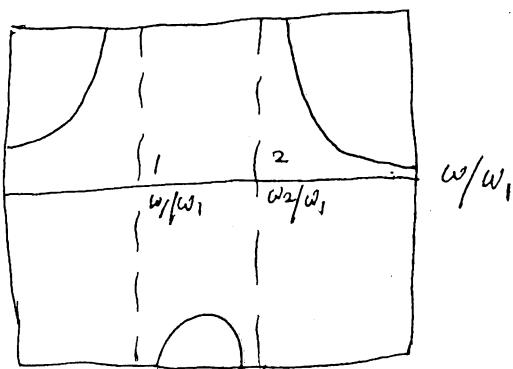
(note: $\omega_1 < \omega_2$)

$m_1 = m_2$
(TMD mass m_1 is $\sqrt{2}$)

$$\omega/\omega_1 = \sqrt{2} \quad \text{or} \quad \omega_1 = \frac{\omega}{\sqrt{2}}$$

the entire basis
of TMD

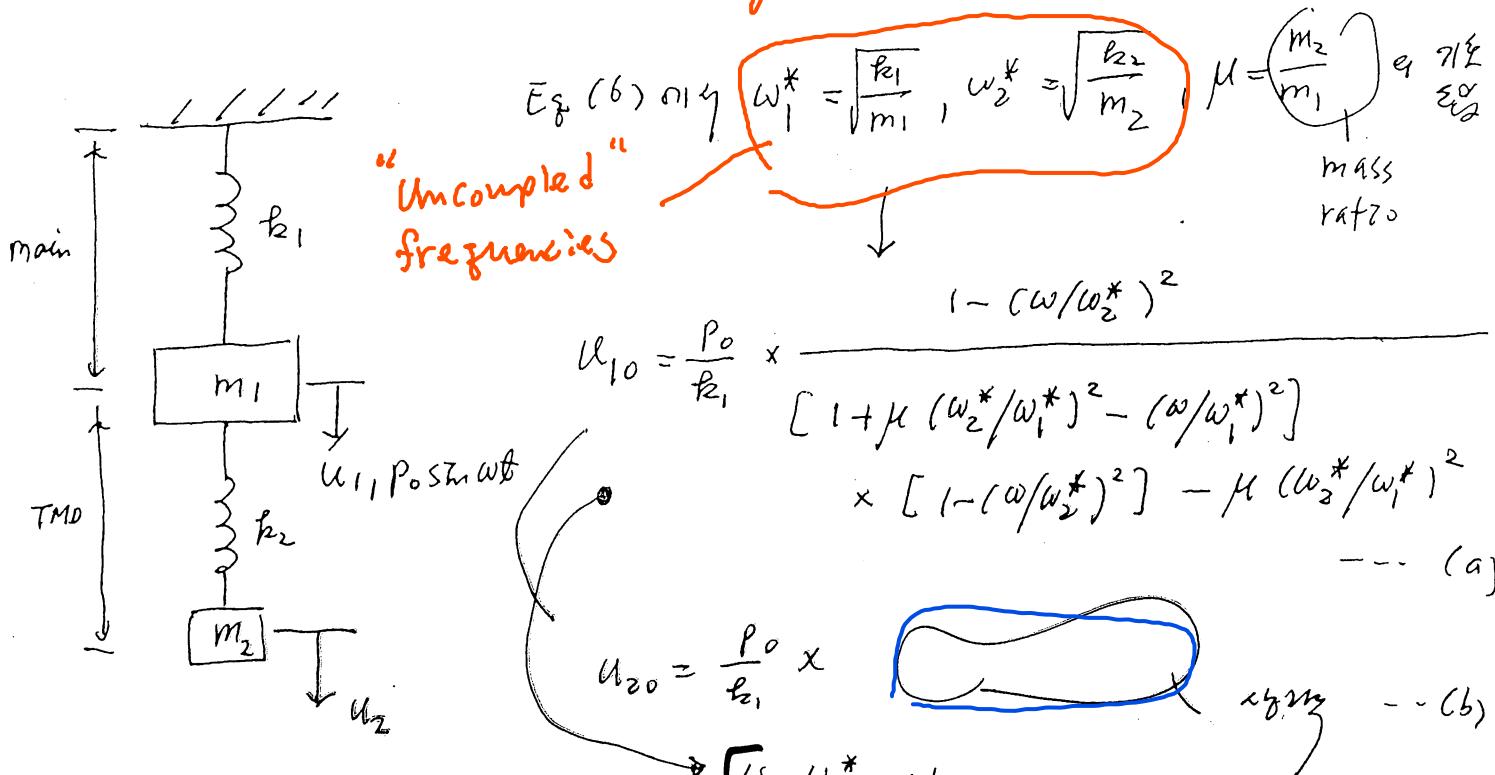
TMD response $\frac{u_{20}}{(u_{2st})_0}$



(3)

The basic principle of a vibration absorber

without getting into the many important aspects of its practical design.
"symbol 4228"



Specialization)

$\mu = 0, 1, 2$, $\omega_1^* = \omega_2^*$ \Rightarrow TMD $\frac{1}{2}$ main system or $\frac{1}{2}$ system tuning

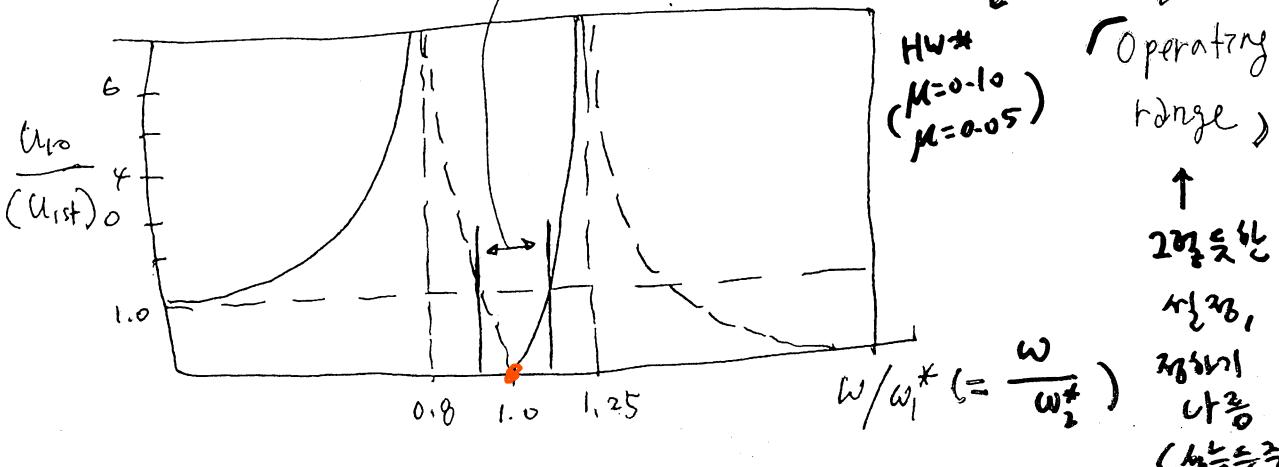
($\frac{1}{5}$) \rightarrow Not good, too heavy!

\hookrightarrow enough weight

$$\frac{U_{10}}{(U_{1st})_0} = \frac{1 - (\omega/\omega_1^*)^2}{[1 + \mu - (\omega/\omega_1^*)^2] [1 - (\omega/\omega_1^*)^2]} - 0.2$$

Operating range: $U_{10}/(U_{1st})_0 < 1.0$

mass ratio: $\mu = 0.1$ to 0.2



Note: ① At $\omega = \omega_1^*$, the response amplitude of the main mass alone is unbounded.

TMD이란 뜻

Thus, if exciting frequency is close to the natural frequency of the main system ω_1^* and operating restrictions make it impossible to vary either one, the TMD can be used to reduce the response of the main system to near zero.

"TMD"

1) 2) effective size " "

$$\begin{aligned} & \text{운동량 } \frac{\partial}{\partial t} m_1 u_1 \\ & \text{main system } \frac{\partial^2}{\partial t^2} (m_1 u_1) + k_1 u_1 \\ & \text{은 } \frac{d}{dt} m_1 \dot{u}_1 = m_1 \ddot{u}_1 \\ & (\approx \text{여기서 } \omega_1 \approx \omega \text{ 일 때}) \\ & \frac{d}{dt} (m_1 \dot{u}_1) = -k_1 u_1 \end{aligned}$$

of the main system to near zero,

② What should be the size of the absorber mass?

To answer this question, use Eq. (b)

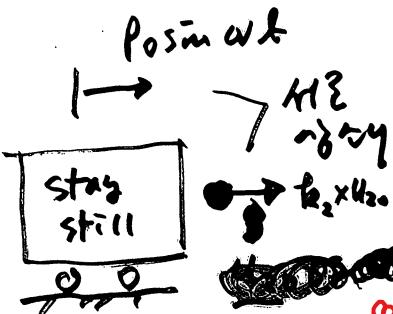
To answer this question, use Eq. (b) to determine the motion of the TMD at $\omega = \omega_2^*$

$$u_{20} = -\frac{P_0}{k_2}$$

The force acting on the absorber mass is

$$k_2 u_{20} = \omega^2 m_2 u_{20} = -P_0$$

$$k_2 u_2(t) = k_2 u_{20} \sin \omega t = -P_0 \sin \omega t$$



This implies that the absorber exerts a force equal and opposite to the exciting force.

Since $m_2 \neq k_2$, $u_2 \neq u_{20}$ $\frac{u_{20}}{u_2}$ is $\frac{m_2}{k_2}$ times $\sin \omega t$.

Mass ratio가 $\frac{m_2}{k_2}$ 일 때 u_2 는 u_{20} 의 $\frac{m_2}{k_2}$ 배인 $\frac{m_2}{k_2} \sin \omega t$ 이다.

따라서 TMD는 ω_1 일 때 주파수를 ω_2 로 하는同步振動을 일으킨다. ω_2 는 ω_1 과 같은 크기의 주파수이다.

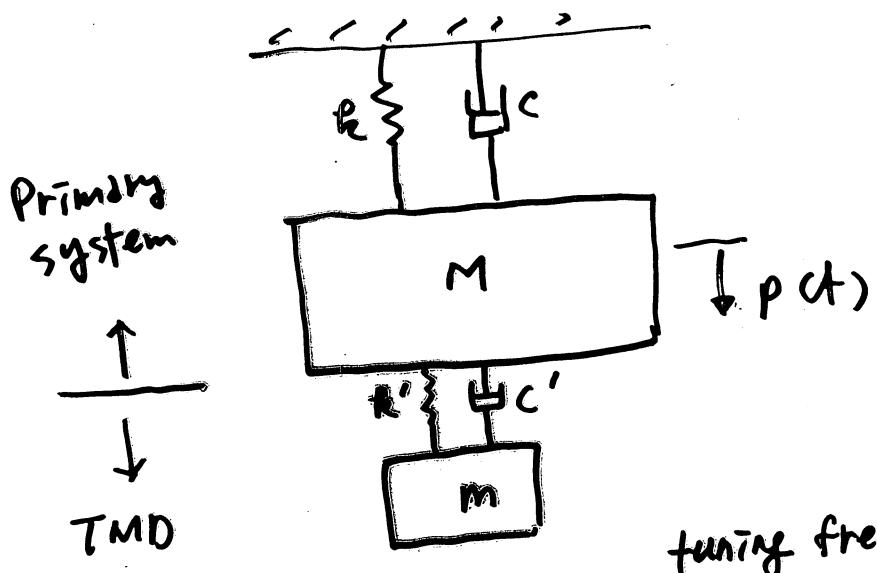
The use of Multiple TMD!

따라서 주파수를 일정하게 유지하는 경우에만 효과적이다.

However, vibration absorbers are also used in situations where the excitation is not nearly harmonic.

ex): Wind-induced vibration / floor vibration

"TMD를 활용한 주제 응집 및 저감의 예시:"



Find optimal mass (m) and damping parameters of 2 TMD.

Harmonic input

Ex) 조화적 흔震动시 가속도 응답을 최저화하기 위한 조건

$$\text{i)} \frac{\omega_{TMD}}{\omega} = \frac{1}{\sqrt{1+\mu}}$$

$$\text{ii)} \xi_{TMD} = \sqrt{\frac{3\mu}{8(1+\mu/2)}}$$

where $\mu = m/M = \text{mass ratio}$

* 실제를 고려할 때
통상적 시장 조건은
조건 tuning 조건