445.204

Introduction to Mechanics of Materials

(재료역학개론)

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Chapter 2

Concept of Stress

Outline

- Internal Axial Forces
- Normal Stress
- Bearing Stress in Connections
- Shearing Stress
- Stresses in Simple Structures
- Allowable Stress and Factor of Safety
- Design of Bars for Axial Loading
- Case Studies
- General Stress Tensor

Load

- Load is a general term that can mean either a force, or a torque or a moment, or any combination of these

- Depending on how it is applied, a force can cause either an axial load or a torsion load or a bending load or any combinations of these loads on a member

Load-Examples

Force P causes compressive load in members AB and BC and tension load in member AC of the truss shown in Figure 1.4 (a) & (b).

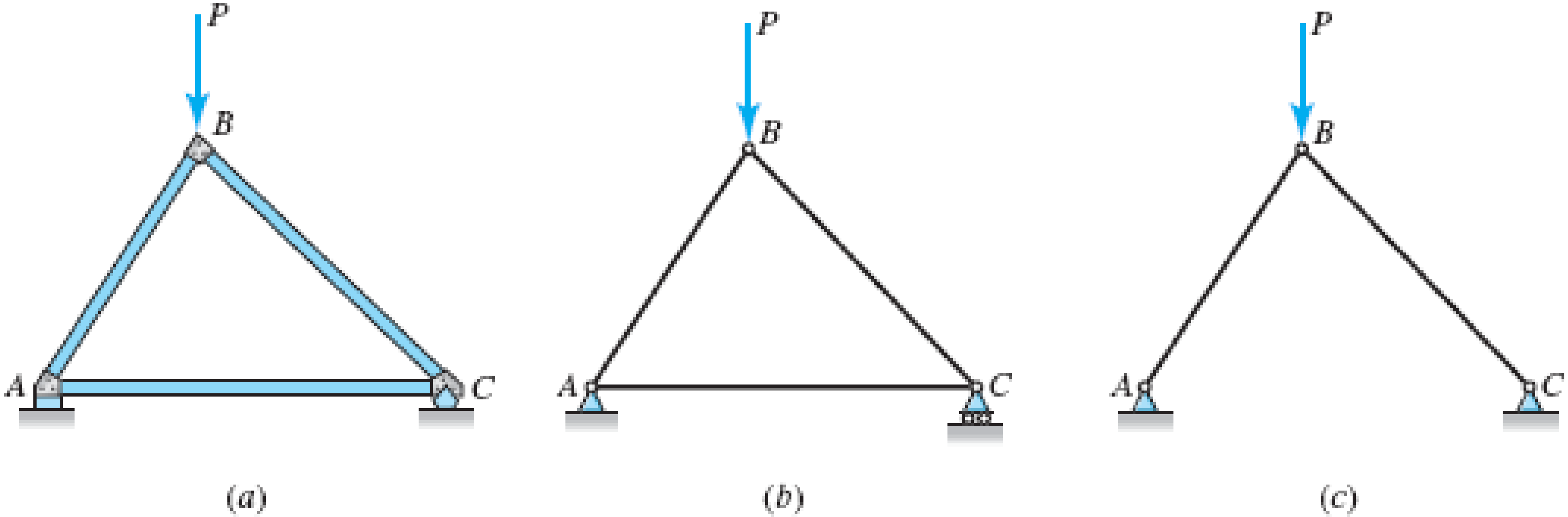


FIGURE 1.4 Basic truss with a load at joint B: (a) triangular and (b) its idealized model; (c) two-bar.

The combined „loads“ P_A and T_A at section A cause equivalent moment and shear loads at various locations of the L-bar shown in Figure 1.9 (a) & (b).

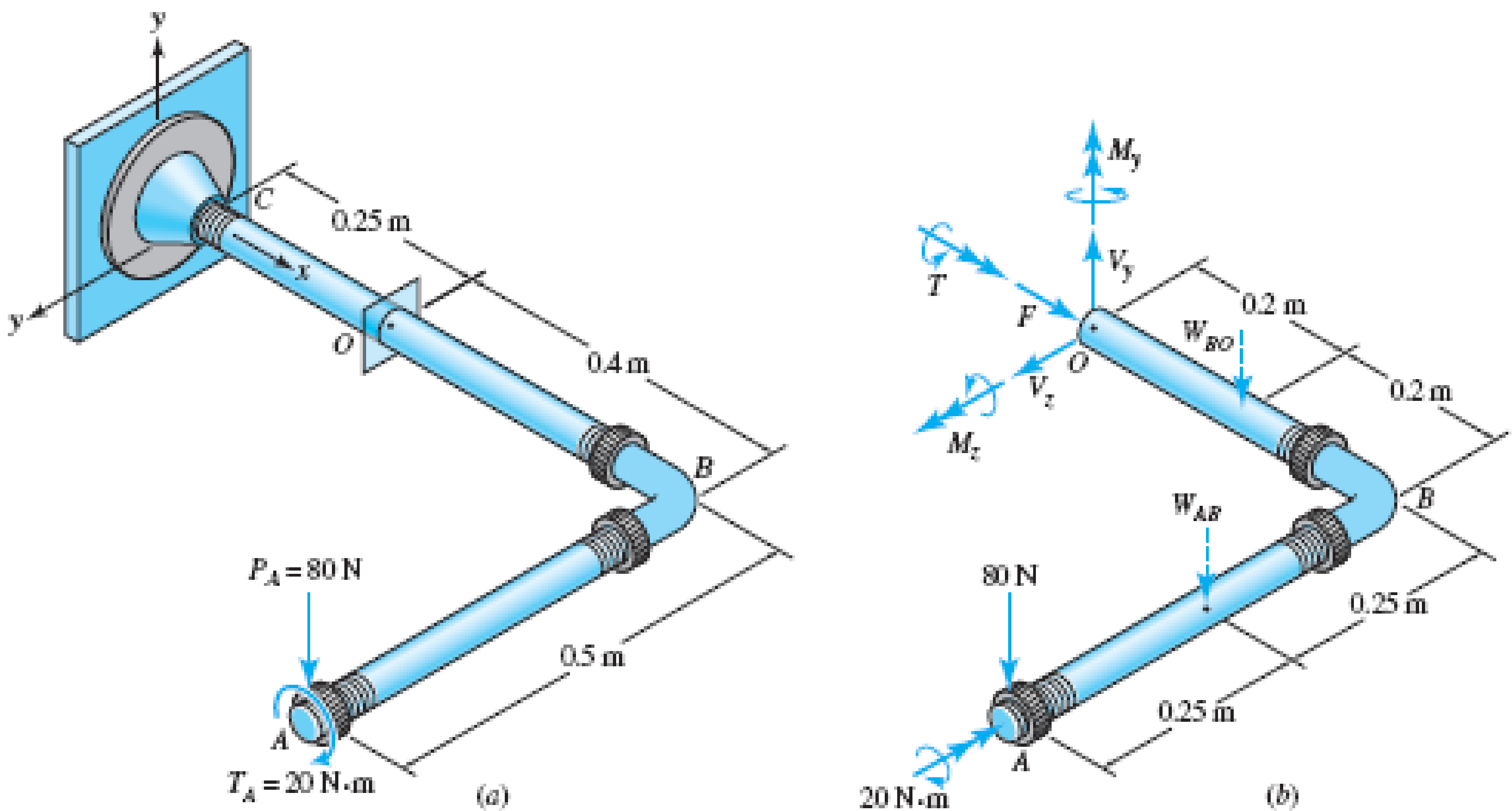
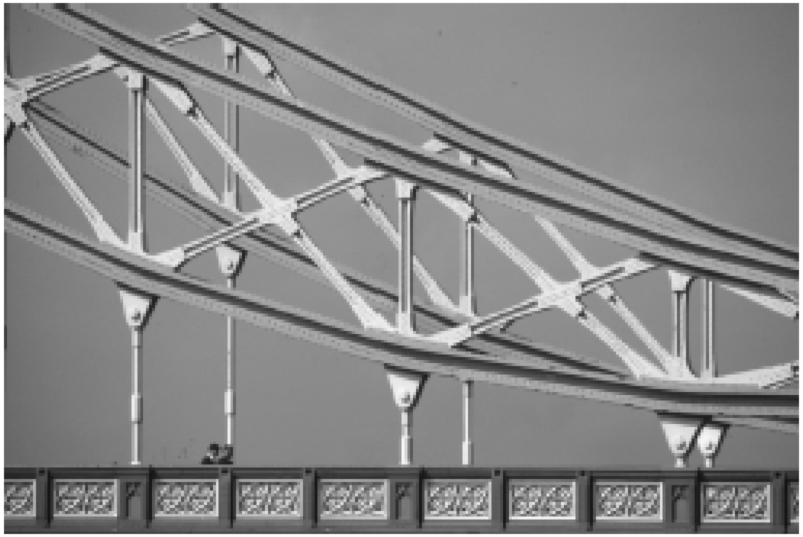


FIGURE 1.9 (a) Pipe assembly; (b) free-body diagram of part ABO.

Internal axial forces

- Method of sections together with equations of equilibrium are used to determine the internal axial forces
- See example next page: stepped bar



These steel tie rods, used to support a portion of a bridge, are subjected to tensile forces. *Michael S. Yamashita/Corbis Images.*

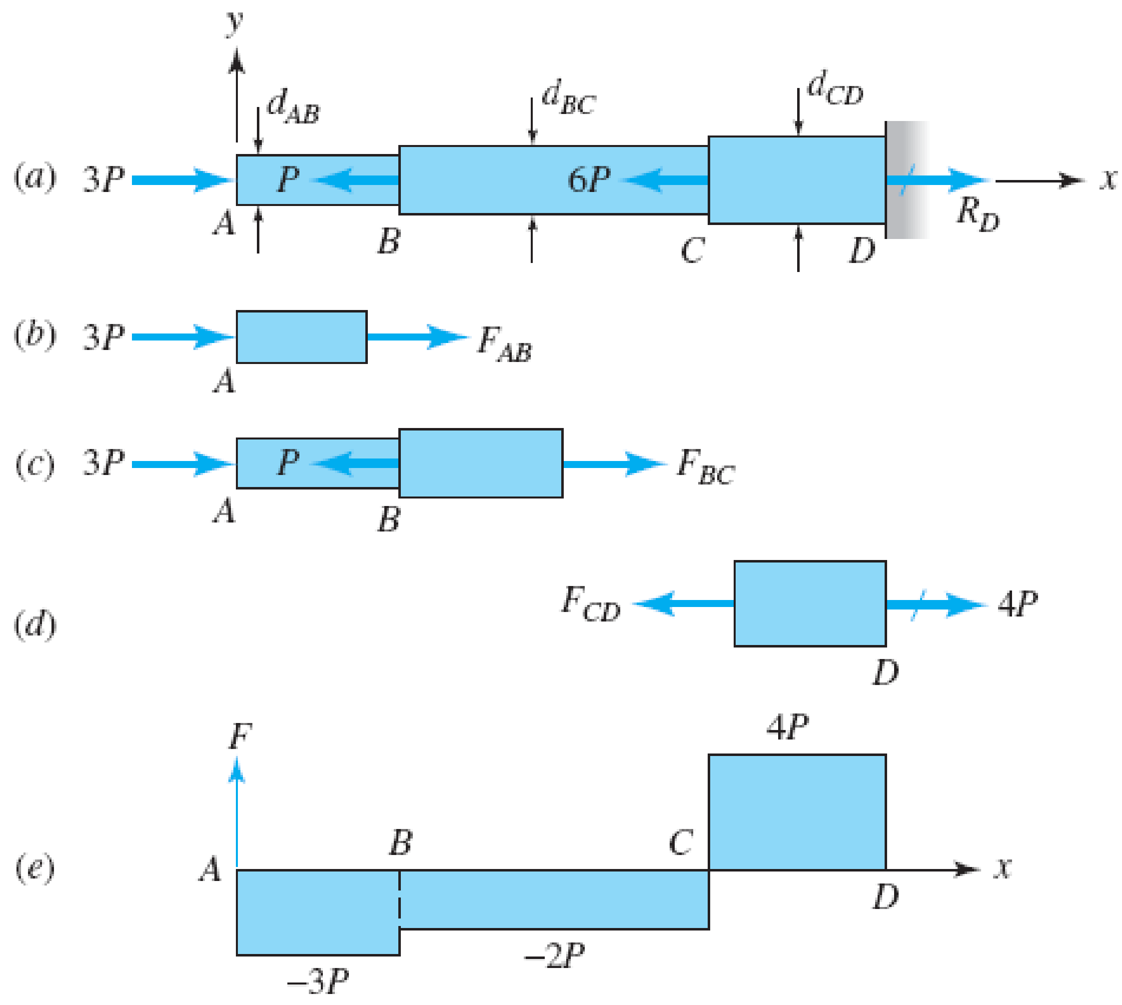


FIGURE 2.1 (*a*) Bar consisting of three prismatic parts; (*b–d*) free-body diagrams of the bar segments; (*e*) axial-force diagram.

Normal stress

- Load intensity = load per unit area = stress, σ

$$\sigma = P / A$$

Question:
Why we use „Stress“ instead of „Load“?

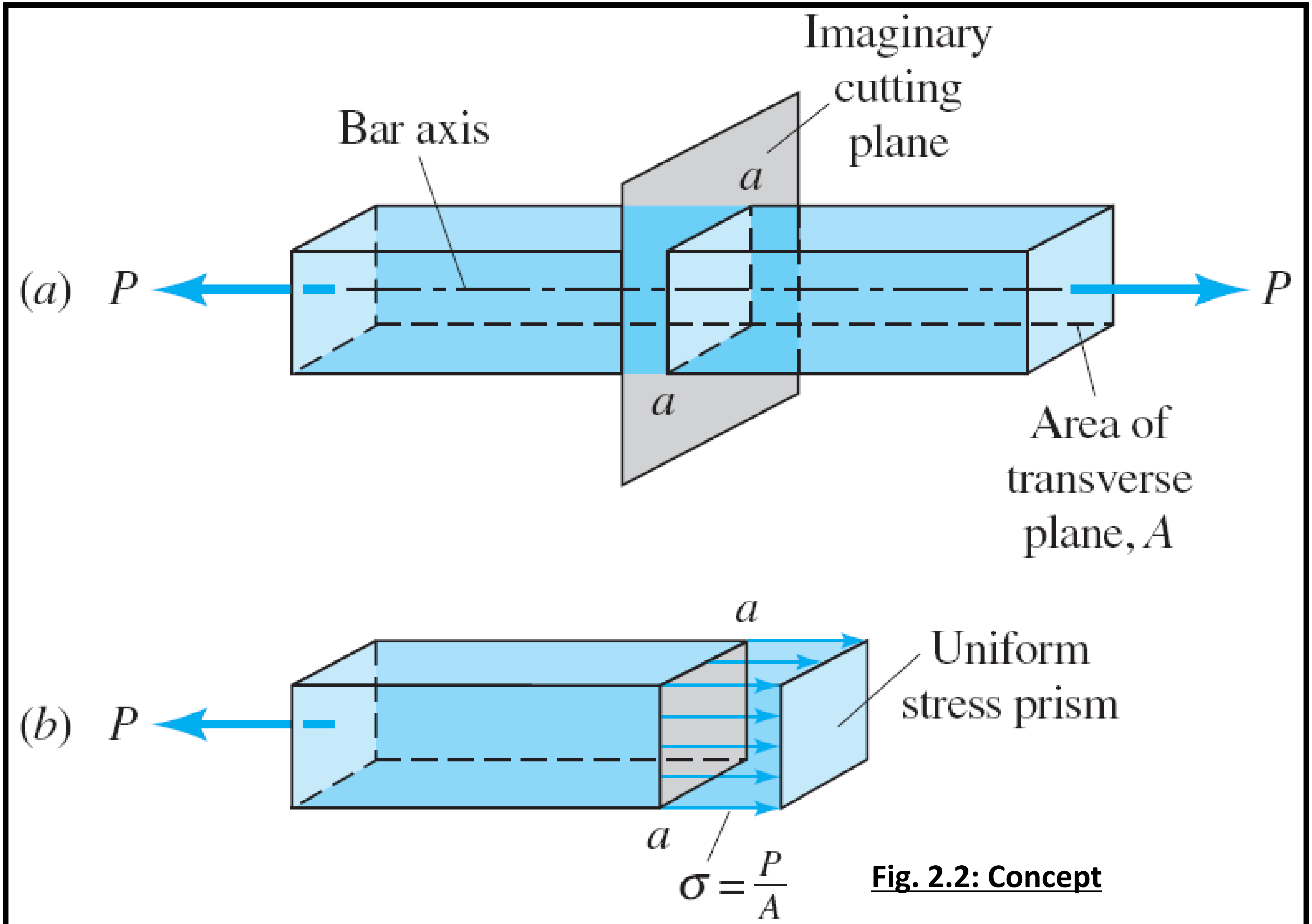
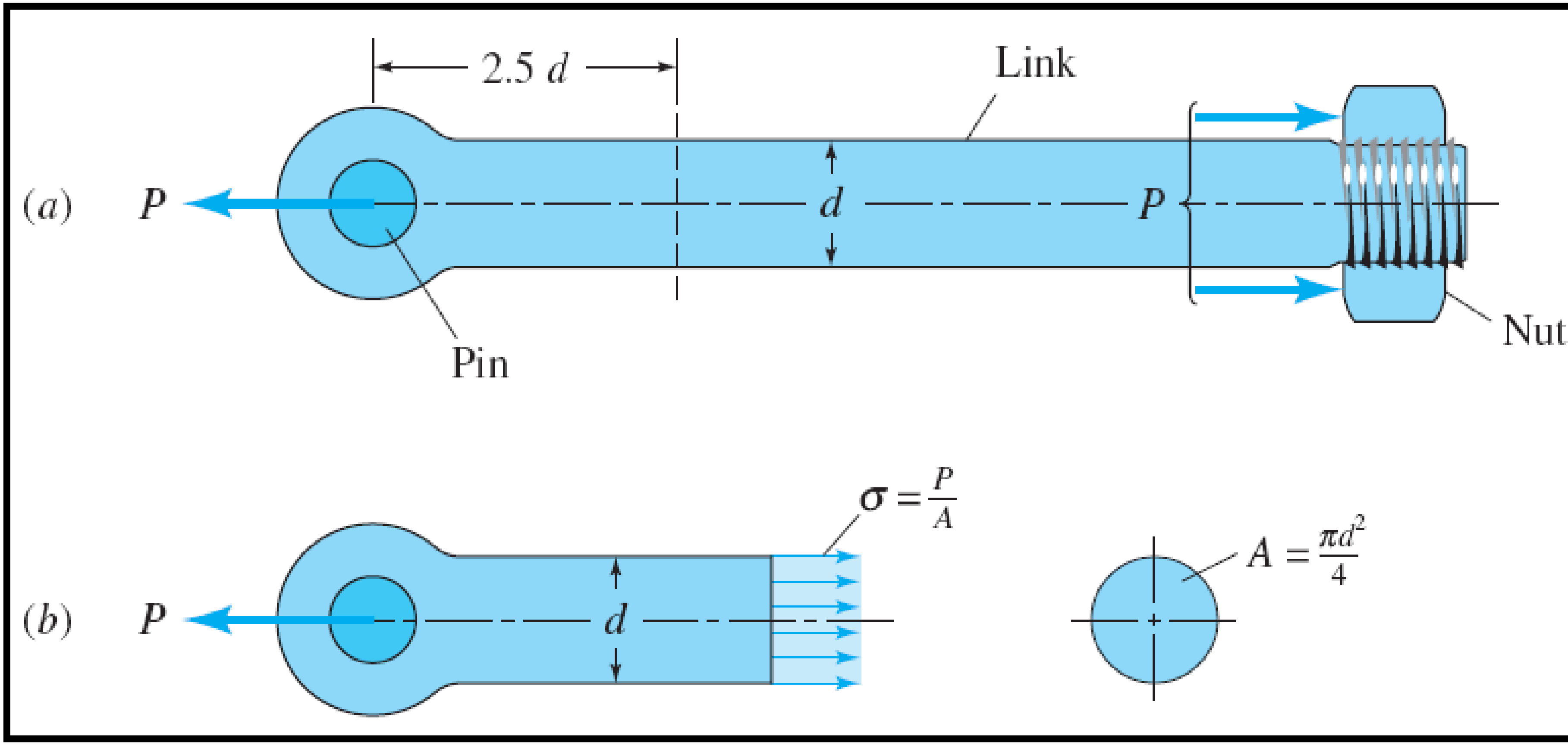


Fig. 2.2: Concept



Application: FIGURE 2.3 (a) A cylindrical tensile link (or eye-bar) loaded through a pin at the left end and a nut at the other end. (b) Free-body diagram of the left part showing uniform stress distribution at the cutting plane.

Assumptions

- The material of the bar is homogeneous (uniform density) and isotropic (same directional properties) cf. anisotropic
- The bar is prismatic (uniform cross section), with no stress raisers such as holes, notches, or threads, etc.
- The bar should have no residual stresses and should not be subjected to temperature changes. cf. thermal stress
- The axial force P acts through the centroid of the cross section (centric loading, to avoid buckling)
- The section (where stress is computed) is remote from a loaded end (Saint – Venant´s principle)

Example 2.2. Maximum normal stress

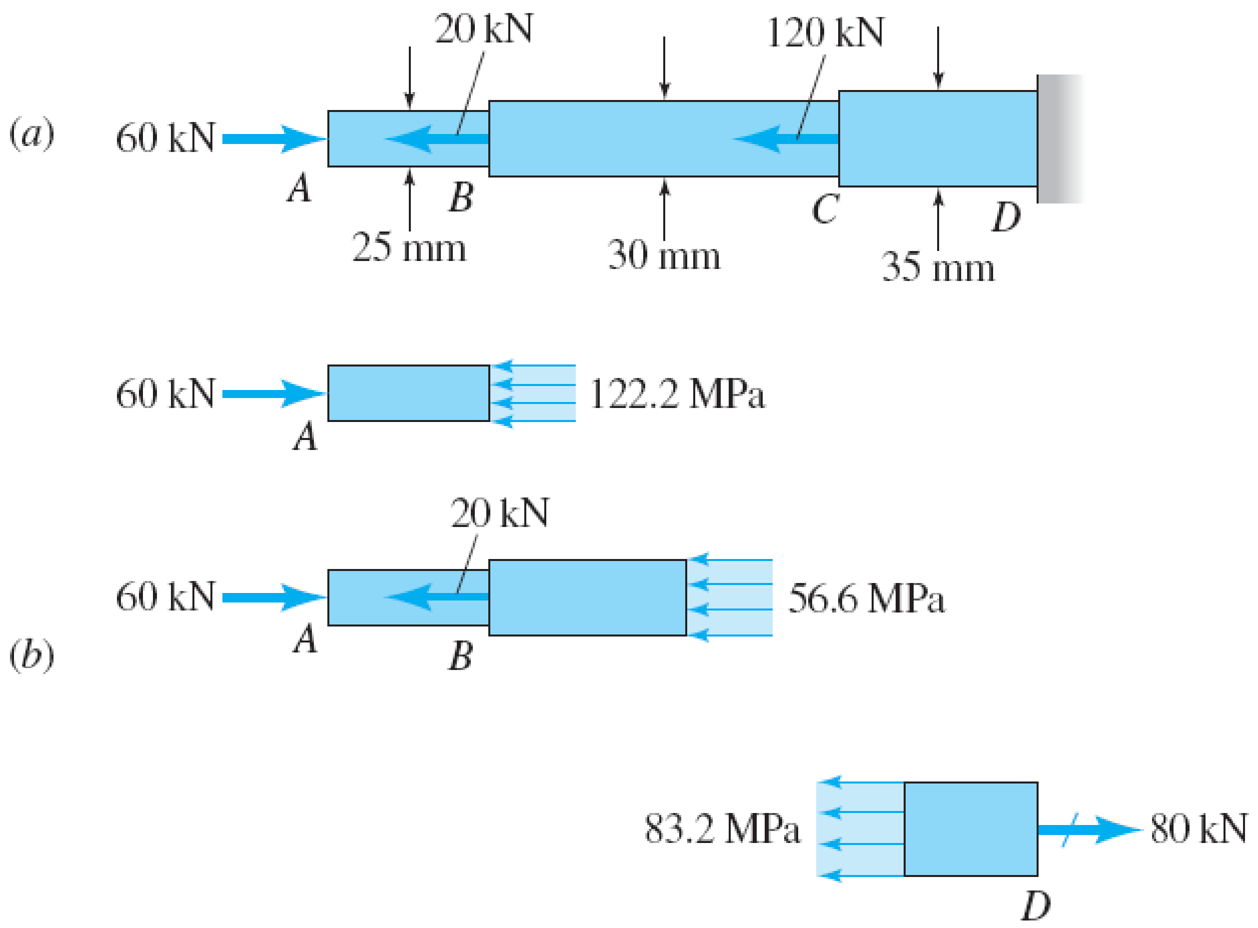


FIGURE 2.4 (*a*) A bar consisting of three parts; (*b*) free-body diagrams showing variation of stress along the length of the bar.

Example 2.3: Maximum normal stress

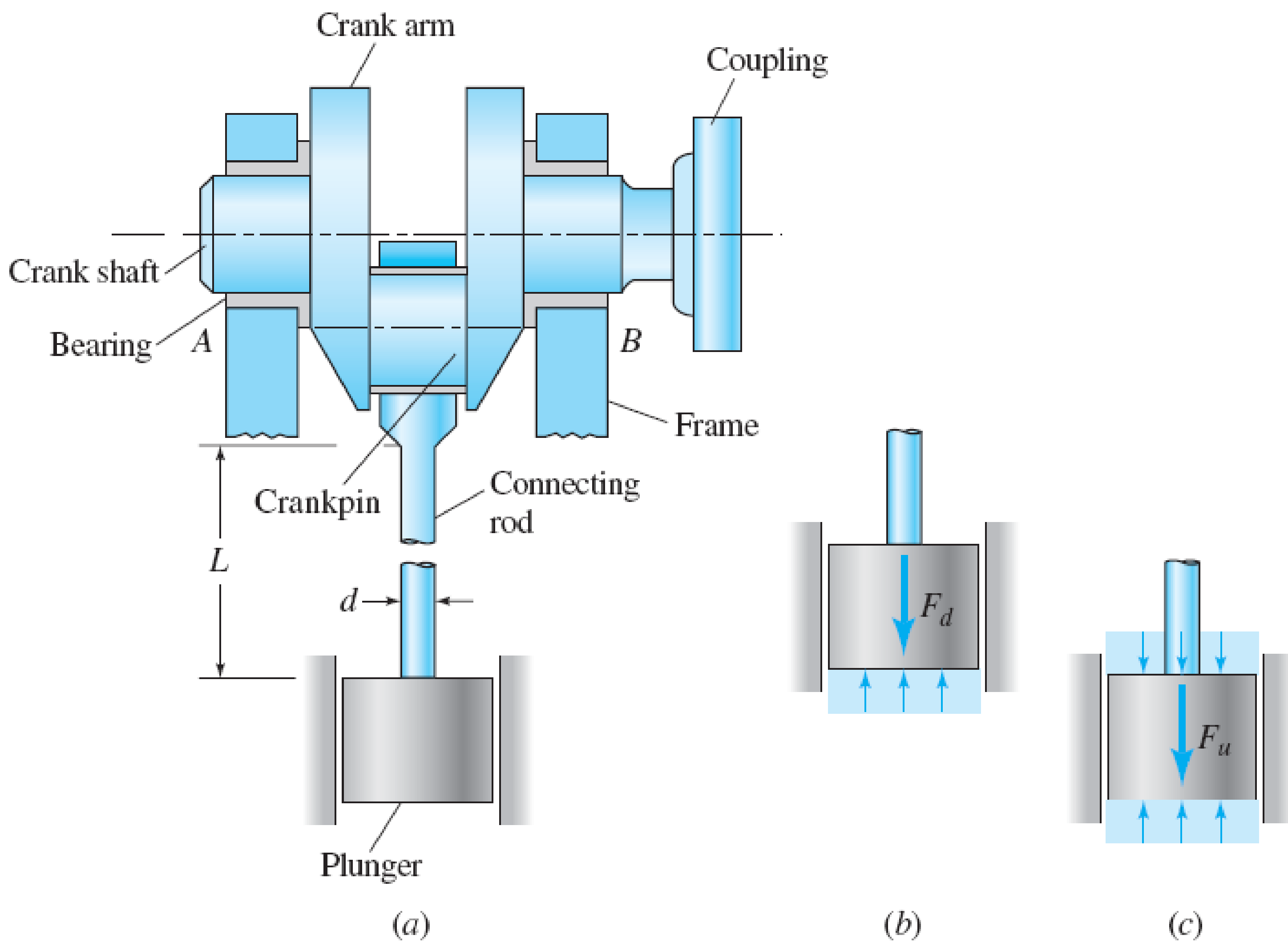


FIGURE 2.5 (*a*) Schematic representation of a well pump. (*b*) Plunger moving down. (*c*) Plunger at the start of upward motion.

Example 2.4: Axial load location

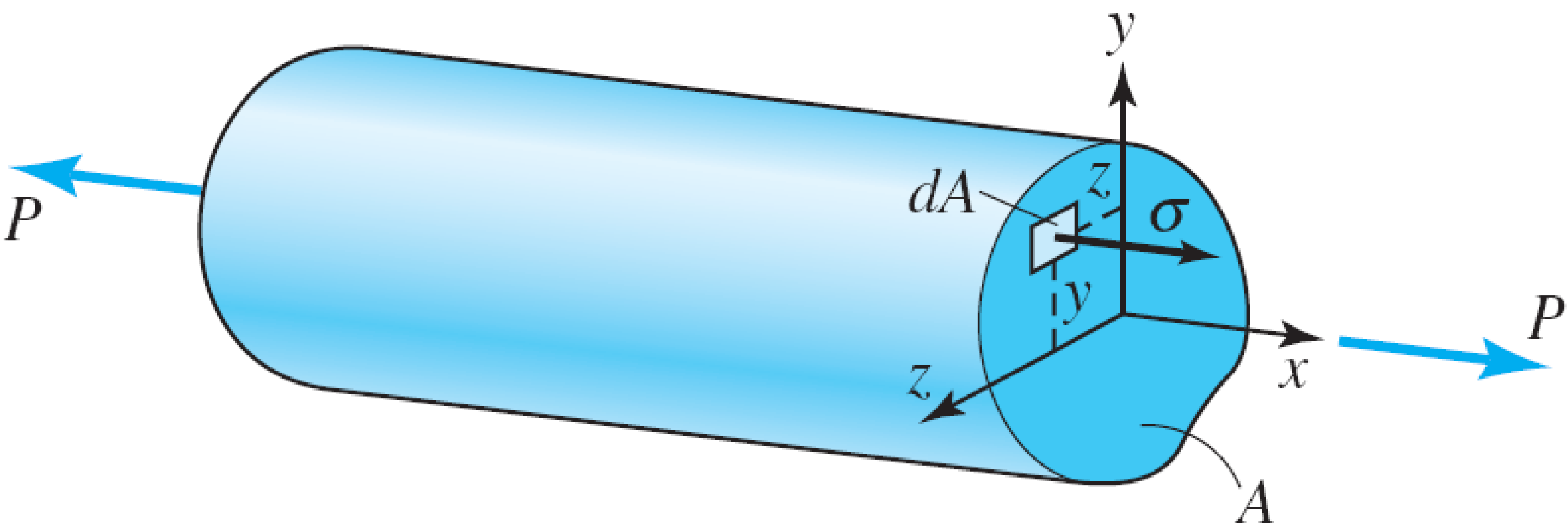


FIGURE 2.6 Axial force P acting at the cross section of a bar.

Bearing stress in connections

- Bearing stress is a compressive stress that occurs as a result of contact (point or surface) between two loaded members

$$\sigma_{bearing} = \text{Axial load, P / Projected area, } A_P$$

Concept and application:

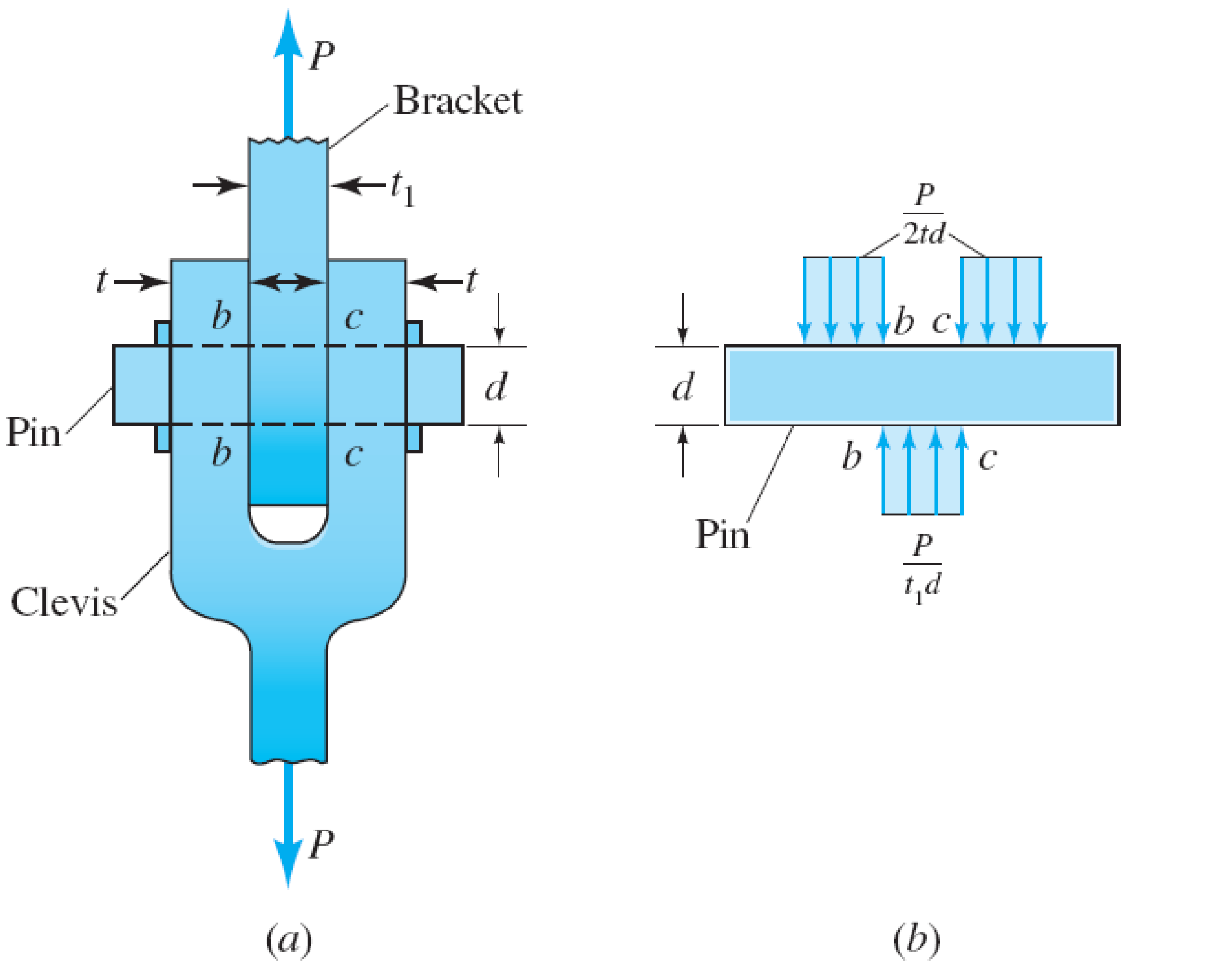


FIGURE 2.7 (*a*) Clevis–pin connection. (*b*) Pin in bearing.

$$\sigma_b = \frac{P}{A_p}$$

Pin is also in shear

Bearing stress:

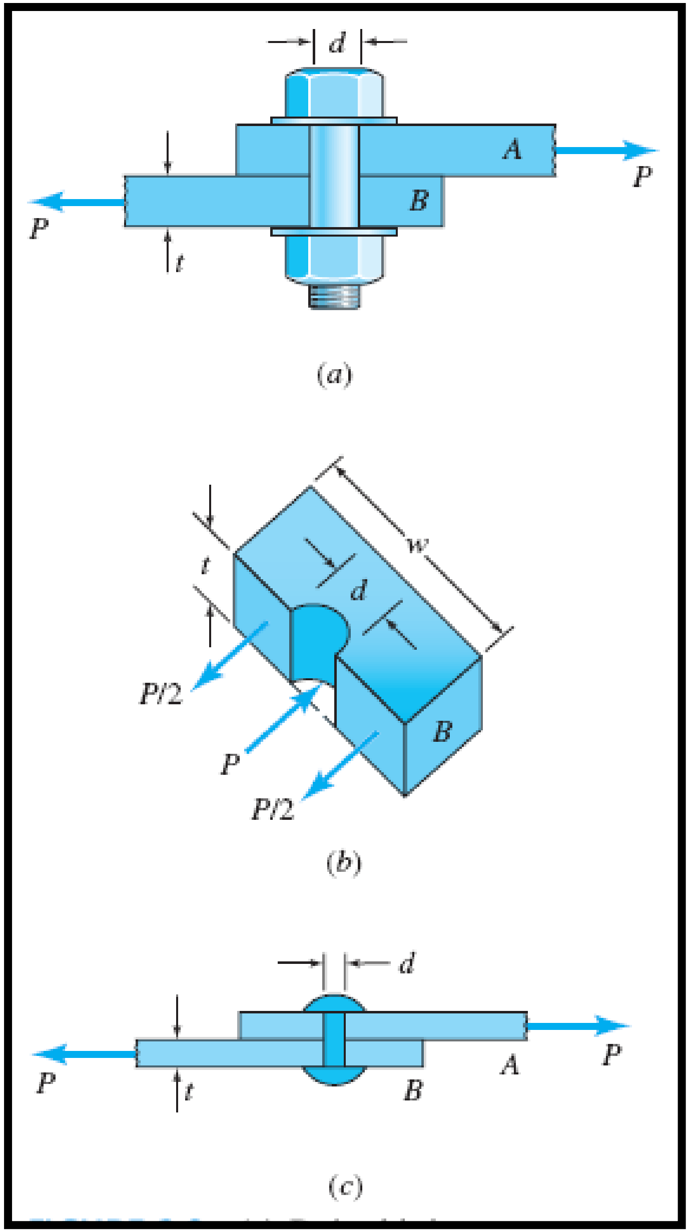
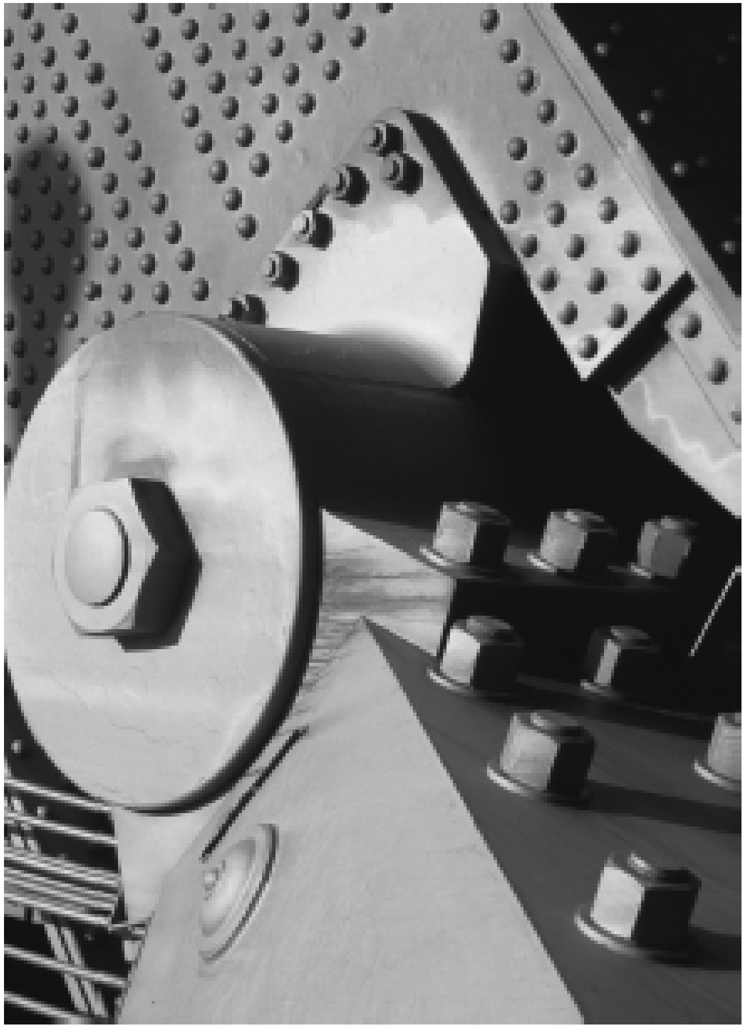


FIGURE 2.8 (*a*) Bolted joint.
(*b*) Plate *B* in bearing.
(*c*) Riveted joint.

Real world example



The connections that join a part of a bridge, built in the 1930s, required a large number of rivets and bolts. *Glenn Beanland/ Lonely Planet Images.*

Shearing (or shear) stresses

Three kinds of shearing stresses:

- Direct shear (mostly due to normal loads) – discussed in this chapter
- Torsional shear (mostly due to torsional loads) – discussed later
- Shear stress or flexural shear (due to transverse loads) – discussed later

Direct shearing stress

$\tau_{average}$ = Shear force, V / A_{shear}

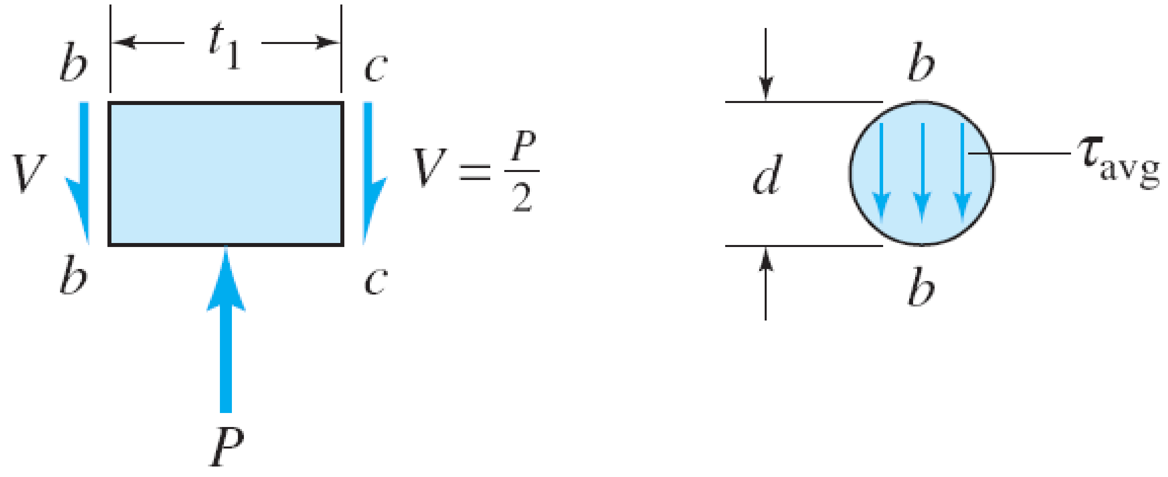


FIGURE 2.9 Portion of pin shown in Fig. 2.7 in double shear.

$$\tau_{avg} = \frac{V}{A}$$

Direct shearing stress

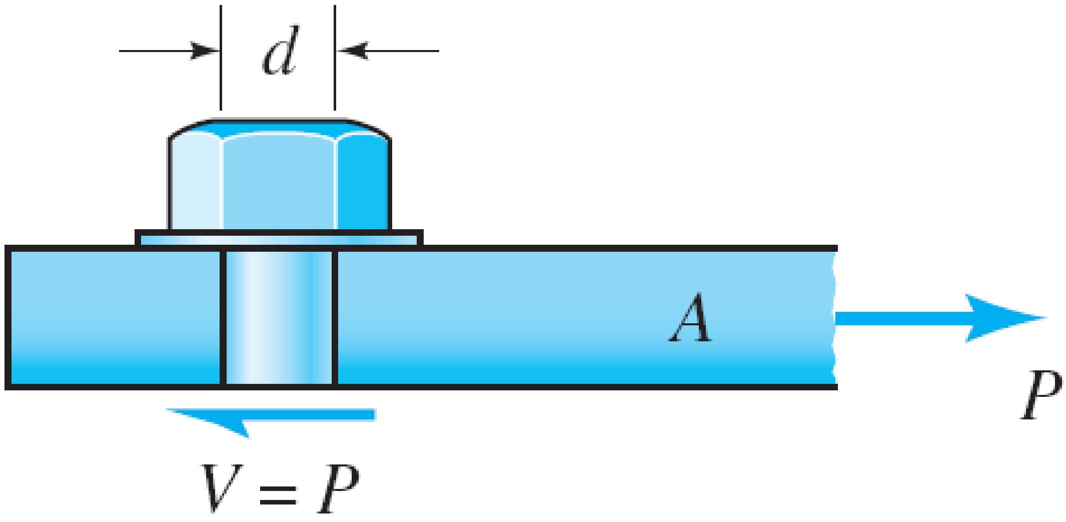


FIGURE 2.10 Free body, upper portion of the bolted joint (Fig. 2.8*a*), in single shear.

$$\tau_{avg} = \frac{V}{A}$$

Direct shearing stress

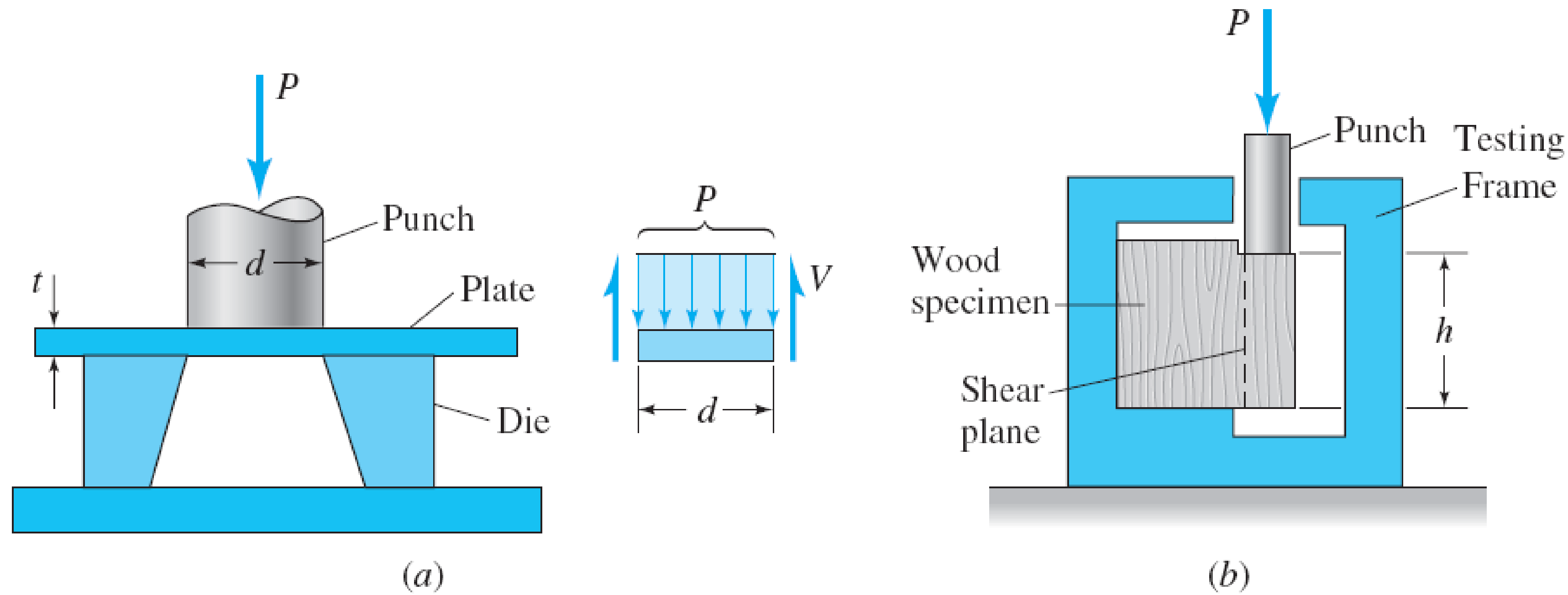


FIGURE 2.11 Transverse shear tests: (*a*) cutting fixture for metals and free body diagram of punched plate; (*b*) shear tool for wood.

Example 2.5

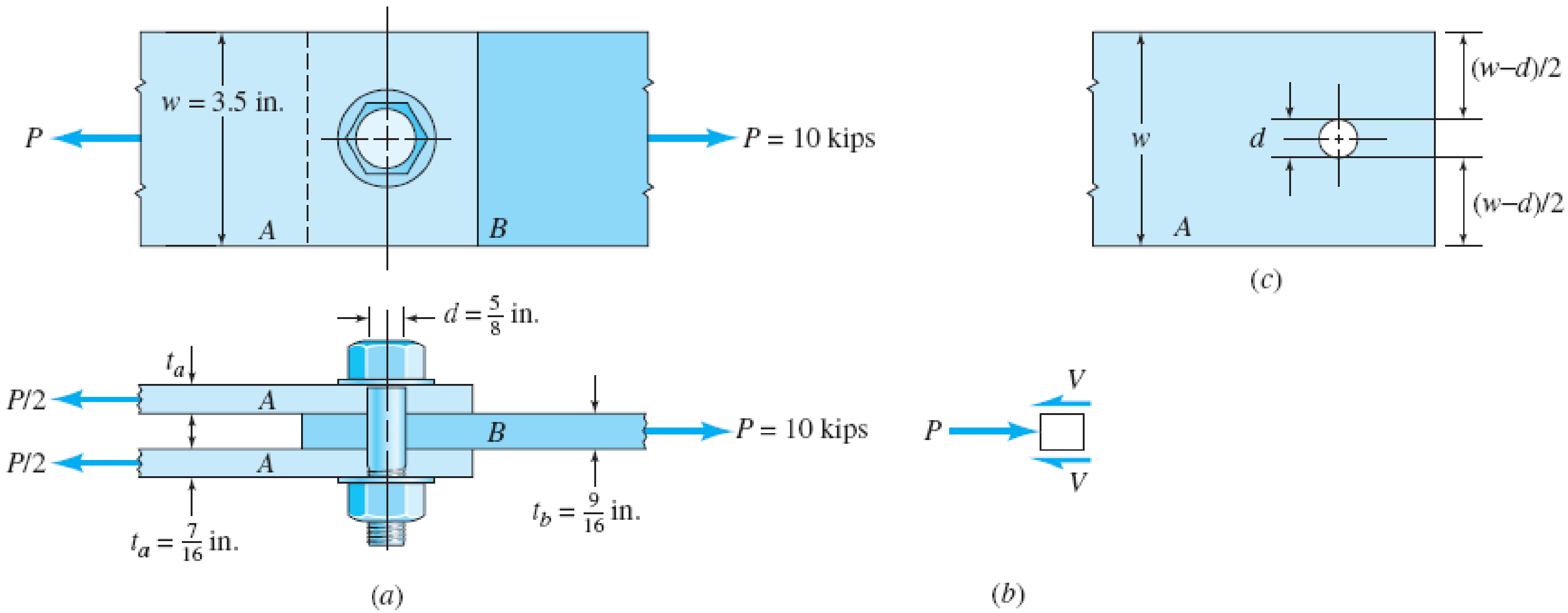


FIGURE 2.12 (*a*) Bolted assembly of three plates; (*b*) free-body diagram of bolt segment; (*c*) plate A.

Example 2.6

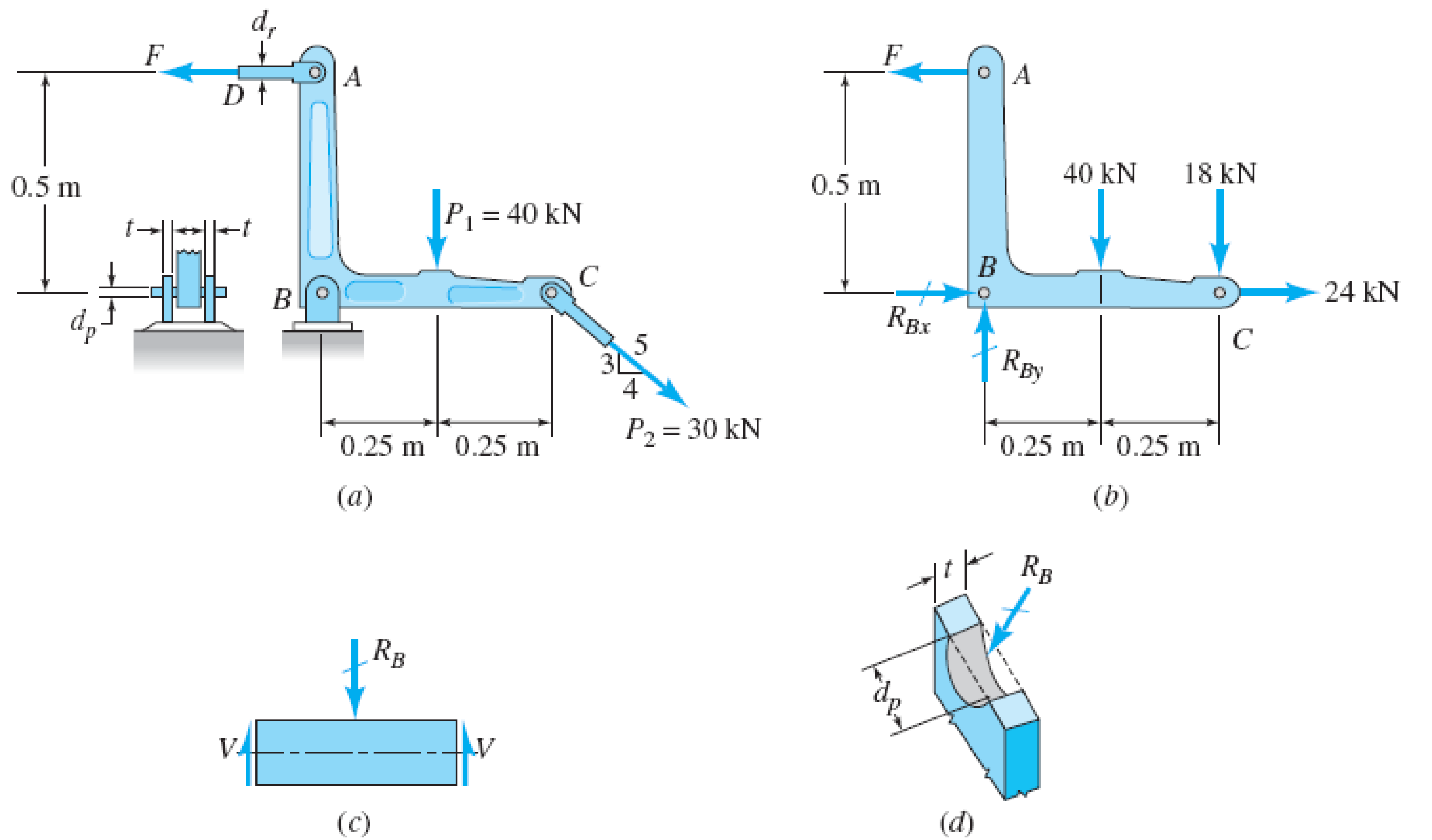


FIGURE 2.13 (*a*) A variously loaded bracket; (*b–d*) free-body diagrams of the angle bracket, pin segment, and part of a support bracket, respectively.

FACTOR OF SAFETY

n_s = failure load/allowable load

n_s = material strength/allowable stress

Factor of safety

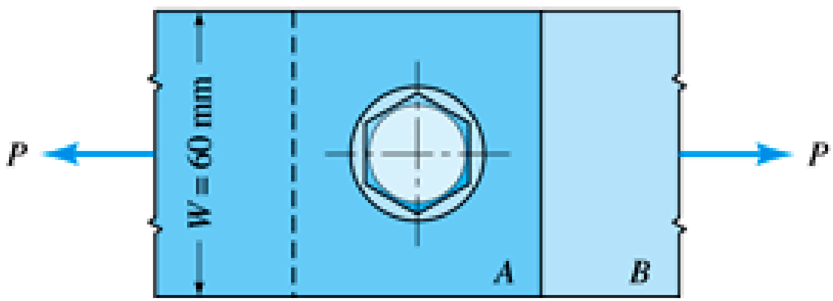
Guidelines:

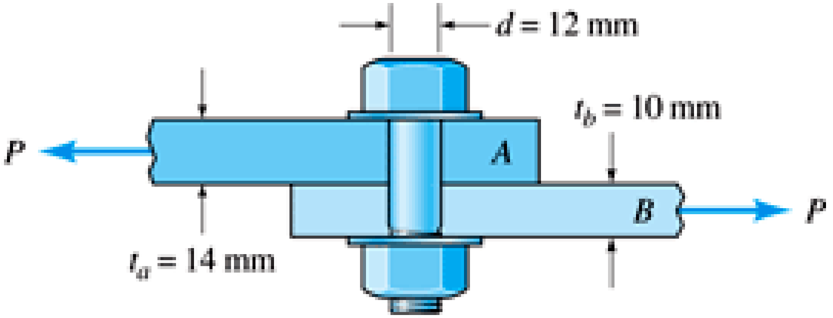
1. 1:25 to 2 for known materials used under controllable conditions and subjected to loads and stresses that can be readily determined with certainty. It is used where low weight is a particularly important consideration.
2. 2 to 3 for average materials operated in ordinary environments and subjected to loads and stresses that can be determined.
3. 3 to 4 for average materials used in uncertain environments or subjected to uncertain stresses.

Codes and standards

AA	Aluminum Association
AISC	American Institute of Steel Construction
AISI	American Iron and Steel Institute
ANSI	American National Standards Institute
ASCE	American Society of Civil Engineers
ASM	American Society of Metals
ASME	American Society of Mechanical Engineers
ASTM	American Society for Testing of Materials
NFPA	National Forest Products Association
ISO	International Standards Organization
SAE	Society of Automotive Engineers
SESA	Society for Experimental Stress Analysis
SPE	Society of Plastic Engineers
UBC	Uniform Building Code

Example 2.9





Example 2.10

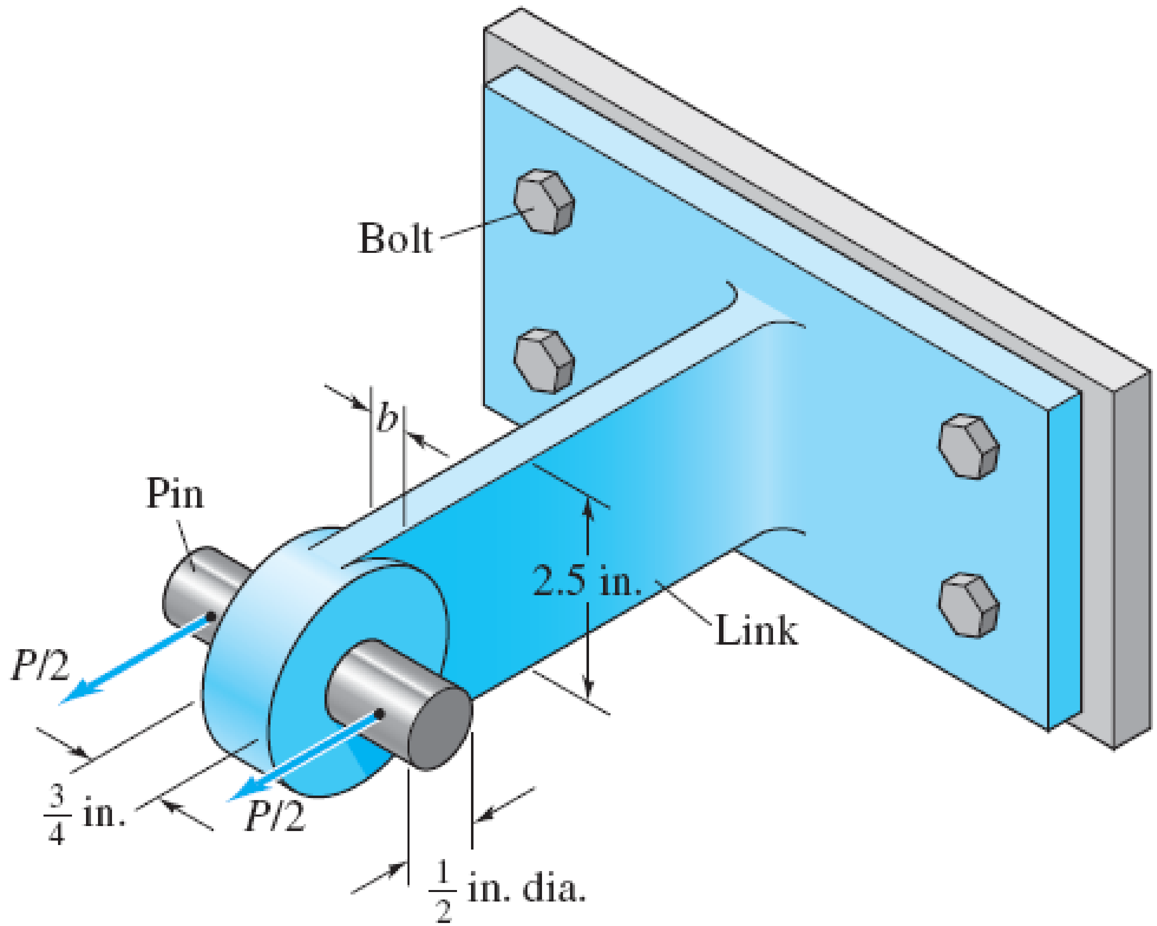
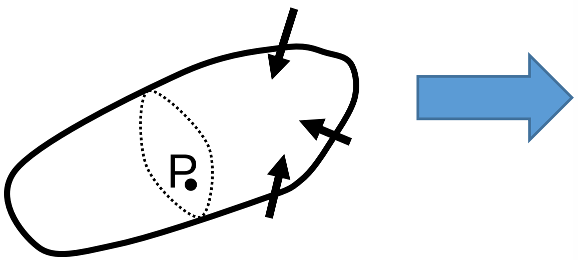


FIGURE 2.17 Bolted assembly.

General stress tensor

Stress on different cuts in the material

Continuum body



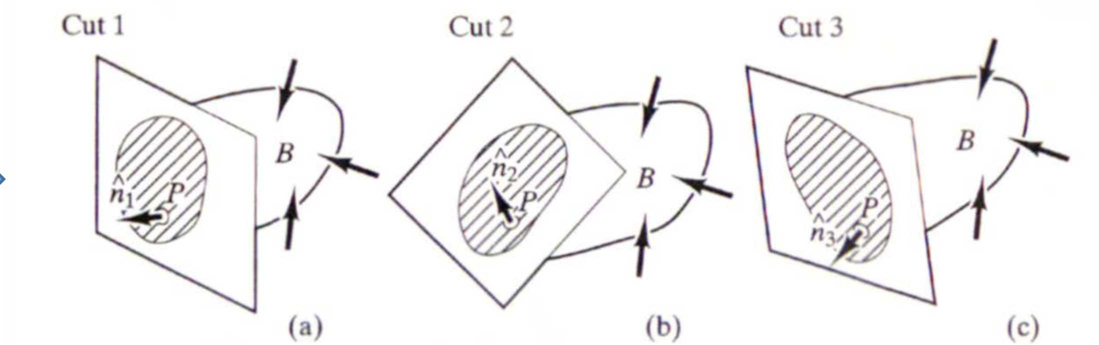


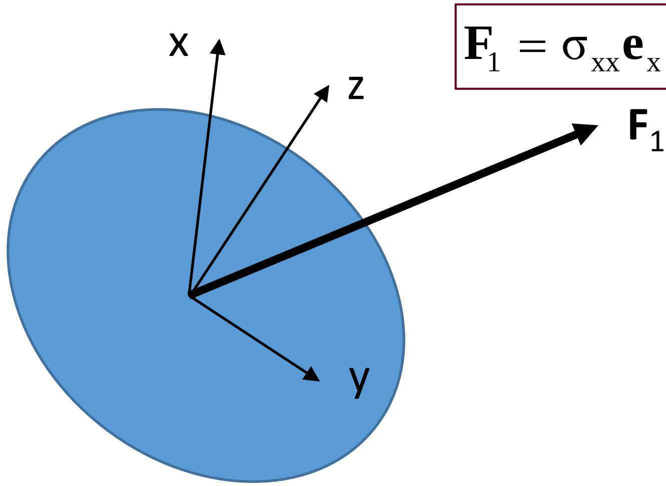
Figure 2.21. Different cuts through the point *P*.

- There are numerous acting planes including point P
- All different acting planes have different acting force
- Because of different acting forces, normal and shear stresses are also different
- To describe stress at point P, both acting plane normal direction and stress direction are needed

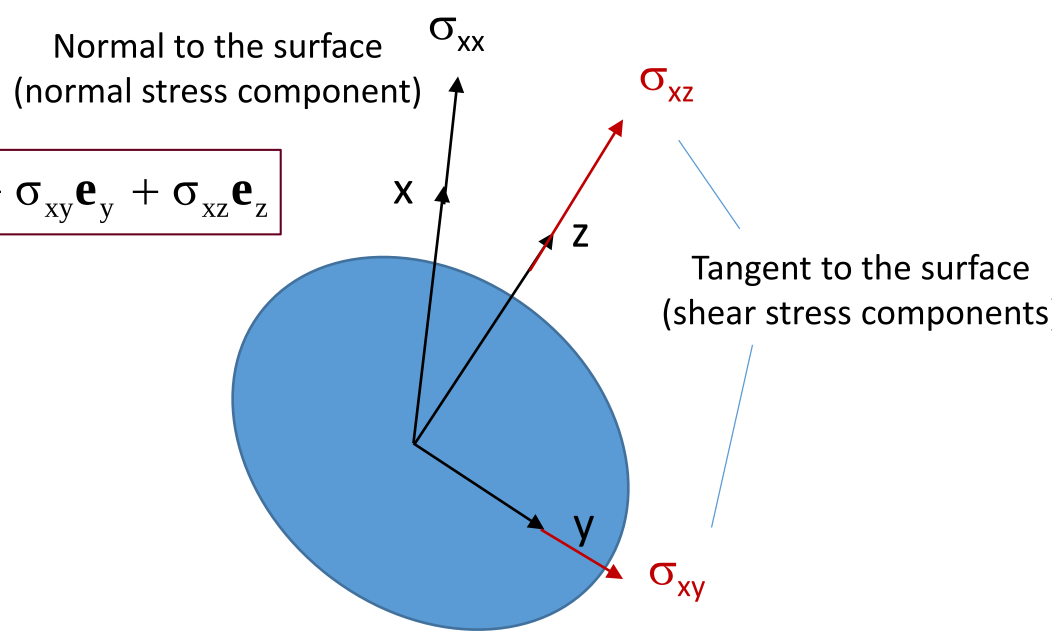
Stress tensor

- Suppose a force $\mathbf{F}_1$ acts on a surface of area A
- This force acts uniformly over the surface
- The force can be resolved in three directions; one acting in the normal and two in the directions parallel to the surface

$$\mathbf{F}_1 = \sigma_{xx}\mathbf{e}_x + \sigma_{xy}\mathbf{e}_y + \sigma_{xz}\mathbf{e}_z$$



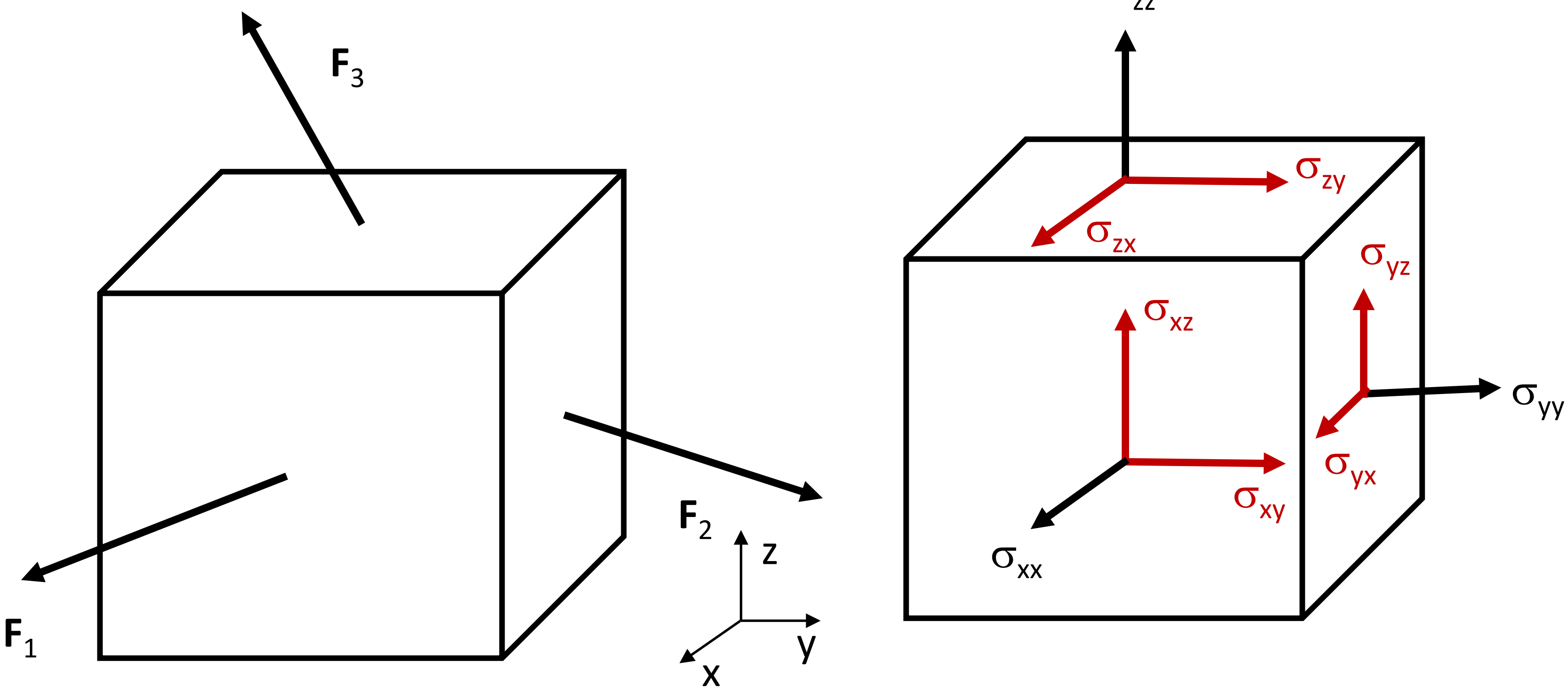
Force on the surface normal to x coordinate



Three components of **force per unit area (= stress)**

Stress tensor

Cartesian coordinate system

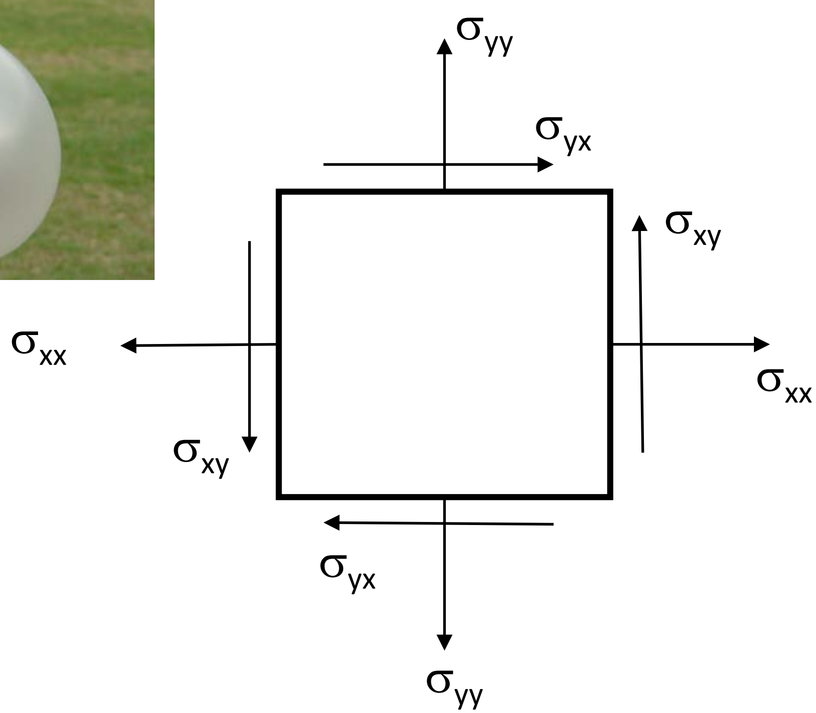


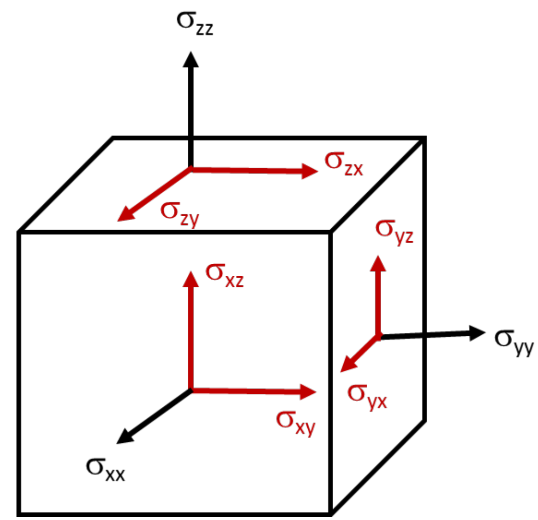
Nomenclature of stress component

σ_{xy}

- y: In the 'y' direction
- x: On 'x' surface

Stress tensor





c.f. Textbook

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

Stress tensor

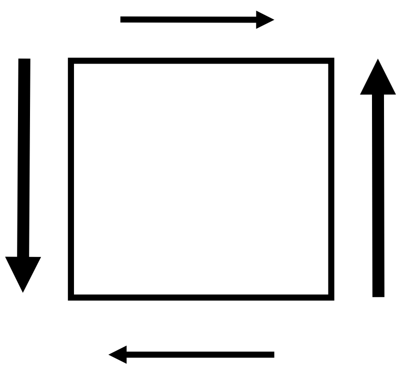
$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$$

2D simplification

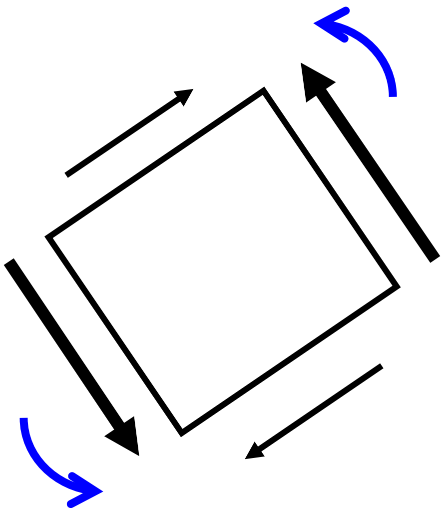
$$\tau_{ij} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}$$

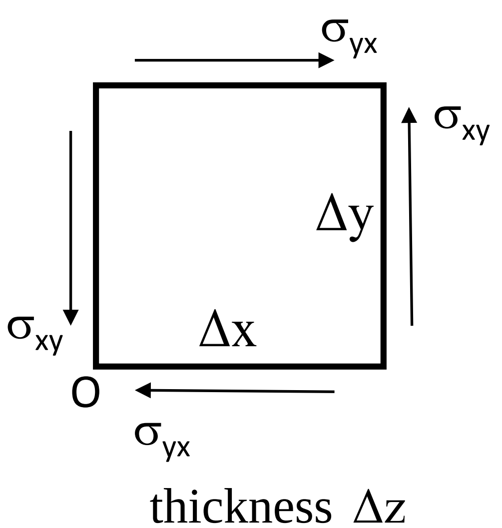
Stress tensor

<u>Symmetry of Tensor using moment equilibrium</u>



What happens?





Moment (or couple) at the reference O should be zero

$$\sigma_{xy}(\Delta y \Delta z)\Delta x - \sigma_{yx}(\Delta x \Delta z)\Delta y = 0$$

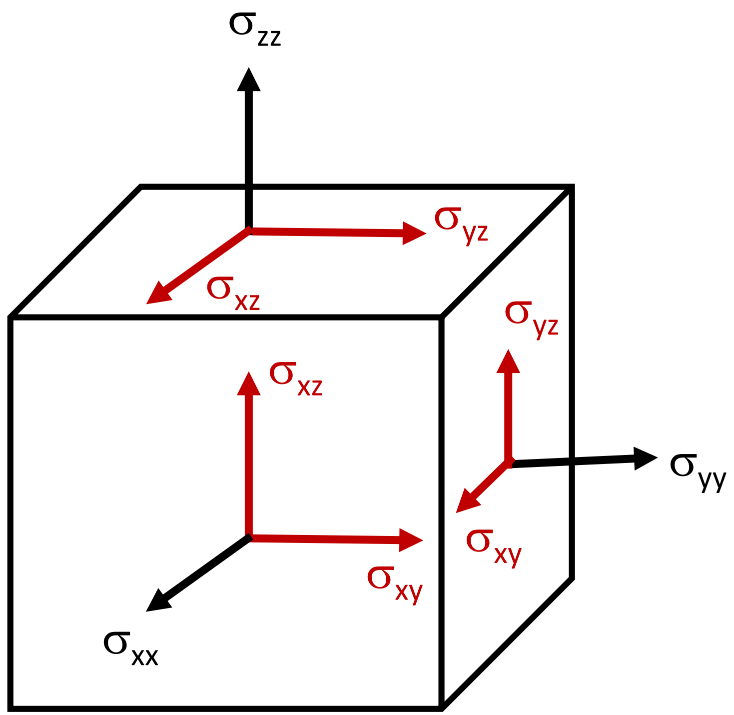
$$\boxed{\sigma_{xy} = \sigma_{yx}}$$

Likewise,

Therefore, only 6 components are independent !

Stress tensor

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$



Stress tensor is symmetric with 6 independent components

1. Three normal components are placed in the diagonal terms (Normal stresses)
2. Three shear components are placed in the off-diagonal terms and they are symmetric (Shear stresses)