Chapter 1

Introduction

Outline

- Methods of Analysis
- Engineering Design
- Static Equilibrium
- Internal Force Resultants
- Example Problem Formulation and Solutio

METHODS OF ANALYSIS

Two common approaches

- Mechanics of materials theory (a.k.a. technical theory, or solid mechanics approach)
- Theory of elasticity approach

METHODS OF ANALYSIS

- **The solid mechanics** approach is simpler and makes assumptions that are based upon experimental evidence and the lessons of engineering practice.
- Reasonably quick solution of the basic problem is possible, for example, determination of strain.

METHODS OF ANALYSIS

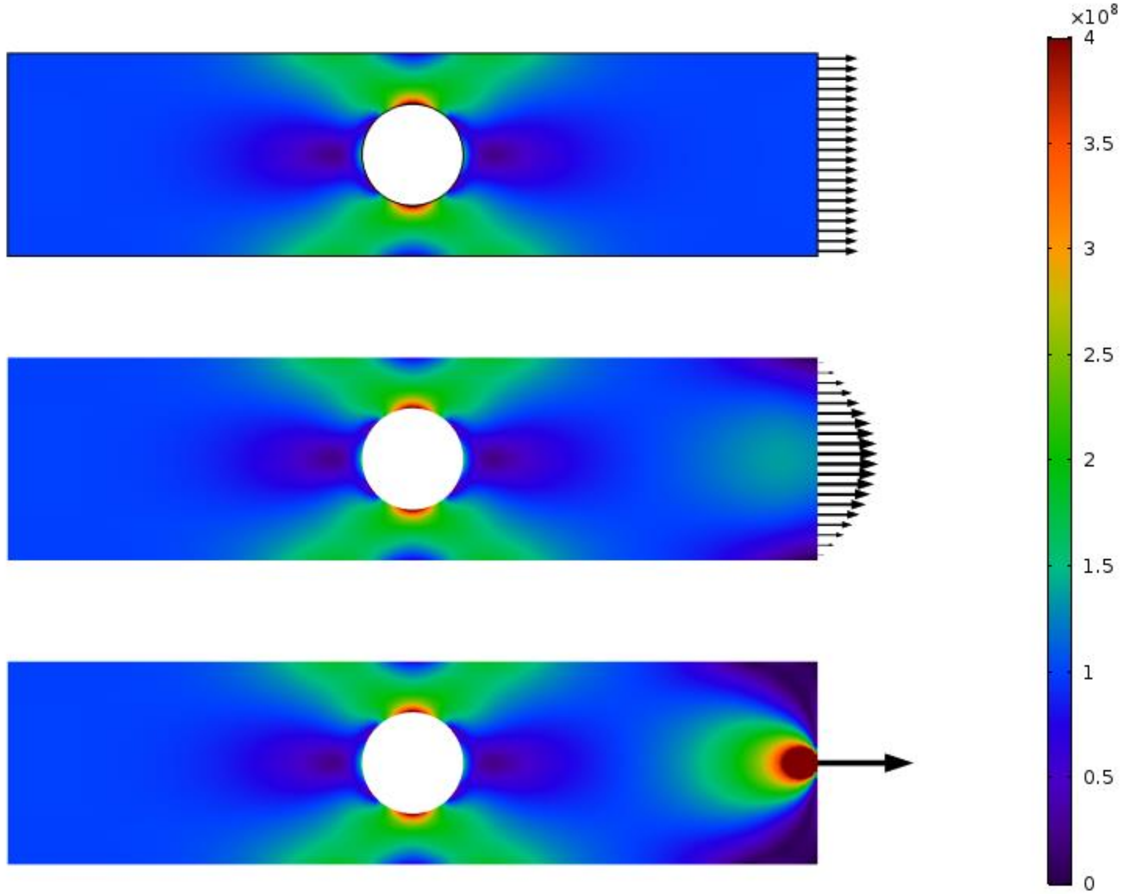
- The **theory of elasticity** approach establishes every step rigorously from the mathematical point of view and hence seeks to verify the validity of the assumptions introduced to determine the quantities, for example, strains.

- This technique can provide **"exact"** results where configurations of loading and shape are simple.

- However, this approach yields solutions with considerable **difficulty**.

METHODS OF ANALYSIS

Saint-Venant’s Principle



Basic Principles of Analysis

1. Equilibrium
   - The equations of static equilibrium of forces (and moments) must be satisfied throughout the member.
2. Material Behavior or **constitutive equations**
   - The stress–strain or force–deformation relations (Hooke's law) must apply to the behavior of the material of which the member is made of.
3. Geometry of Deformation or **compatibility conditions**
   - The conditions of compatibility of deformations must be satisfied: that is, each deformed portion of the member must fit together with adjacent portions.

Boundary conditions are used in the method of analysis.

Basic Principles of Analysis

Alternative to the equilibrium method

- Energy methods: based on strain energy theory

- Numerical methods: finite element method (FEM), finite difference method (FDM), boundary element method (BEM) etc.

Major Steps in Engineering Design Procedure

1. Evaluate the mode of possible failure of the member.
2. Determine a relationship between the applied load and the resulting effect such as stress or deformation.
3. Determine the maximum usable value of a significant quantity such as stress or deformation that could conceivably cause failure.
4. Employ this value in connection with the equation found in step 2 or, if required, in any of the formulas associated with the various theories of failure.
5. Determine the safety factor or verify if the design is safe.

STATIC EQUILIBRIUM

Types of loads: External loads and Internal loads

- **External loads**: surface forces and body forces
  - Surfaces forces can be for example, a concentrated load acting at a point or a distributed load both acting on the surface of a body
  - Body forces act on a volumetric portion of the body, forexample, magnetic force or gravitational force
  - Reaction forces caused by the supports
- **Internal loads:** forces of interaction between the constituent material particles of the body

CONDITIONS OF EQUILIBRIUM

- The equations of static equlibrium require:

$$\sum F_x = 0;\ \sum F_y = 0, \text{ and } \sum F_z = 0$$
$$\sum M_x = 0;\ \sum M_y = 0, \text{ and } \sum M_z = 0$$

- For a body to be in static equilibrium, the sum of all forces acting upon a body in any direction is zero and also the sum of all moments taken about any axis is also zero.

Planar Equations of Equilibrium

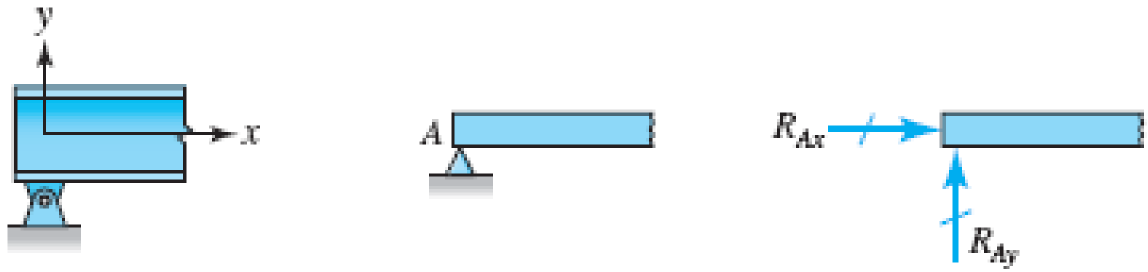
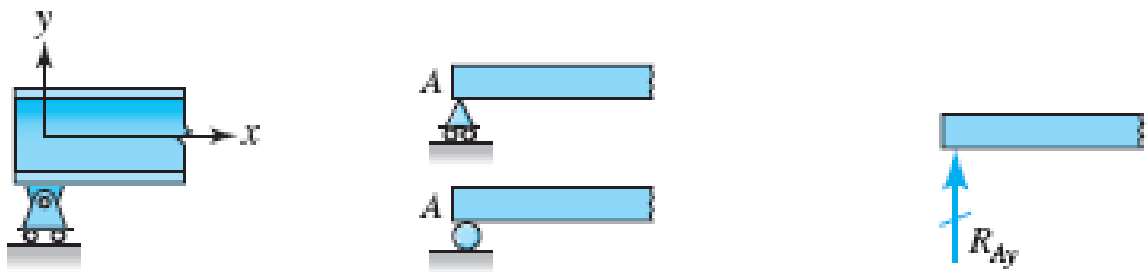
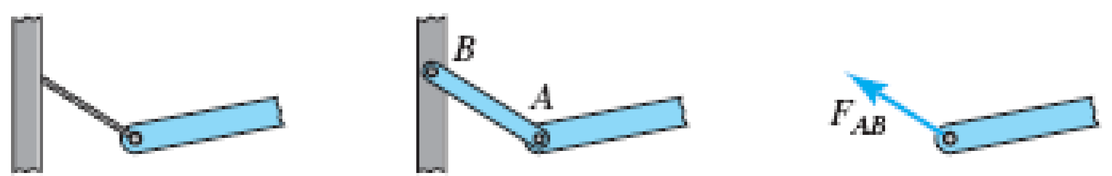
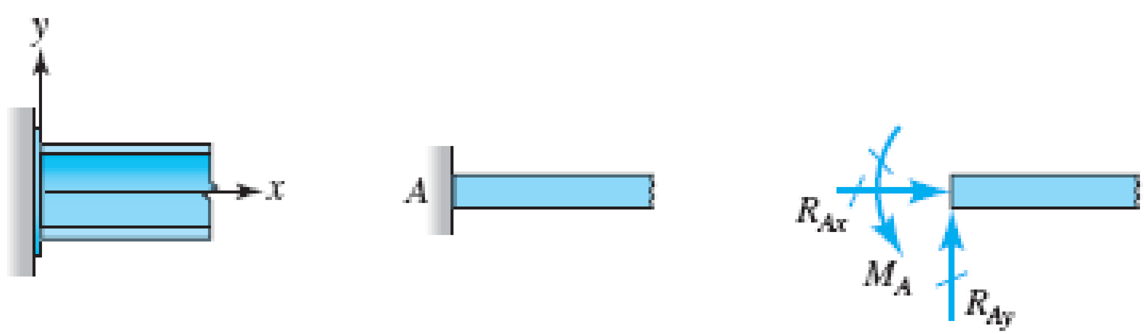
For a planar body to be in equilibrium, any one of the following sets of 3 equations may be used to solve for the unknown variables.

1. $\sum F_x = 0$, $\sum F_y = 0$, and $\sum M_A = 0$, where the resultant moment is with respect to any axis z or any point A in the xy-plane, or

2. $\sum F_x = 0$, $\sum M_A = 0$, and $\sum M_B = 0$, provided that the line connecting the points A and B is not perpendicular to the x axis, or

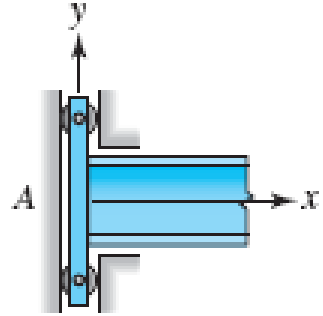
3. $\sum M_A = 0$, $\sum M_B = 0$, and $\sum M_C = 0$, where points A, B, and C are not collinear

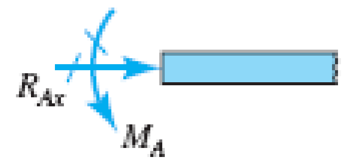
Support reactions and applications of equilibrium of planar bodies

Type of Support or Connection	Reaction*
1. Hinge or pin	
2. Roller	
3. Cable or link	
4. Fixed or clamped	

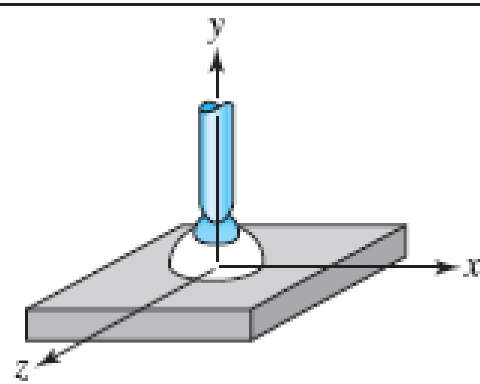
R_{Ay}

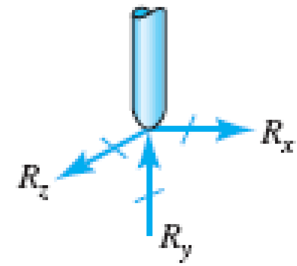
5. Guided



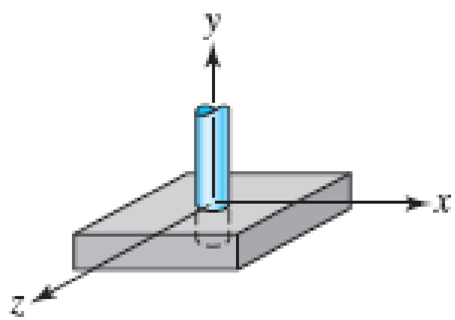


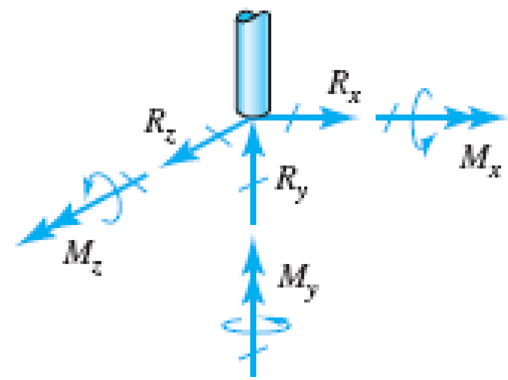
6. Ball and socket (nonplanar)





7. Fixed (nonplanar)





*Usually the reaction will be identified in this text with a *slash* drawn through its vector as shown.

Freebody diagrams

1. Select the free body to be used.
2. **Detach** this body from its supports and **separate** it from any other bodies. If internal force resultants are to be found, use the method of sections.
3. Show on the sketch all of the external forces acting on the body. Location, magnitude, and direction of each force should be marked on the sketch.
4. Label significant points and include dimensions.

INTERNAL FORCE RESULTANTS

1. Isolate the bodies. Sketch the isolated body and show all external forces acting on it: draw a free-body diagram.
2. Apply the equations of equilibrium to the diagram to determine the unknown external forces.
3. Cut the body at a section of interest by an imaginary plane, isolate one of the segments, and repeat step 2 for that segment. If the entire body is in equilibrium, any part of it must be in equilibrium. That is, there must be internal forces transmitted across the cut sections.

Excercise examples

External vs. internal forces

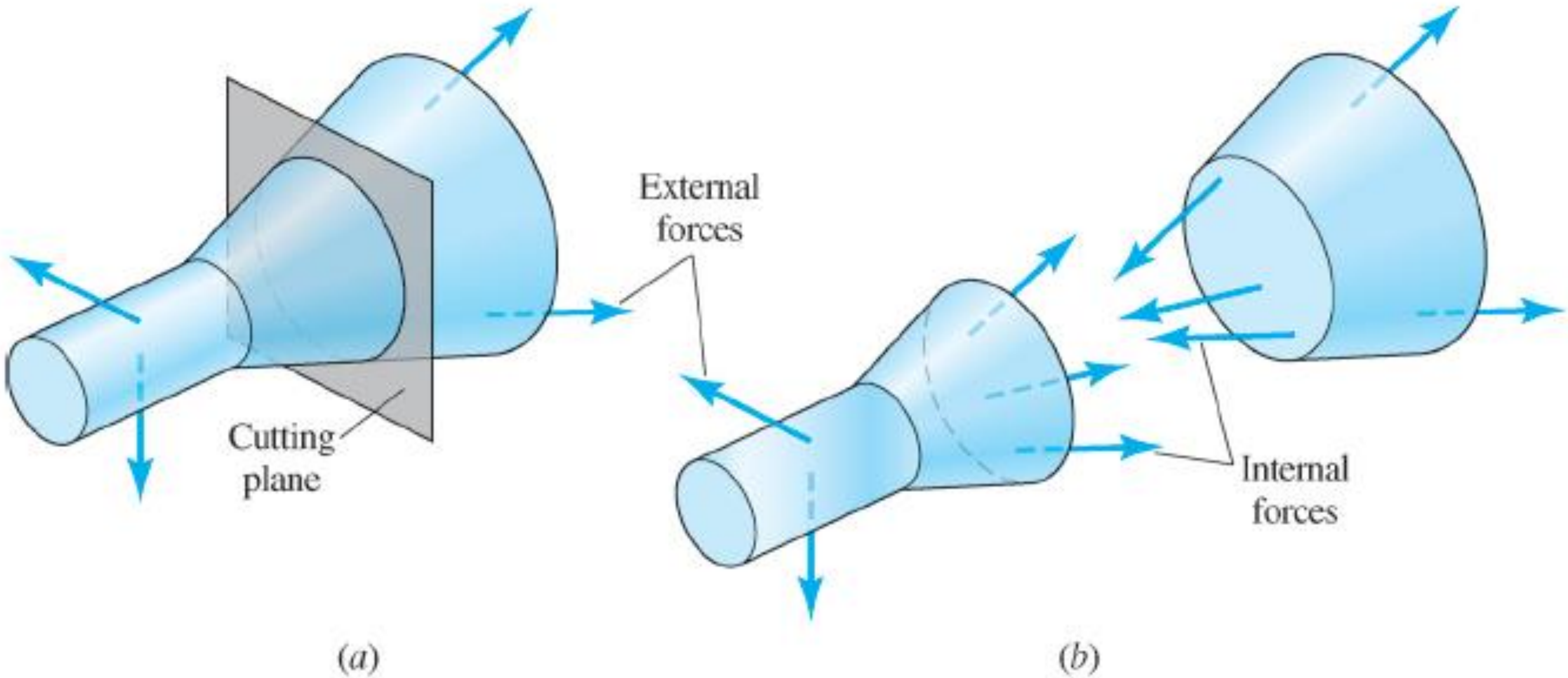


FIGURE 1.1 Method of sections: (*a*) body acted on by external forces; (*b*) internal forces acting on a plane.

Components of internal forces

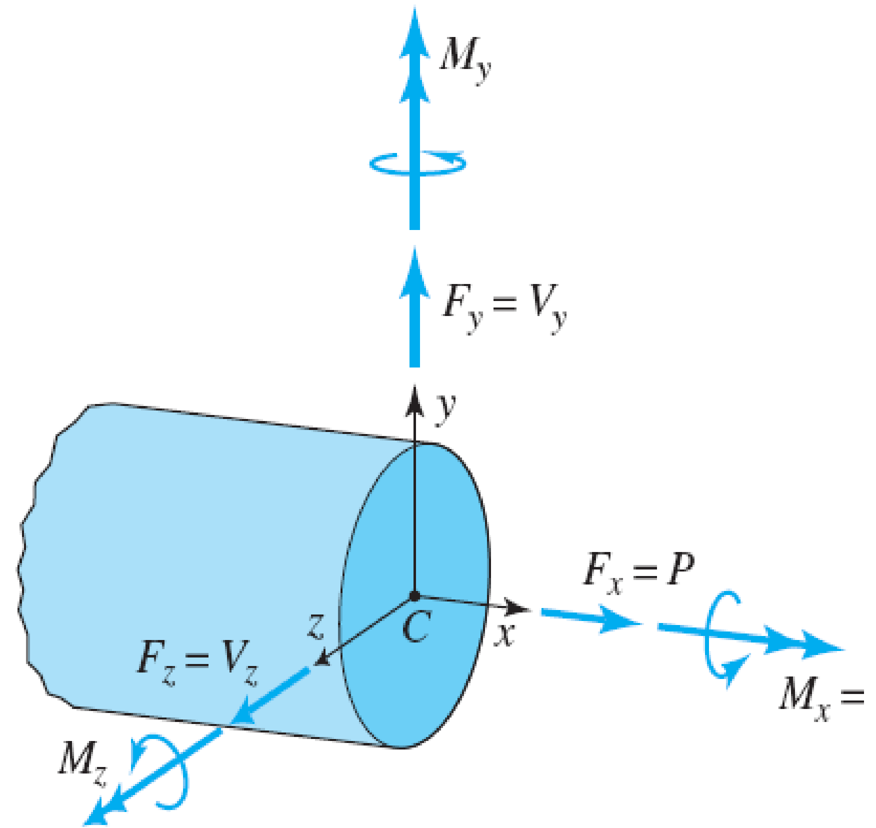
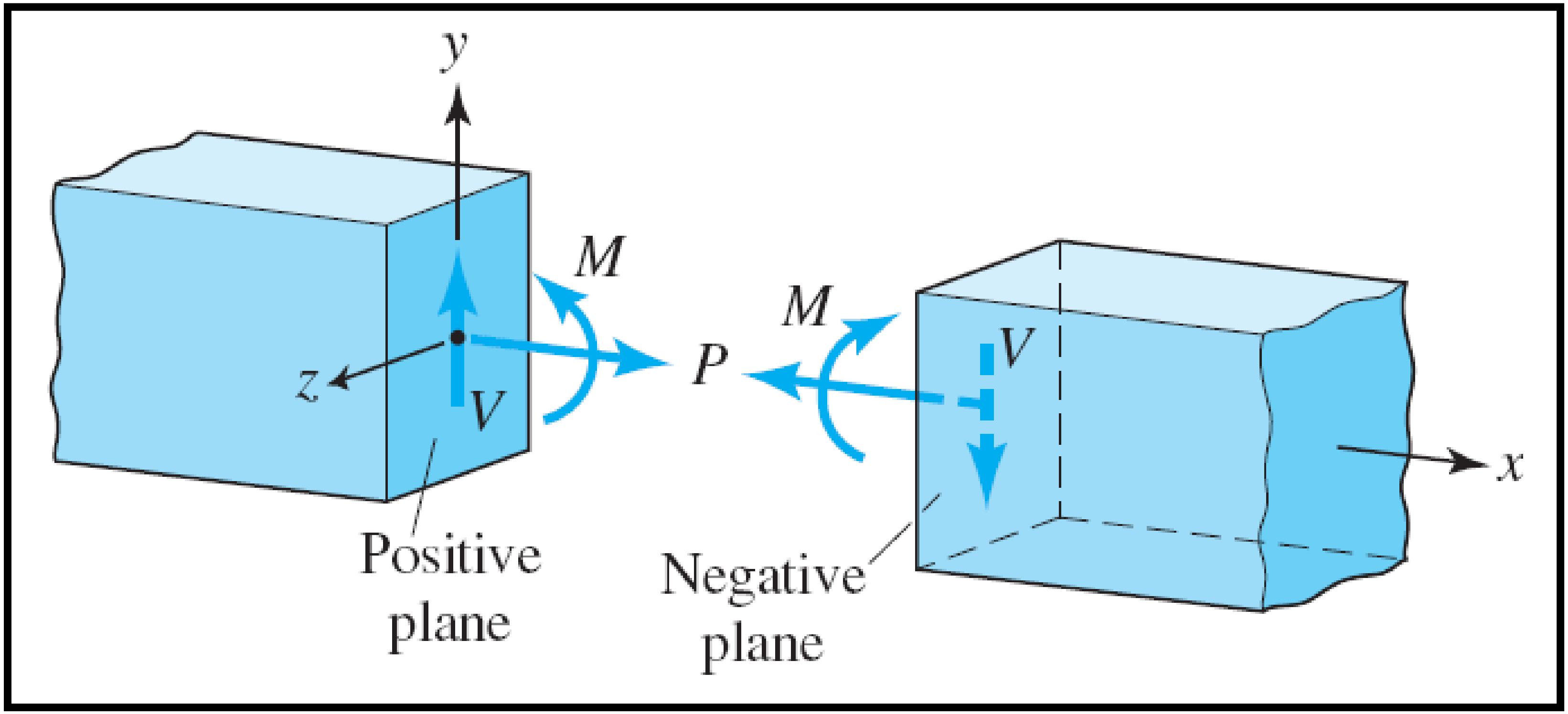


FIGURE 1.2 Three-dimensional forces and moments acting on a section of a member.

Force and moment components



Units

TABLE 1.2 Basic Units

	SI Unit		U.S. Unit	
Quantity	**Name**	**Symbol**	**Name**	**Symbol**
Length	meter	m	foot	ft
Force*	newton	N*	pound force	lb
Time	second	s	second	s
Mass	kilogram	kg	slug	$lb \cdot s^2/ft$
Temperature	degree Celsius	°C	degree Fahrenheit	°F

*Derived unit ($kg\ m/s^2$).

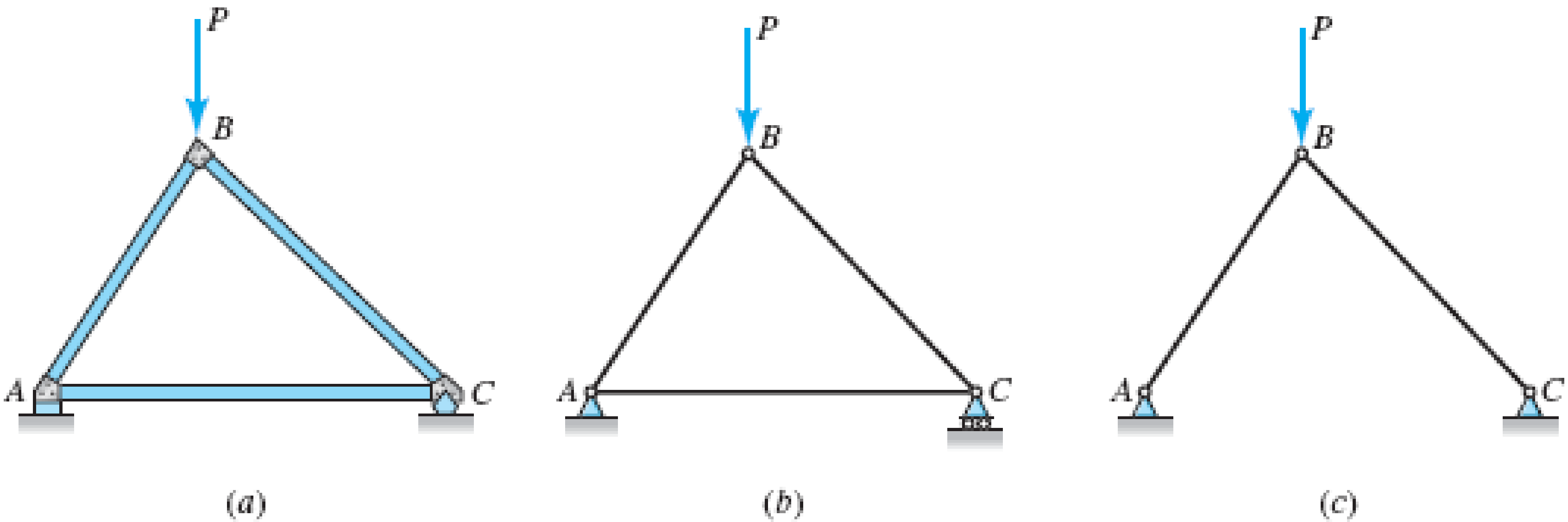


FIGURE 1.4 Basic truss with a load at joint B: (a) triangular and (b) its idealized model; (c) two-bar.

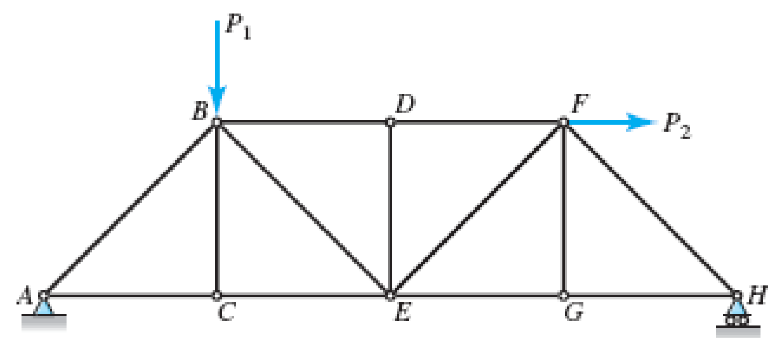


FIGURE 1.5 Plane truss subjected to loads at joints B and F.

Example 1.1.

SOLUTION: See Fig. 1.6 and Table 1.1.

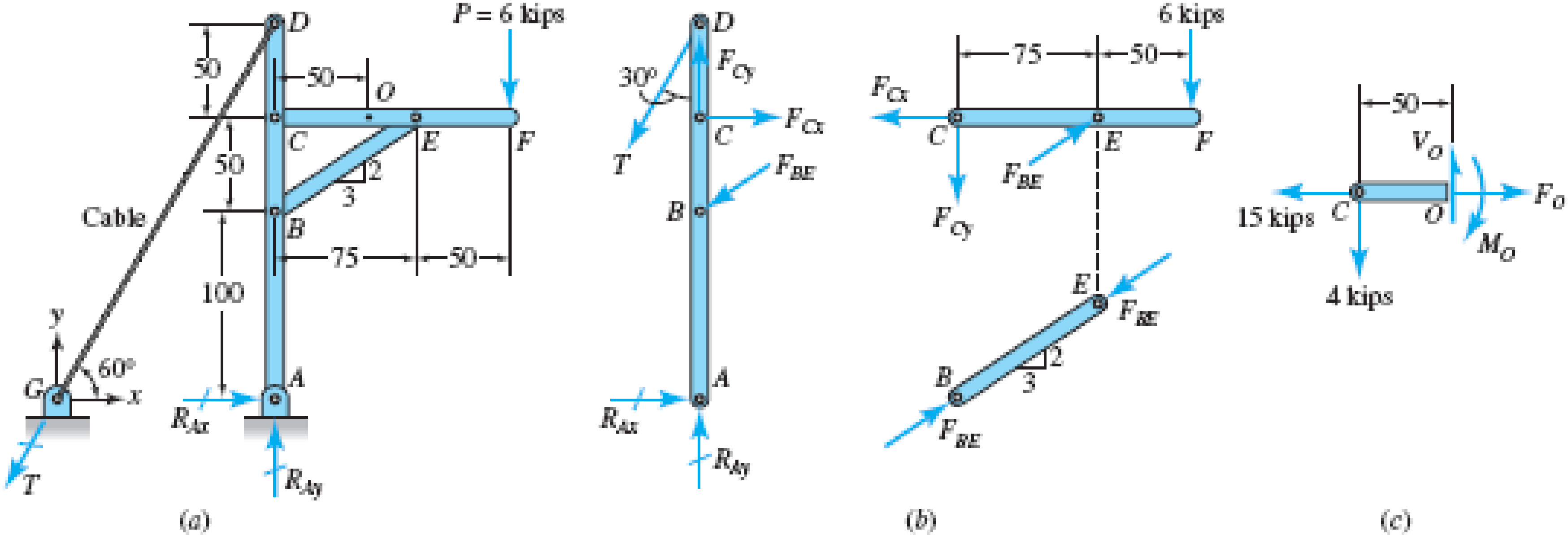


FIGURE 1.6 (*a*) Structural assembly; (*b* and *c*) free-body diagrams of members and part *CO* of beam *CEF*.

Free-Body: Entire Frame. There are two components R_{Ax} and R_{Ay} of the reaction at A and the force T exerted by the cable at D. So, we can find the reactions by considering the free-body diagram of the entire frame (Fig. 1.6*a*):

$$\sum M_A = -6(125) + T\sin 30^\circ(200) = 0 \qquad T = 7.5\text{ kips}$$

$$\sum F_x = R_{Ax} - 7.5\sin 30^\circ = 0 \qquad R_{Ax} = 3.75\text{ kips}$$

$$\sum F_y = R_{Ay} - 7.5\cos 30^\circ - 6 = 0 \qquad R_{Ay} = 12.5\text{ kips}$$

(a) The frame is now dismembered. Inasmuch as only two members are connected at each joint, equal and opposite components are shown on each

SOLUTION: See Fig. 1.6 and Table 1.1.

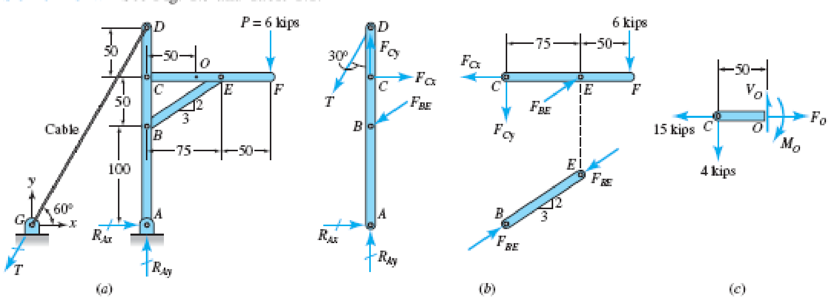


FIGURE 1.6 (*a*) Structural assembly; (*b* and *c*) free-body diagrams of members and part *CO* of beam *CEF*.

member at each joint (Figure 1.6*b*). Observe that BE is a two-force **member**—that is, a bar subjected to forces having the same line of action, same magnitude, and opposite senses at only two points.

Free-Body: Member *CEF*

$$\sum M_C = -6(125) + \frac{2}{\sqrt{13}}F_{BE}(75) = 0 \qquad F_{BE} = 18.03\text{ kips }(C)$$

$$\sum M_E = -6(50) + F_{Cy}(75) = 0 \qquad F_{Cy} = 4\text{ kips}$$

$$\sum F_x = \frac{3}{\sqrt{13}}(18.03) - F_{Cx} = 0 \qquad F_{Cx} = 15\text{ kips}$$

Free-Body: Member *ABCD*. All internal force resultants have been found. To check the results, by applying equations of statics we verify that the member *ABCD* is in equilibrium.

Comment: The positive values found indicate that the directions shown for the force components are correct; member BE is in compression (C).

(b) Free-Body: Part *CO*. The beam *CEF* is cut at point O. Selecting the free-body diagram of the portion *CO* (Fig. 1.6*c*), we obtain

$$M_O = 4(50) = 200\text{ kip}\cdot\text{in.} \qquad F_O = 15\text{ kips} \qquad V_O = 4\text{ kips}$$

Comment: The internal load resultants at O are equivalent to a couple, an axial force, and a shear force, acting as shown in the figure.

Example 1.2.

EXAMPLE 1.2

Forces in Members of a Truss

Given: The truss shown in Fig. 1.7*a* is constructed of seven bars, each having length L. The loads $P_1 = 100$ N and $P_2 = 50$ N act at joints B and C, respectively.

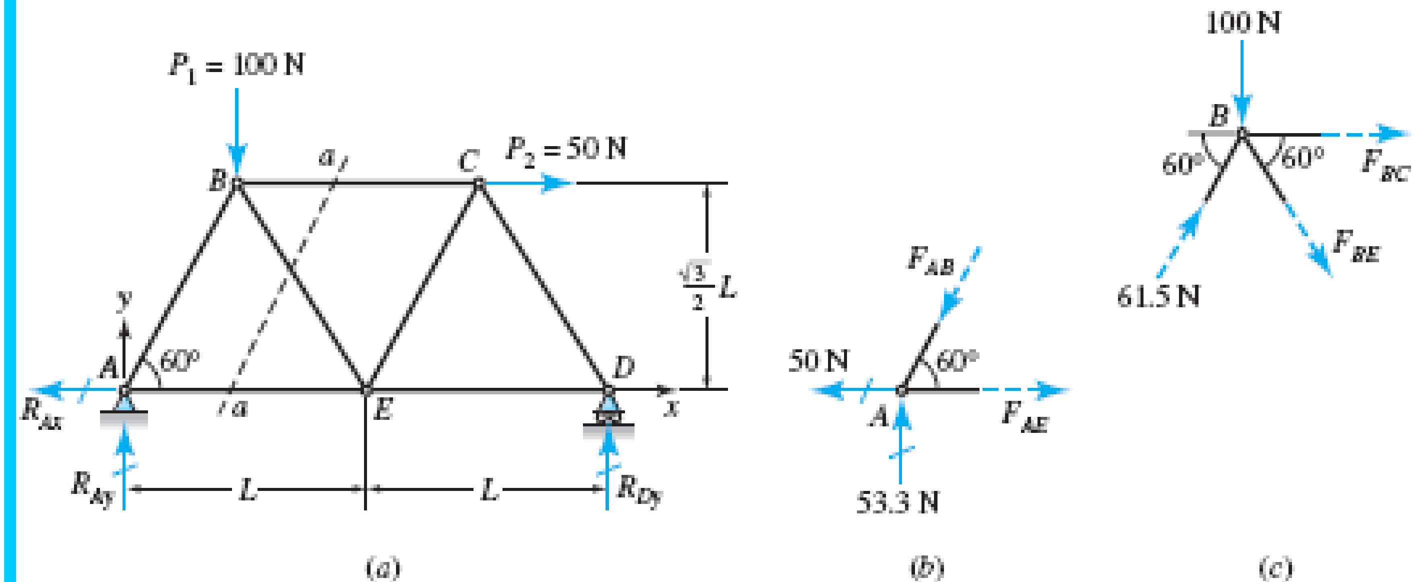


FIGURE 1.7 (*a*) A seven-member truss; (*b* and *c*) free-body diagrams of joint *A* and joint *B*.

Find:

(a) The axial forces F_{AB}, F_{AE}, F_{BC}, and F_{BE} using the method of joints.

(b) The axial force F_{BC} using the method of sections.

Assumption: Friction in pinned joints and the weight of the members are neglected.

SOLUTION: The support reactions are indicated in the figure.

Free-Body: Entire Truss. Reactions at the supports can be calculated by applying the equilibrium conditions to the free-body diagram of the entire truss (Fig. 1.7*a*):

$$\sum F_x = 0: \quad -R_{Ax} + 50 = 0 \qquad R_{Ax} = 50\,\mathrm{N} \leftarrow$$

$$\sum M_A = 0: \quad -100\left(\frac{L}{2}\right) - 50\left(\frac{\sqrt{3}}{2}L\right) + R_{Dy}(2L) = 0 \qquad R_{Dy} = 46.6\,\mathrm{N}\uparrow$$

$$\sum M_D = 0: \quad -R_{Ay}(2L) + 100\left(\frac{3}{2}L\right) + 50\left(\frac{\sqrt{3}}{2}L\right) = 0 \qquad R_{Ay} = 53.4\,\mathrm{N}\uparrow$$

Note, as a check, that $\sum F_y = 0$.

(a) Method of Joints. We first draw free-body diagrams of the joints. The unknown forces are usually shown as being directed away from a joint if they are in tension and toward the joint for compression. Occasionally, it is possible to tell by inspection if the forces are in tension or in compression. If the assumed direction is correct, the force calculated from equations of equilibrium will be positive; when the direction is not correct, the force will be negative. In the method of joints, the equations of statics are applied to each joint separately. Great care must be exercised in handling the signs of the forces obtained for a joint when setting up the free-body diagrams of subsequent joints.

Free Body: Joint *A* (Fig. 1.7*b*)

$$\sum F_y = 0: \quad 53.4 - F_{AB}\sin 60^\circ = 0 \qquad F_{AB} = 61.5\,\mathrm{N}$$

$$\sum F_x = 0: \quad -50 + F_{AE} - F_{AB}\cos 60^\circ = 0 \qquad F_{AE} = 80.8\,\mathrm{N}$$

Free Body: Joint *B* (Fig. 1.7*c*)

$$\sum F_y = 0: \quad -100 + 61.5\sin 60^\circ - F_{BE}\sin 60^\circ = 0 \qquad F_{BE} = -54\,\mathrm{N}$$

$$\sum F_x = 0: \quad F_{BC} + 61.5\cos 60^\circ + F_{BE}\cos 60^\circ = 0 \qquad F_{BC} = -3.8\,\mathrm{N}$$

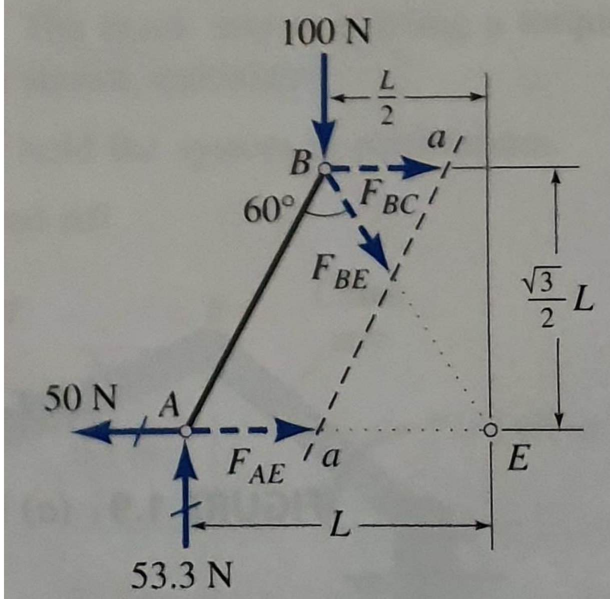


FIGURE 1.8 Method of sections for determining the forces F_{AE}, F_{BE}, and F_{BC}.

(b) Method of Sections. An imaginary cut through *a-a* (Fig. 1.7*a*) exposes the unknown forces F_{AE}, F_{BC}, and F_{BE} as shown in Fig. 1.8. To eliminate the two forces passing through the point *E*, we write

$$\sum M_E = 0: \quad -53.3\,L - F_{BC}\left(\frac{\sqrt{3}L}{2}\right) + 100\left(\frac{L}{2}\right) = 0 \qquad F_{BC} = -3.8\,\mathrm{N}$$

This corresponds to the result found in part (a).

Example 1.3.

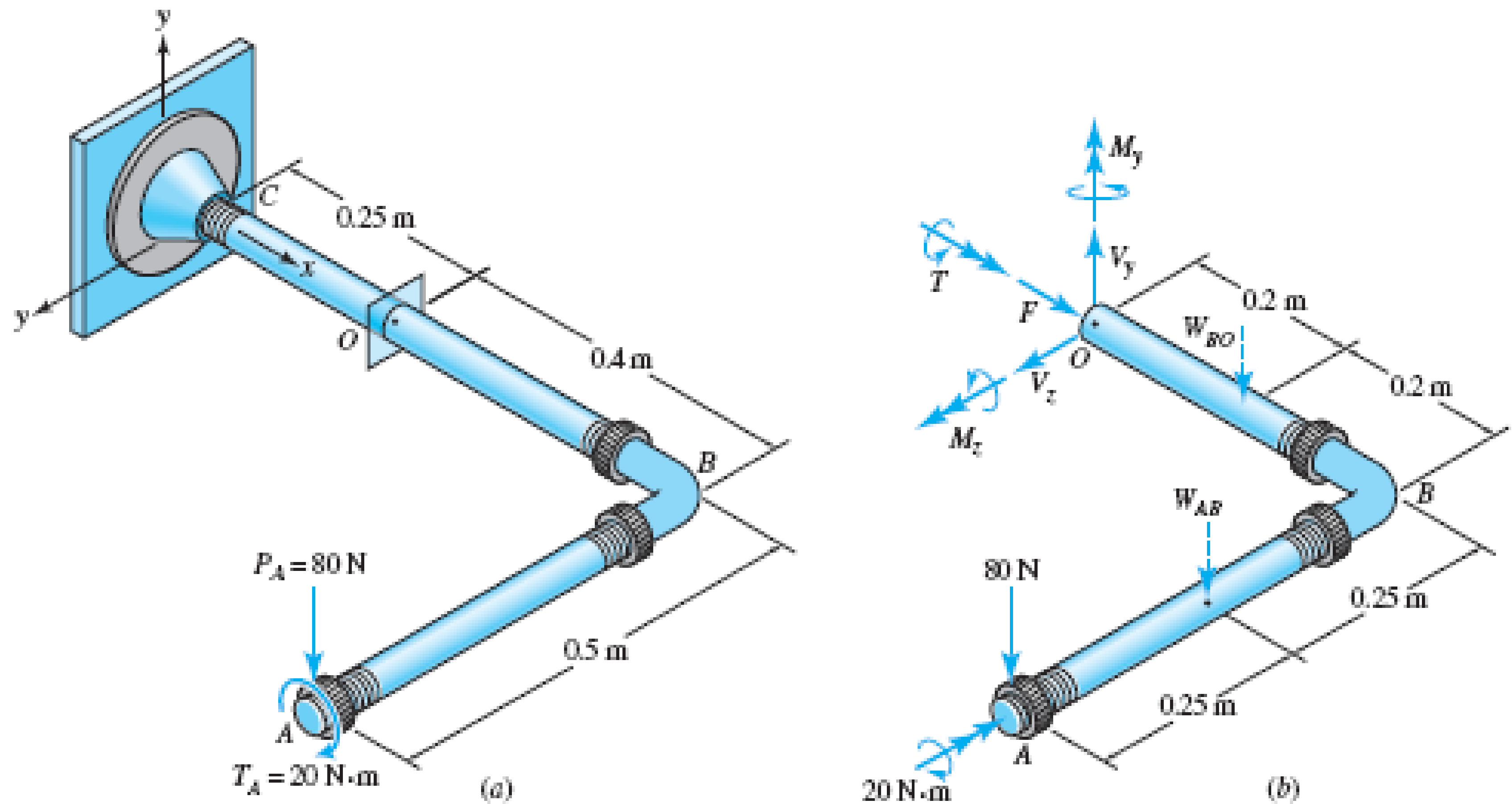


FIGURE 1.9 (*a*) Pipe assembly; (*b*) free-body diagram of part *ABO*.

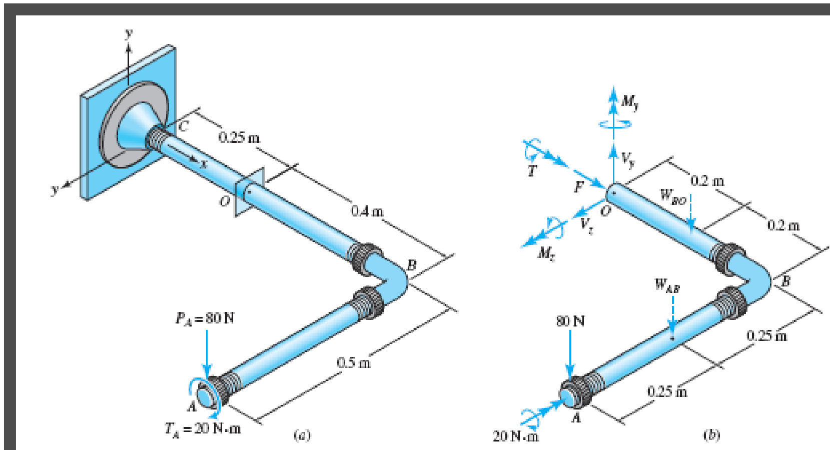


FIGURE 1.9 (*a*) Pipe assembly; (*b*) free-body diagram of part *ABO*.

of Eqs. (1.1) relate the applied loads to the internal forces on the pipe at point O:

$$\sum F_x = 0: \qquad\qquad F = 0$$

$$\sum F_y = 0: \quad V_y - 12.02 - 9.61 - 80 = 0 \qquad V_y = 101.6\,\text{N}$$

$$\sum F_z = 0: \qquad\qquad V_z = 0$$

Using the last three of Eqs. (1.1), the moments about point O are

$$\sum M_x = 0: \quad T + (12.02)(0.25) + 80(0.5) = 0 \qquad T = -43\,\text{N·m}$$

$$\sum M_y = 0: \qquad\qquad M_y = 0$$

$$\sum M_z = 0: \quad M_z - 20 - 80(0.4) - (12.02)(0.4) - (9.61)(0.2) = 0 \qquad M_z = 58.7\,\text{N·m}$$

Comment: The negative value obtained for T indicates that the torque vector is directed opposite to that shown in the figure.