3. Discontinuity orientation

▶ Graphical representation of orientation data

① Rose diagram

- Showing dip direction and joint count (percentage).
- Useful for the joints whose dips are similar to each other (ex. dip>60°)
- Radial length (value) of frequency axis may present the total length of joints as well as count (number) / percentage.
- Using strike instead of dip direction is not recommended due to duplication of information across the rose diagram. Ex) N15E/30SE and N15E/30NW
- Pros: Simplicity in presentation of joint orientation and frequency.
- Cons: 1) Lack of dip information → Select data with a specific range of dip angle or show the histogram of dip with the rose diagram as in Fig. 3.2. Even with the dip histogram, however, dip direction and dip of each joint cannot be recognized.

  2) Frequency bars of the rose diagram become wider as their length becomes longer, making their values seem to be exaggerated in terms of areal ratio.

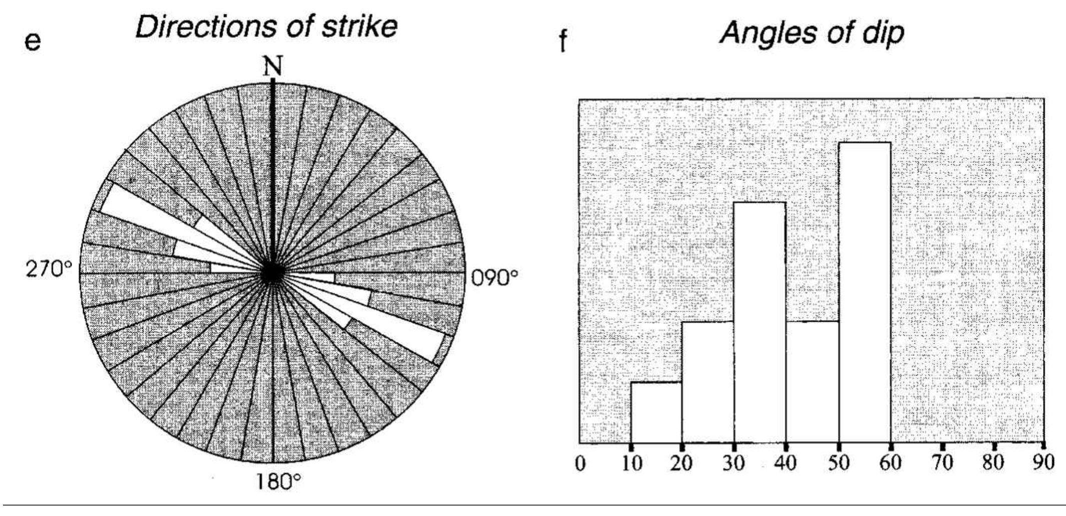


Fig. Rose diagram with dip angle histogram

② Stereographic projection (refer to another handout)

-Features of equal-angle projection: A small circle on the sphere remains a circle in a projection plane even though its size changes.

-Meaning of 'equal-angle': an angle measured at the origin becomes half when it is measured at focus regardless of its location (plunge), while length of a line on the sphere changes by its location in a projection plane.

-Features of equal-area projection: An area of a small circle on the sphere is kept constant in a projection plane regardless of its location even though their shape changes.

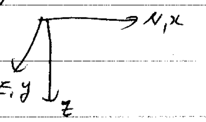
▶ Orientation sampling bias due to a linear survey

- Obtain $\cos\delta$ when trend/plunge of joint normal and scanline are given as α_n/β_n , α_s/β_s, respectively (δ means an acute angle between the joint normal and scanline).

절리법선벡터를 $\vec{e}_n$, scanline 방향벡터를 $\vec{e}_s$ 라 할때

$\cos\delta = |\vec{e}_n \cdot \vec{e}_s|$ (δ를 예각으로 한다면)

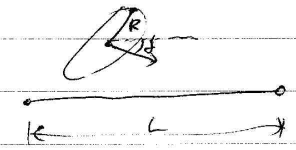
$\vec{e}_n$ 과 $\vec{e}_s$ 를 직각좌표계 성분으로 나타내 보자 (p.72)

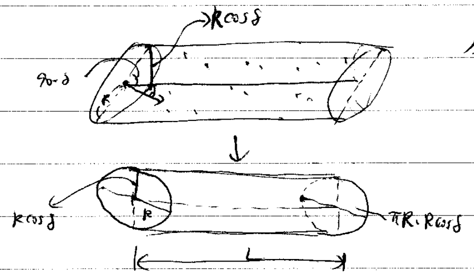


$$\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \begin{pmatrix} \cos\alpha_n \cos\beta_n \\ \sin\alpha_n \cos\beta_n \\ \sin\beta_n \end{pmatrix} \quad \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = \begin{pmatrix} \cos\alpha_s \cos\beta_s \\ \sin\alpha_s \cos\beta_s \\ \sin\beta_s \end{pmatrix}$$

$$\begin{aligned} \therefore |\vec{e}_n \cdot \vec{e}_s| &= |\cos\alpha_n \cos\beta_n \cdot \cos\alpha_s \cdot \cos\beta_s \\ &\quad + \sin\alpha_n \cos\beta_n \cdot \sin\alpha_s \cdot \cos\beta_s \\ &\quad + \sin\beta_n \cdot \sin\beta_s | \\ &= |\cos\beta_n \cos\beta_s \left(\cos\alpha_n \cos\alpha_s + \sin\alpha_n \cdot \sin\alpha_s \right) + \sin\beta_n \cdot \sin\beta_s | \\ &= |\cos\beta_n \cos\beta_s \left(\tfrac{1}{2}\left(\cos(\alpha_n - \alpha_s) + \cancel{\cos(\alpha_n + \alpha_s)}\right) + \tfrac{1}{2}\left(\cos(\alpha_n - \alpha_s) - \cancel{\cos(\alpha_n + \alpha_s)}\right)\right) + \sin\beta_n \sin\beta_s | \\ &= \underline{|\cos\beta_n \cos\beta_s \cdot \cos(\alpha_n - \alpha_s) + \sin\beta_n \sin\beta_s |} \end{aligned}$$

-Correction (weighting) factor for bias due to the orientation of a scanline: Let scanline length L, radius and volumetric frequency of joints R and λ_V, respectively, and the acute angle between joint normal and scanline δ.





실린더의 부피 $V = \pi R^2 \cos\delta \cdot L$

$N_\delta = \rho_V \cdot V = \rho_V \pi R^2 L \cos\delta$

$N_\delta \propto \cos\delta$ (다른 조건이 일정할때)

$\therefore$ Weight $= W = \dfrac{1}{\cos\delta}$

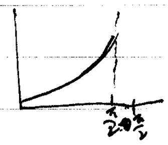
$W \cdot N_\delta = \rho_V \pi R^2 L$ (모든 절리가 scanline에 직교할때의 조사개수)

-Influence of orientation measurement error on weighting factor:

$W = \dfrac{1}{\cos\delta}$, Measurement error : ϵ 라 하면

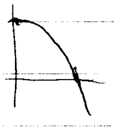
error 가 포함된 weight : $W' = \dfrac{1}{\cos(\delta+\epsilon)}$

$\therefore$ Weight error(%) $= \dfrac{W'-W}{W} \times 100$



$= \left(\dfrac{\cos\delta}{\cos(\delta+\epsilon)} - 1\right) \times 100$

$\left(= \left(\dfrac{\cos\delta}{\sin(90-(\delta+\epsilon))} - 1\right) \times 100 \right.$: Yow (1987)의 제안)



Cosine 함수의 변화율(기울기)이 0~90° 구간중 90°에서 최고이며 0→90°에 따라 증가한다. 따라서 δ가 90°에 가까워질수록 Weight factor의 크기는 크게 변화한다. : Scanline과 절리면이 평행할 수록 환산된 값(빈도?)의 불확실성도 커진다.

$\Rightarrow$ weighting 제한 : 84° $(9.6) = \dfrac{1}{\cos\delta}$

▶ Identifying and delimiting sets

-When joint sets show very complicated patterns of orientation or they are overlapped each other around boundaries, delimiting sets by humans may give different results according to personal experience and knowledge about the site. Here introduced is an objective strategy or algorithm to delimit joint sets.

-Clustering algorithm by Shanley & Mahtab (1976) / Mahtab & Yegulalp (1982)

①Assumption: Joints belonging to a set shows higher degree of clustering, which means their poles are relatively close to each other.

②Strategy: Calculating the degree of clustering of poles in a search cone that moves on a hemisphere → combining 'dense' poles together into a set.

③Theory: Poisson process - probability by which an event occurs t times in a fixed range of time or space (v) can be calculated using an average number of the event occurrences in that time or space range.

$$P(t,v)=\frac{e^{-\lambda v}(\lambda v)^t}{t\,!}=\frac{e^{-m}(m)^t}{t\,!},$$

where λ is a frequency of the event in a unit time or space.

Assumptions of Poisson process (sequence) - 1) Probability of the event occurrence is constant throughout all the time (space) intervals, which means that the event occurs randomly and the probability of the event is proportional to the size of the interval. 2) Events occur independently.

Ex.) We know from the 20 years record of rainstorms over a province that the rainstorm occurs 4 times a year on average. What is the probability of no rainfalls next year?

$$P(0,1)=\frac{e^{-4}(4)^0}{0\,!}=e^{-4}=0.018$$

④Algorithm: 1) Surface area of a hemisphere is set 1.

2) Put a cone whose angle from center to edge is ψ on the sphere.

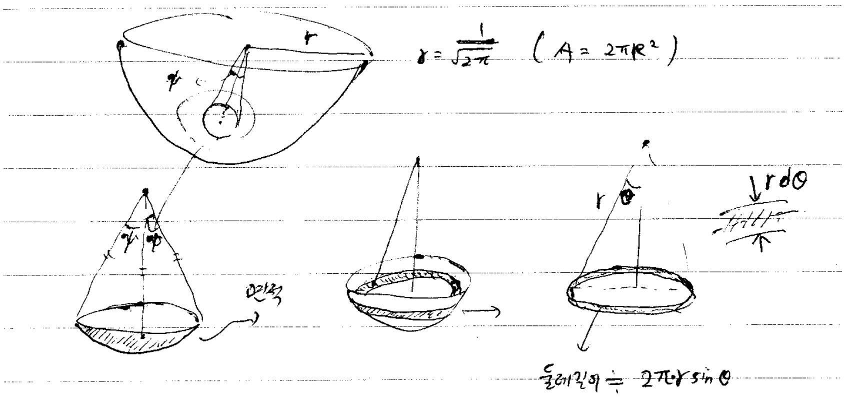
3) Move the cone to a pole so that its center coincides with the pole, and record the number of poles in the cone or the sum of weight from each pole on the cone center.

4) The pole in the center is set 'dense' if the number of poles in the

cone is greater than a critical value (t_{crit}).

5) Poles within an angle of ψ from the 'dense' pole are set to belong to the same set.

⑤ Surface area of a cone



∴ 띠 면적 ≒ $2\pi r^2 \sin\theta \cdot d\theta$

Cone 면적 : $C = \int_0^{\psi} 2\pi r^2 \sin\theta \, d\theta = -2\pi r^2 \cos\theta \Big|_0^{\psi} = 2\pi r^2 - 2\pi r^2 \cos\psi$

$r = \frac{1}{\sqrt{2\pi}}$ 이므로 $C = 1 - \cos\psi$ (2)

⑥ Critical value for ‘Dense’

1) With n poles on a sphere whose area is 1, the probability that t poles are included in a cone whose area is C, P(t,C), can be obtained as follow.

$$P(t, C) = \frac{e^{-nC}(nC)^t}{t\,!}.$$

2) Cumulative probability of P(t,C), P(≤t,C), can be calculated by

$$P(\le t, C) = \sum_{j=0}^{t} \frac{e^{-nC}(nC)^j}{j\,!}.$$

3) Limiting probability S = P(>t_{crit},C) is a monotonically decreasing function as follow.

$$P(> t_{crit}, C) = 1 - \sum_{j=0}^{t_{crit}} \frac{e^{-nC}(nC)^j}{j\,!}$$

S − 0.05: Shanley & Mahtab (1976)

└ C : Mahtab & Yegulalp (1982)

S → t_{crit} can be obtained by interpolation: real number

(required for the cases weighting factors are involved)

Ex.) Example 3.5(p.78)

t_{crit} increases as the cone size increases. With S = 0.05 t_{crit} increases more quickly than with S = C as the cone size increases.

⑦ Cone size

Find a cone size with which following objective function, F, becomes minimum.

$$F = \frac{F_1}{F_2}$$

$$F_1 = \frac{1}{M}\sum_{j=1}^{M}\frac{1}{N_j}\sum_{i=1}^{N_j} d^2(X_j^i, \overline{X_j})$$

$$F_2 = \frac{2}{M(M-1)}\sum_{j=i+1}^{M}\sum_{i=1}^{M-1} d^2(\overline{X_i}, \overline{X_j})$$

where X_j^i indicates a direction vector of the i th joints belonging to the j th joint set; $\overline{X_j}$ is the mean direction vector of the j th joint set; N_j is the number of the j th joint set; M is the number of joint sets; $d(X_j^i, \overline{X_j})$ is the distance between X_j^i and $\overline{X_j}$. Therefore, F_1 indicates the mean distance between the mean direction of the j th set and each joint direction vector while F_2 indicates the mean distance between the mean direction vectors of joint set i and joint set j. Small F means high density of each joint set and large distance between joint set i and j.

⑧ Features of the algorithm

1) With a bigger search cone: The number of poles designated to a joint set increases while the number of 'dense' poles rarely changes. → more chances for the joint sets to overlap each other.

2) Overlap of clusters: lately defined cluster (set) breaks into the previously defined clusters (Fig.3.8).

※ Fig.3.8 and Fig.3.9 show result of clustering with or without weighting.

▶ Representative orientation for a set

-Normalized weighting factor: When weights are assigned to poles, each weight is normalized (W_{ni}) so that the sum of all weights equals to the number of poles (N).

$$N_W = \sum_{i=1}^{N} w_i$$

$$W_{ni} = \frac{w_i N}{N_W}$$

-Mean orientation of a joint set: Obtained by a vector sum of all the joint direction vectors (weighted/unweighted). If some of the direction vectors (poles) are located across the hemisphere edge, their direction vectors should be adjusted to opposite directions before summing the vectors.

-Magnitude of the vector sum: The magnitude of vector sum, $|r_n|$, is always less or equal to the joint number, N. As $|r_n| \rightarrow N$, the degree of clustering becomes higher.

-Relation between the weight and scanline orientation:
Example 3.6 (p84) shows a case to determine the mean orientation of a joint set from 2 scanlines (refer to Table 3.4). There are 31 virtual joint poles in Fig.3.10. We can see that both scanlines are of almost 90° to each other and that the mean orientations with and without weighting are almost same. This is because 1) the bias from scanline orientation has been greatly offset through their relationship in orientation and 2) the poles are comparatively dense so that the difference in weight of each pole becomes small.

▶ Fisher distribution

-Probability density function: (random) variable-frequency(도수) →
variable-relative frequency(probability) → variable-PDF

Ex.) Variable: Uniaxial strength of granite (70~120MPa) = x,
Frequency: No. of each strength division(8,15,22,25,18,12) = frequency

-Assumption of Fisher distribution:

① Poles of a set are isotropically distributed for the mean orientation.

② Areal density of poles on a sphere is proportional to $e^{K\cos\theta}$ so that it becomes smaller as it is farther from the mean orientation.

-Probability in F.D.: Probability that a pole exists at θ from the center.

f(Θ)dΘ (f(Θ) is PDF)

-No. of poles at Θ: C · $e^{K\cos\Theta}$ $\sin\Theta d\Theta$

-Derivation of PDF:

$$\int_0^{\pi} Ce^{K\cos\theta}\sin\theta d\theta = 1 \quad \text{when} \quad C = \frac{K}{e^K - e^{-K}}$$

$$\text{therefore, } f(\theta) = \frac{K\sin\theta\, e^{(K\cos\theta)}}{e^K - e^{-K}}$$

-Estimation of K by Maximum Likelihood Method (최우추정법):

It is assumed that sampled/observed data follow a certain (given) probability function. A joint probability function is built up by setting the unknown parameters independent variables. The independent variables making the joint probability function maximum are calculated. The maximization of the joint probability function means that the sampled data are assumed to show the most probable case.

Applying MLM to F.D. → The angle of each pole from the center Θ_i is an observed data and the distribution of poles is assumed to follow Fisher distribution. The unknown parameter is K which indicates the degree of clustering. Maximizing the joint probability function and obtaining the estimator of K is as follows.

$$Q(\theta) = \prod_i^M \frac{K\sin\theta_i\, e^{K\cos\theta_i}}{e^K - e^{-K}}$$

$$\ln Q = \sum_i^M \left(\ln K\sin\theta_i + K\cos\theta_i\right) - M\ln\left(e^K - e^{-K}\right)$$

$$\frac{\partial \ln Q}{\partial K} = \sum_i^M \left(\frac{1}{K} + \cos\theta_i\right) - M\frac{e^K + e^{-K}}{e^K - e^{-K}} = 0.$$

$$\frac{M}{K} + \sum_i^M \cos\theta_i \rightarrow |r_n| = M\frac{e^K + e^{-K}}{e^K - e^{-K}}$$

$$\therefore \frac{|r_n|}{M} = \frac{e^K + e^{-K}}{e^K - e^{-K}} - \frac{1}{K}$$

-Truncated Fisher (Arnold) distribution:

Fisher distribution defined in a hemisphere

$$f(\theta) = \frac{K\sin\theta\, e^{(K\cos\theta)}}{\left(e^K - 1\right)}$$

- Generalized truncated Fisher distribution

The maximum deviation angle from the center is set Θ_0.

$$f(\theta) = \frac{K \sin \theta \, e^{K \cos \theta}}{\left(e^{K} - e^{K \cos \theta_0}\right)}$$

-Simplification of the estimator of K obtained by M.L.M.:

① Assuming K is bigger than 5.

$$k \approx \frac{M}{M - |r_n|}$$

② If M has big enough value,

$$k \approx \frac{M-1}{M-|r_n|} \approx \frac{1 - 1/M}{1 - |r_n|/M} \approx \frac{1}{1 - |r_n|/M}$$

We can see that as $|r_n| \rightarrow M$ k becomes bigger.

※ Refer to equation (3.22)~ (3.29) at p89~p90.