3. Analyzing strain: Definitions and Concepts

3.1 Introduction

- Strain

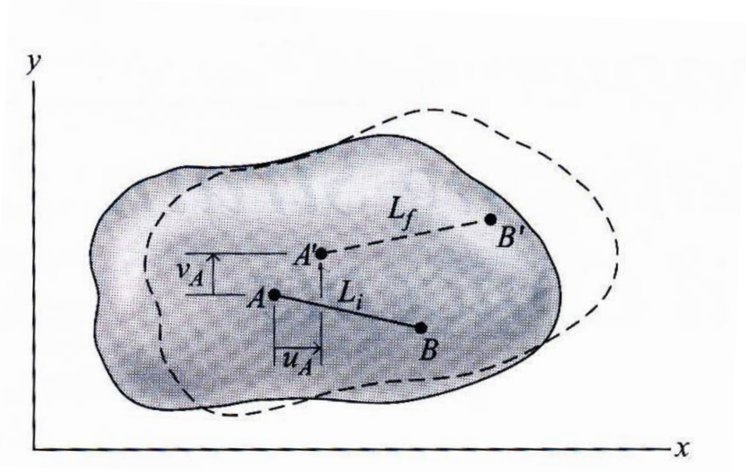
  - The nature of deformations experienced by a real deformable body as a result of internal force or stress distributions will be studied.

  - Methods to measure or compute deformations will be established.

3.2 Displacement, deformation, and strain

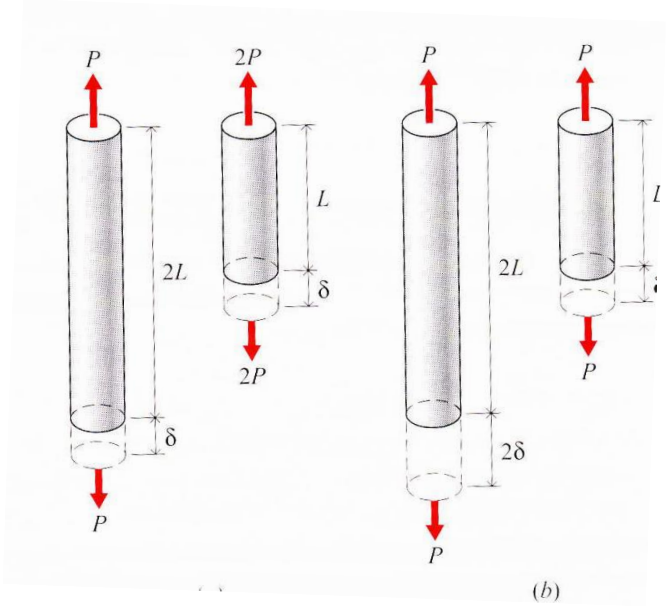
- Displacement
  - Movement of a point with respect to some convenient reference system of axes
  - Vector quantity: from A to A' , (u_A, v_A)
  - Associated with a translation/rotation and change in size/shape

- Deformation
  - Change in any dimension associated with relative displacements (δ)
  - $\delta_{AB} = L_f - L_i$



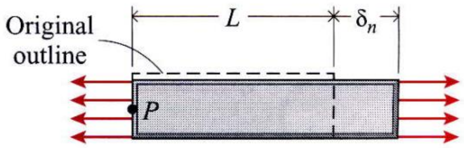
3.2 Displacement, deformation, and strain

- Strain
  - A quantity used to measure the intensity of a deformation
  - Normal strain: rate of elongation or contraction of a line segment (ε)
  - Shear strain: change in angle between two lines that are orthogonal in the undeformed state (γ)



- Average axial strain

$$\varepsilon_{avg} = \frac{\delta_n}{L}$$



- Axial strain at a point

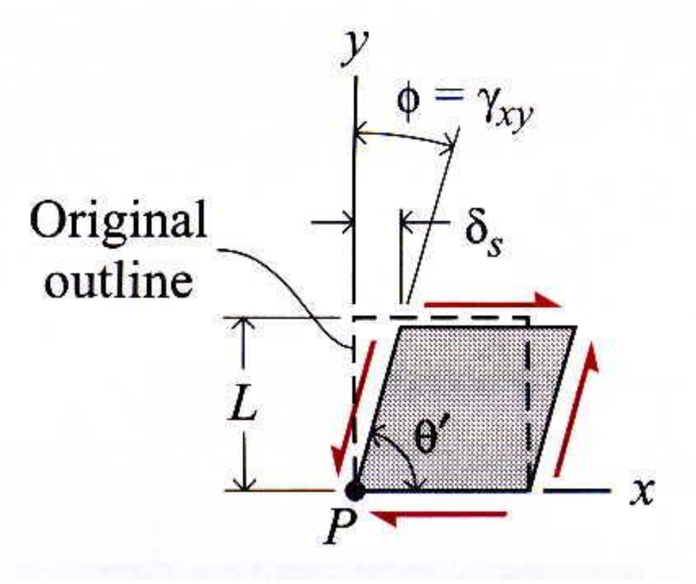
$$\varepsilon(p) = \lim_{\Delta L \to 0} \frac{\Delta \delta_n}{\Delta L} = \frac{d\delta_n}{dL}$$

3.2 Displacement, deformation, and strain

- Shearing Strain

$$\gamma_{avg} = \frac{\delta_s}{L} = \tan\phi \approx \phi$$

$$\gamma_{xy}(p) = \lim_{\Delta L \to 0} \frac{\Delta \delta_s}{\Delta L} = \frac{d\delta_s}{dL} = \frac{\pi}{2} - \theta'$$



- Sign convention of strains
  - Normal strain: tensile strain (+)
    compressive strain (-)
  - Shear strain: increased angle (-)
    decreased angel (+)

3.2 Displacement, deformation, and strain

- Example problem 3-1

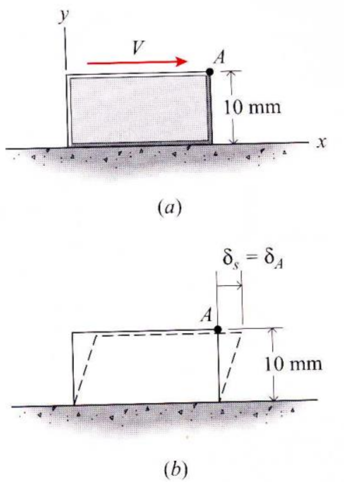
  There is a bar which is of 1-in.-diameter and 8-ft-length. It has 0.5 in.-diameter in a 2-ft central portion. Axial strain in the central portion is 960 μin./in., total elongation of the bar is 0.04032 in., and the diameter of the central portion is 0.49986 in. when an axial load is applied to the ends of the bar.

  - The elongation of the central portion of the bar
  - The axial strain in the end portions of the bar
  - The diametral strain in the central portion of the bar

- Example problem 3-2

  The shear force V produces an average shearing strain γ_{avg} of 1000 μrad.

  - Horizontal displacement of point A



3.3 The state of strain at a point

- State of strain

  - Completely determined by defining 3 normal strains and 3 shear strains on the faces of a rectangular parallelepiped

$$\varepsilon_x = \frac{dx' - dx}{dx} = \frac{d\delta_x}{dx} \qquad \gamma_{xy} = \frac{\pi}{2} - \theta'_{xy}$$

$$\varepsilon_y = \frac{dy' - dy}{dy} = \frac{d\delta_y}{dy} \qquad \gamma_{yz} = \frac{\pi}{2} - \theta'_{yz}$$

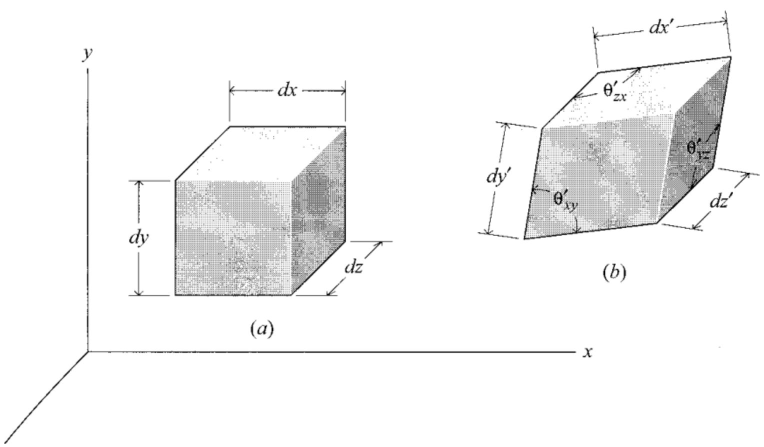
$$\varepsilon_z = \frac{dz' - dz}{dz} = \frac{d\delta_z}{dz} \qquad \gamma_{zx} = \frac{\pi}{2} - \theta'_{zx}$$

$$\longrightarrow$$

$$dx' = (1 + \varepsilon_x)dx \qquad \theta'_{xy} = \frac{\pi}{2} - \gamma_{xy}$$

$$dy' = (1 + \varepsilon_y)dy \qquad \theta'_{yz} = \frac{\pi}{2} - \gamma_{yz}$$

$$dz' = (1 + \varepsilon_z)dz \qquad \theta'_{zx} = \frac{\pi}{2} - \gamma_{zx}$$



$$\varepsilon_n = \frac{dn' - dn}{dn} = \frac{d\delta_n}{dn} \qquad \gamma_{nt} = \frac{\pi}{2} - \theta'_{nt}$$

$$dn' = (1 + \varepsilon_n)dn \qquad \theta'_{nt} = \frac{\pi}{2} - \gamma_{nt}$$

3.4 The strain transformation equations for plane strain

- Normal strain

$$(OB')^2 = (OC')^2 + (C'B')^2 - 2(OC')(C'B')\cos\left(\frac{\pi}{2} + \gamma_{xy}\right)$$

$$\rightarrow \left[(1+\varepsilon_n)dn\right]^2 = \left[(1+\varepsilon_x)dx\right]^2 + \left[(1+\varepsilon_y)dy\right]^2$$

$$- 2\left[(1+\varepsilon_x)dx\right]\left[(1+\varepsilon_y)dy\right]\left[-\sin\gamma_{xy}\right]$$

Substituti ng $dx = dn\cos\theta$ and $dy = dn\sin\theta$

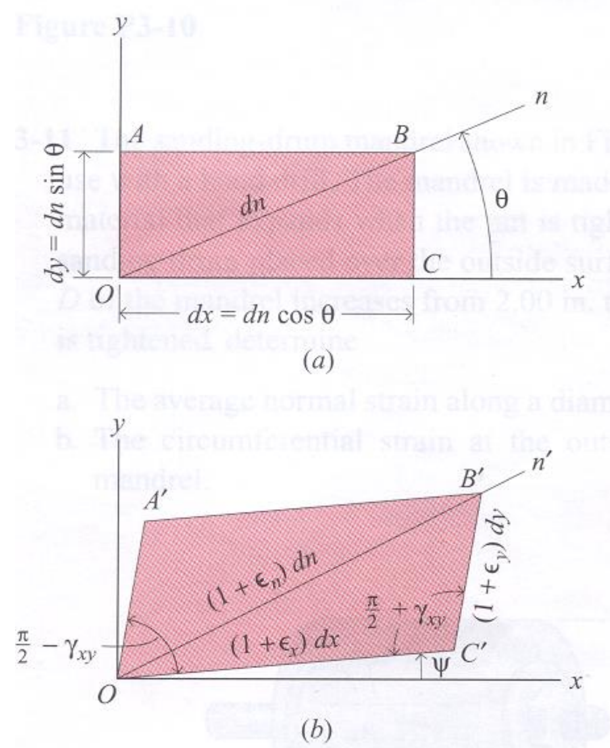
$$(1+\varepsilon_n)^2 dn^2 = (1+\varepsilon_x)^2 dn^2\cos^2\theta + (1+\varepsilon_y)^2 dn^2 \sin^2\theta$$

$$+ 2dn^2 \sin\theta\cos\theta(1+\varepsilon_x)(1+\varepsilon_y)\sin\gamma_{xy}$$

$$\rightarrow 1 + 2\varepsilon_n = (1+2\varepsilon_x)\cos^2\theta + (1+2\varepsilon_y)\sin^2\theta + 2\gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_n = \varepsilon_x\cos^2\theta + \varepsilon_y\sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\rightarrow \varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$



3.4 The strain transformation equations for plane strain

- Shear strain

$$\gamma_{nt} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$\left(\text{c.f. } \tau_{nt} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta\right)$$

- Sign convention

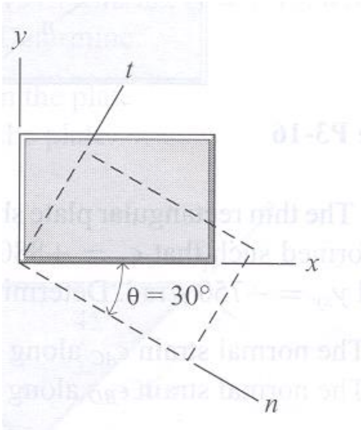
- Tensile strains are positive; compressive strains are negative
- Shearing strains that decrease the angle between two lines are positive
- Angles measured counterclockwise from the x-axis are positive

3.4 The strain transformation equations for plane strain

- Example problem 3-4

$\varepsilon_x = 800\mu, \quad \varepsilon_y = -1000\mu, \quad \text{and} \quad \gamma_{xy} = -600\mu$

Determine ε_n, ε_t, and γ_{nt}

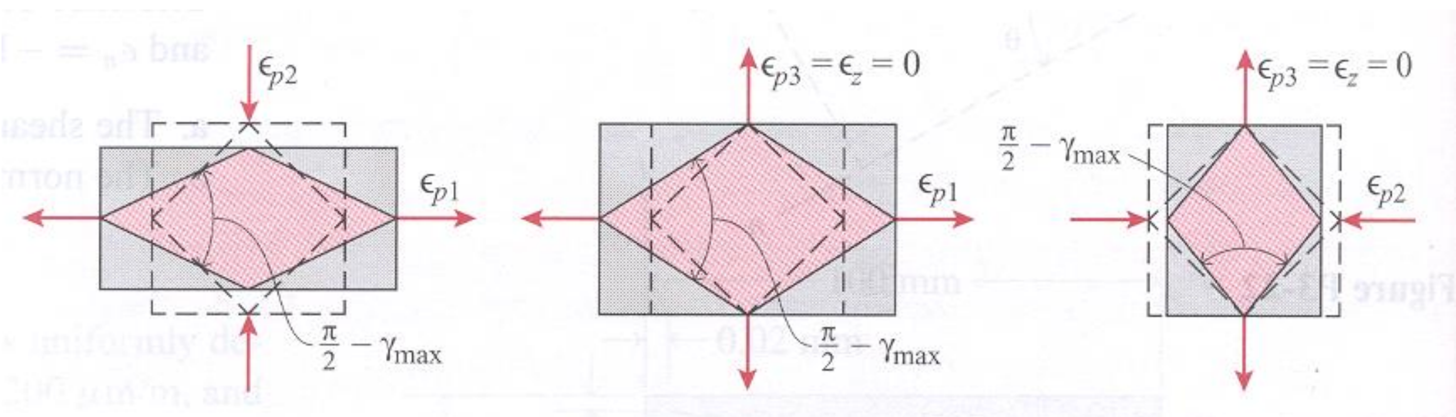


3.5 Principal strains and maximum shear strain

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \quad \left(\text{c.f. } \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right)$$

$$\varepsilon_{p1,p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \quad \left(\text{c.f. } \sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}} \right)$$

$$\gamma_{\max} = 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \quad \left(\text{c.f. } \tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}} \right)$$



3.5 Principal strains and maximum shear strain

- Example problem 3-5

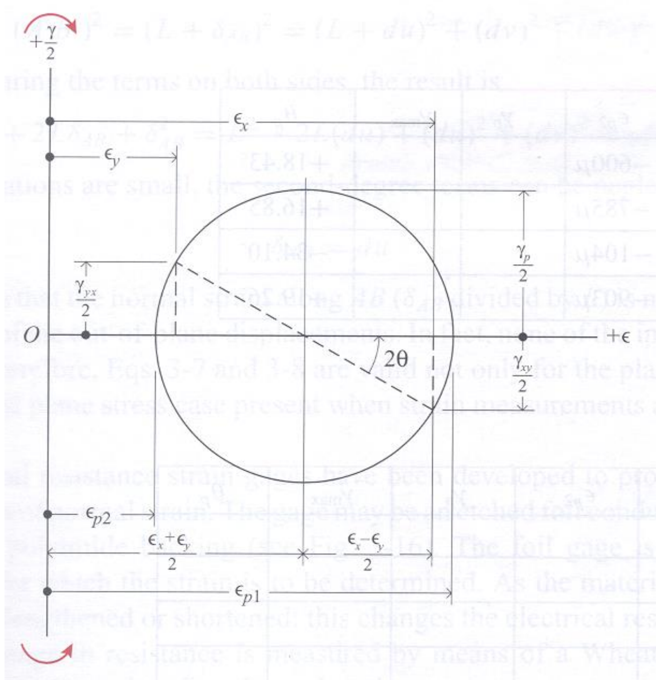
$\varepsilon_x = 1200\mu, \quad \varepsilon_y = -600\mu, \quad \text{and} \quad \gamma_{xy} = 900\mu$

Determine principal strains and the maximum shear strain

3.6 Mohr’s circle for plane strain

$$\left(\varepsilon_n - \frac{\varepsilon_x + \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{nt}}{2}\right)^2 = \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$



3.7 Strain measurement and rosette analysis

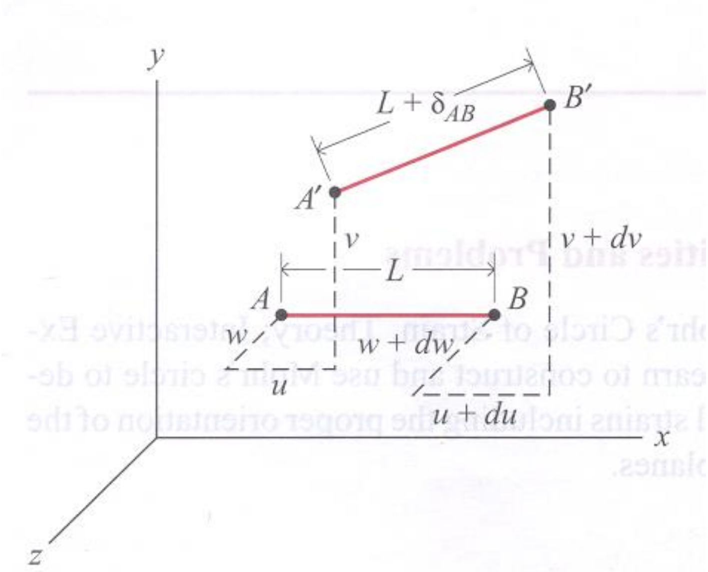
- Normal strain is not affected by the presence of the out-of-plane displacements: strains for a plane strain case are valid for a plane stress case

$$(A'B')^2 = (L + \delta_{AB})^2 = (L + du)^2 + (dv)^2 + (dw)^2$$

$$L^2 + 2L\delta_{AB} + \delta_{AB}^2 = L^2 + 2Ldu + (du)^2 + (dv)^2 + (dw)^2$$

Neglecting the second - degree terms

$$\delta_{AB} = du$$



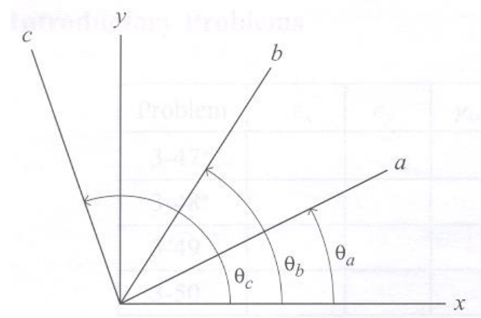
3.7 Strain measurement and rosette analysis

- The electrical resistance strain gages are sensitive only to normal strains: shear strains are obtained by measuring normal strains in two or three different directions

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b$$

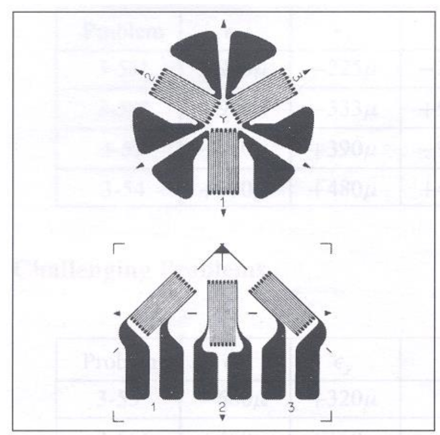
$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c$$



- The out-of-plane principal strain should be considered to obtain the maximum shear strains

$$\gamma_{\max} = \left(\varepsilon_{p\max} - \varepsilon_{p\min}\right) : \left(\varepsilon_{p1} - \varepsilon_{p2}\right), \quad \left(\varepsilon_{p1} - \varepsilon_{p3}\right), \quad \text{or} \quad \left(\varepsilon_{p3} - \varepsilon_{p2}\right)$$

$$\varepsilon_{p3} = -\frac{\nu}{1-\nu}\left(\varepsilon_x + \varepsilon_y\right)$$



3.7 Strain measurement and rosette analysis

- Example problem 3-7

- Principal strains and the maximum shear strain with their orientations on a sketch

Poisson's ratio is 1/3.

$\varepsilon_a = 1000\mu \quad \varepsilon_b = 750\mu \quad \varepsilon_c = -650\mu$