445.204

Introduction to Mechanics of Materials

(재료역학개론)

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Chapter 3

3.1. Concept of Strain

3.2. Material (Mechanical) Properties

Outline

- Deformation
- Definition of Strain
- Components of Strain
- Stress–Strain Diagrams
- True Stress and True Strain
- Elastic versus Plastic Behavior
- Hooke's Law
- Poisson's Ratio
- Generalized Hooke's Law
- Strain Energy
- (optional) Impact Strength
- (optional) Fatigue
- Permanent Deformation
- General Properties of Materials

In this chapter …

- Concept of deformation and strain
- Small deformation theory
  - Hooke´s law
  - Principle of superposition
- Material properties in 1D
  - Tension/compression or axial strains
  - Shear strains
- Hooke‘s law
- Various mechanical properties

3.1. Concept of Strain

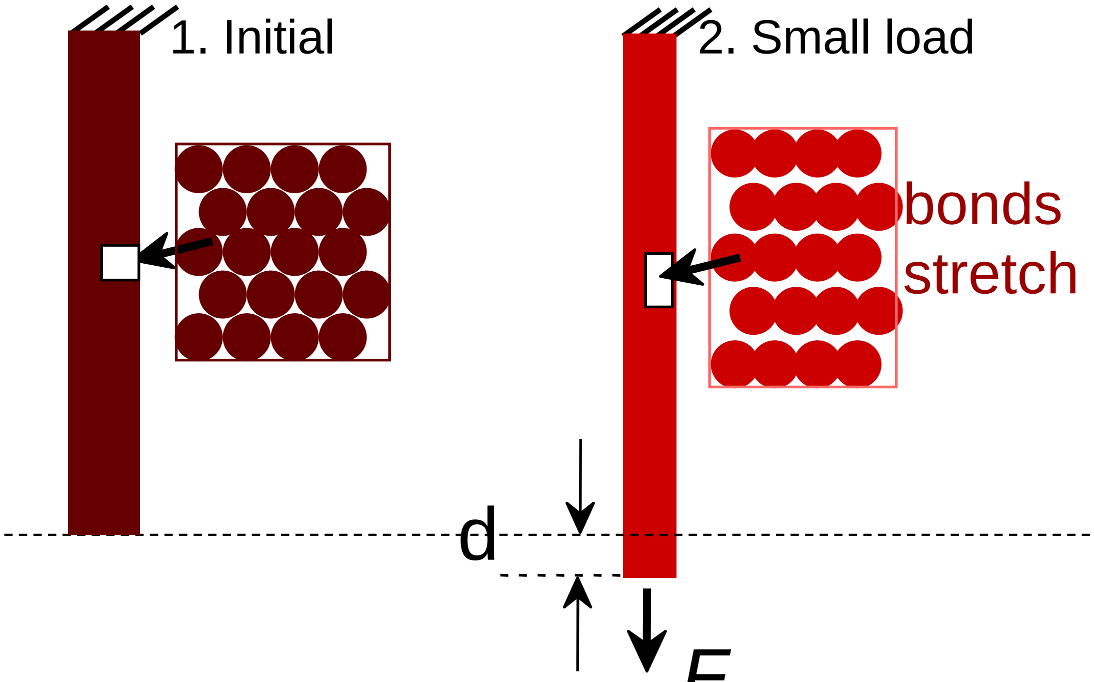
Deformation

- Rigid body displacement
  - Translation
  - Rotation
  - Combination of transloation and rotation

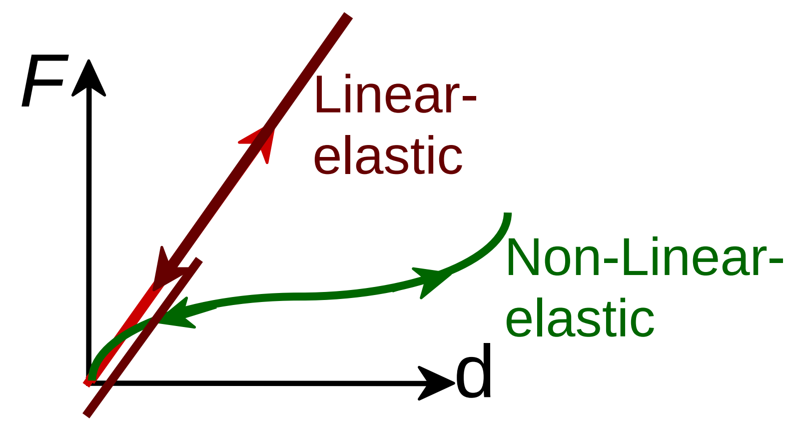
- **Elastic deformation**
  - Occurs due to a change in shape by small enough external load
  - No permanent shape change if the load is removed
  - By elastic lattice distortions (in microscale)
  - Example: axial, bending or torsion (angle of twist), general...

- Plastic deformation
  - Occurs due to a change in shape by large enough external load
  - Permanent shape change if the load is removed
  - Results in elastic lattice distortions + shear of lattice by microscopic defects such as dislocations, twin, phase transformation etc...

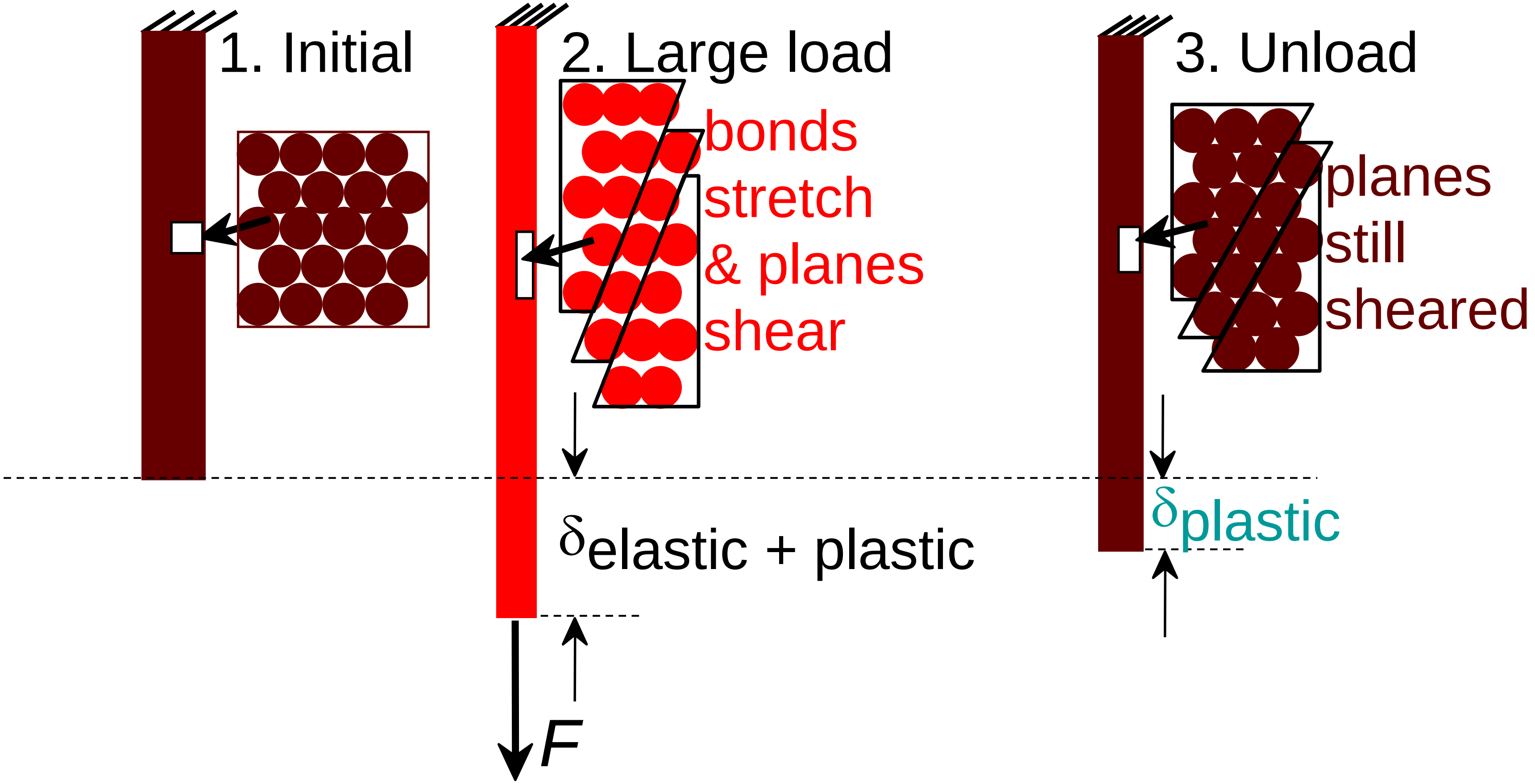
Deformation - elastic



Elastic means reversible!



Deformation - plastic

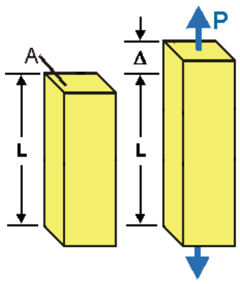


Plastic means permanent!

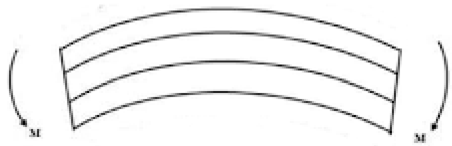
Strain

- Strain (ε or γ)
  - Change in the relative position of two points within a body

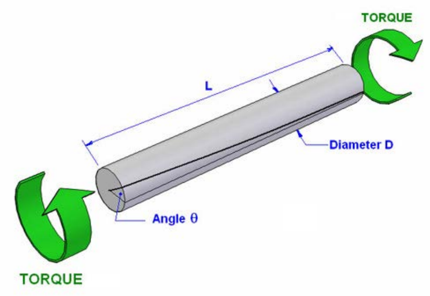
    - Axial strain, ε (elongation (+) or contraction (-))
    - Bending strain, ε (can be zero, positive or negative)
    - Torsional strain, γ (can be positive or negative)
    - Combination of the above



axial

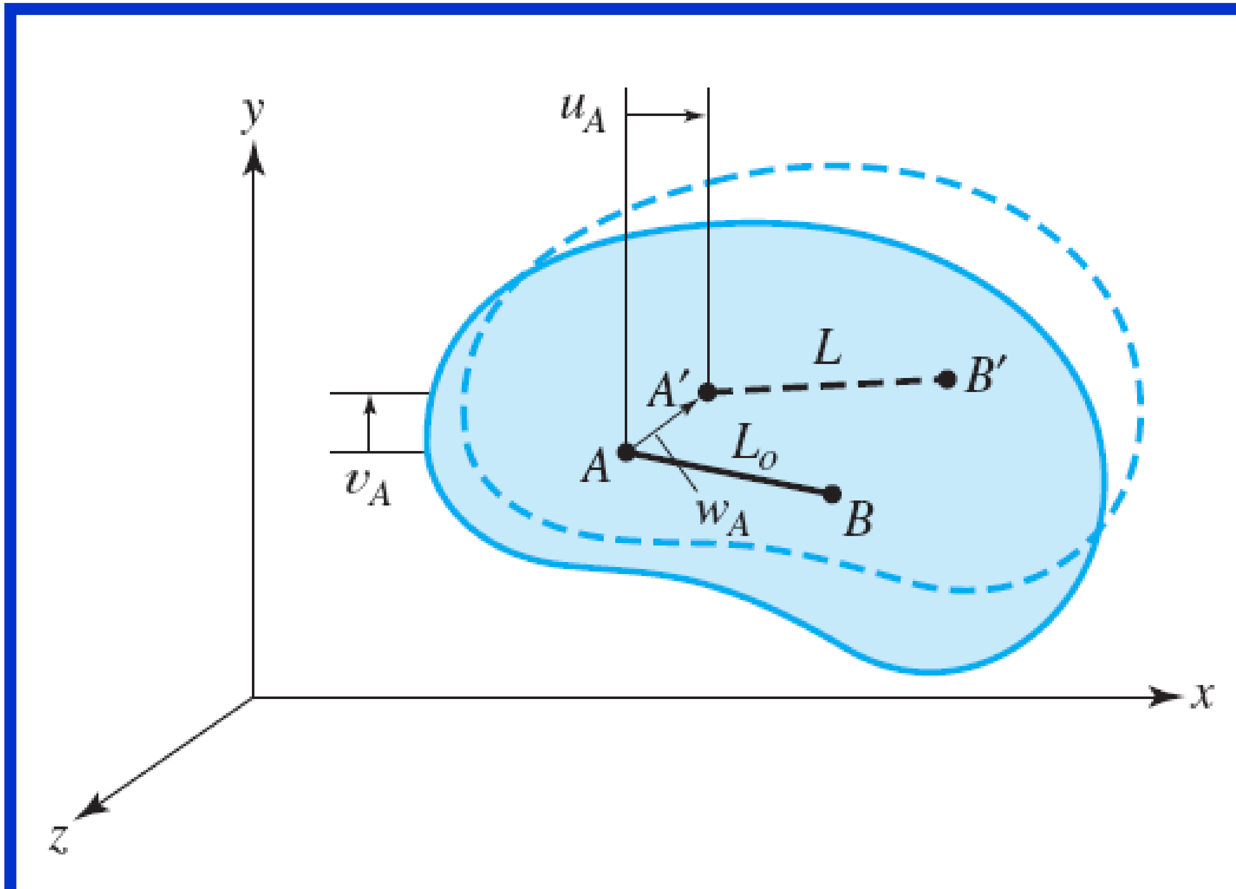


Bending



Torsion

Displacements and strains in a body.



Principle of superposition

- Small deformation theory
  - Valid in the linear elasticity
  - Hooke‘s law or linear relationship between stress and strain diagram

- Small displacement (or deformation) assumption and linear behavior of materials lead to the **principle of superposition**.

- This rule is valid whenever the following conditions are satisfied:

  - The quantity (displacement or stress) to be determined is directly proportional to the loads that produce it (linear elasticity)

  - The loading does not significantly change the size and shape of the member

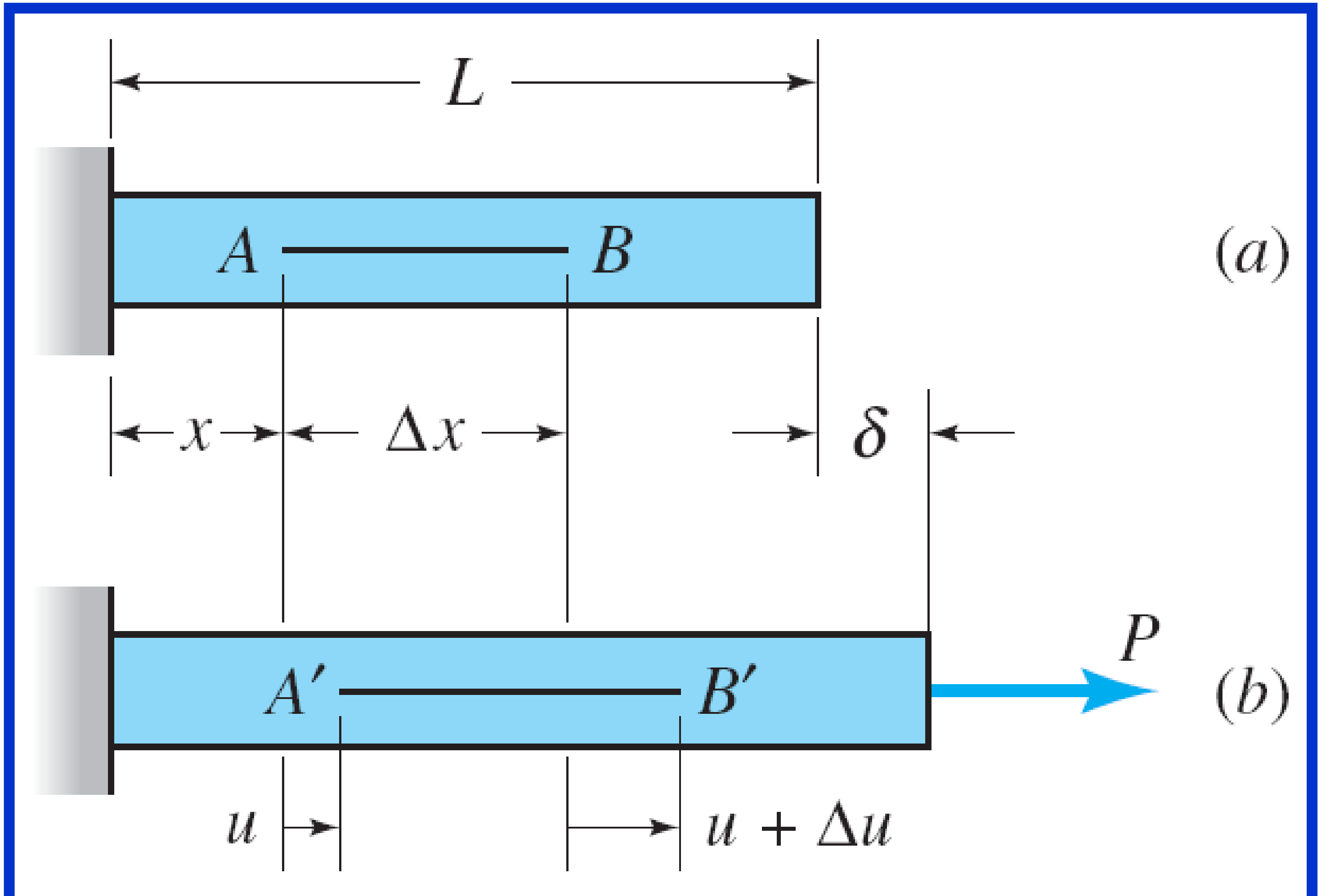
Normal Strain

- Strain in the axial direction

$$\varepsilon = \frac{\text{Change in length } (\delta)}{\text{Original length } (L)}$$

$$\boxed{\varepsilon = \delta / L}$$

Normal strain is positive when there is elongation, otherwise, negative

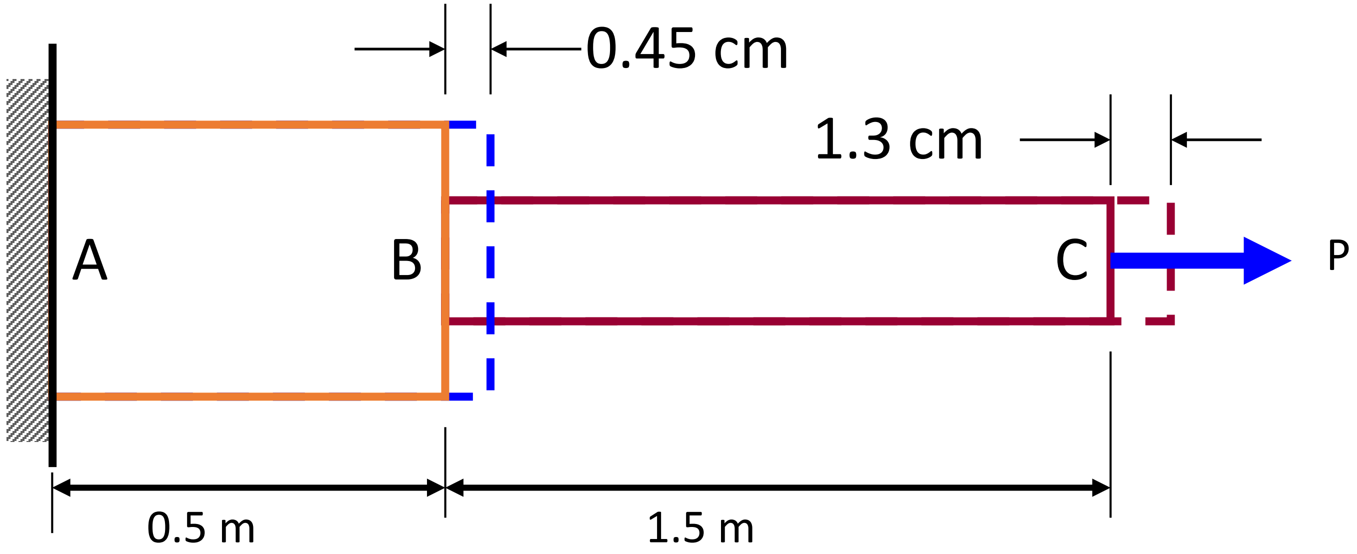


L
A
B
(a)
x
Δx
δ
P
A′
B′
(b)
u
u + Δu

Example

- A stepped round bar is subjected to a load P that produces axial deformation in each portion of the bar.

Q: Calculate the strain in portions AB and BC and the total strain in the bar.



$$\varepsilon_{AB} = 0.0045\ \text{m} / 0.5\ \text{m} = 0.009$$

$$\varepsilon_{BC} = 0.013\ \text{m} / 1.5\ \text{m} = 0.0087$$

Using the superposition principle:

Total strain, $\varepsilon_{AC} = \varepsilon_{AB} + \varepsilon_{BC}$

$= 0.009 + 0.0087$

$=$ **0.0177 (or 1.77%)**

On the otherhand, if we use the definition of strain (change in length / original length) as applied to the entire bar,

$$\varepsilon_{AC} = (0.0045 \text{ m} + 0.013 \text{ m}) / (1.5 \text{ m} + 0.5 \text{ m}) = \mathbf{0.00875 \text{ (or } 0.875\%)}$$

Why is this result different than the previous one?

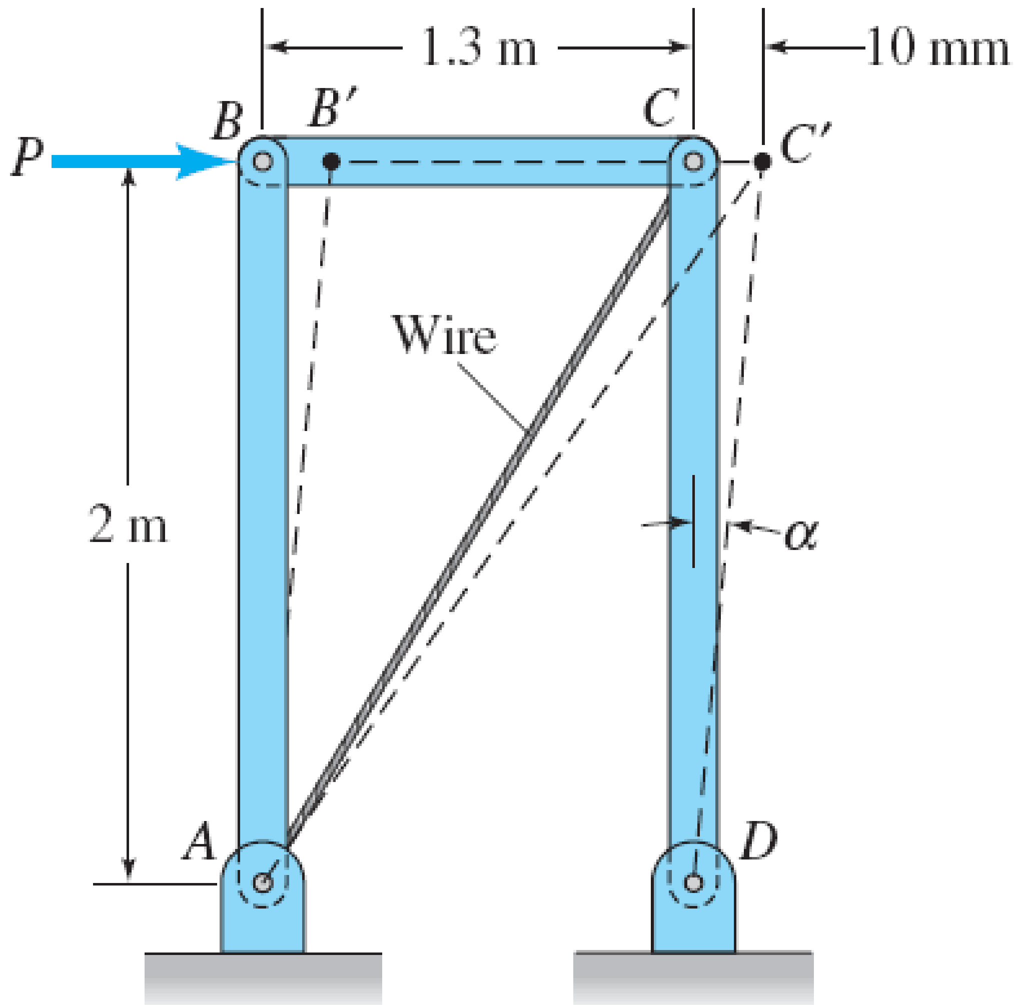
Example 3.1.

- See Figure 3.4 (Next slide)

A pin-connected frame ABCD consists of three bars and a wire. After a horizontal load P is applied at joint B, joint C moves 10 mm to the right.

What is the normal strain in the wire?

Figure 3.4: Displacements of a plane truss



Shear Strain

- Shearing strain is positive if the right angle between the reference lines decrease

  (see Figure 3.3)

$$\gamma_{nt} = \pi/2 - \theta'$$

$$\cong \tan(\pi/2 - \theta')$$

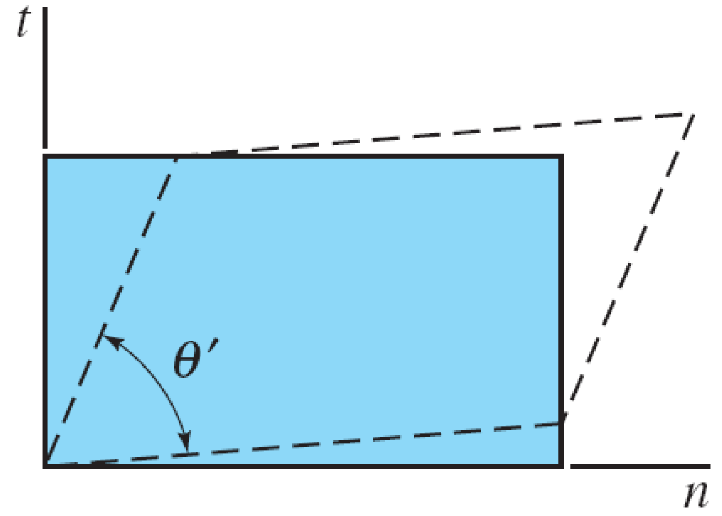


FIGURE 3.3 Distortion of a rectangular plate.

Example 3.2

- A metallic, rectangular plate of length L and width L / 2. When the plate is subjected to stresses acting along the edge faces, it distorts into a parallelogram as illustrated in figure.

Find shearing strain between the edges AB and AD.

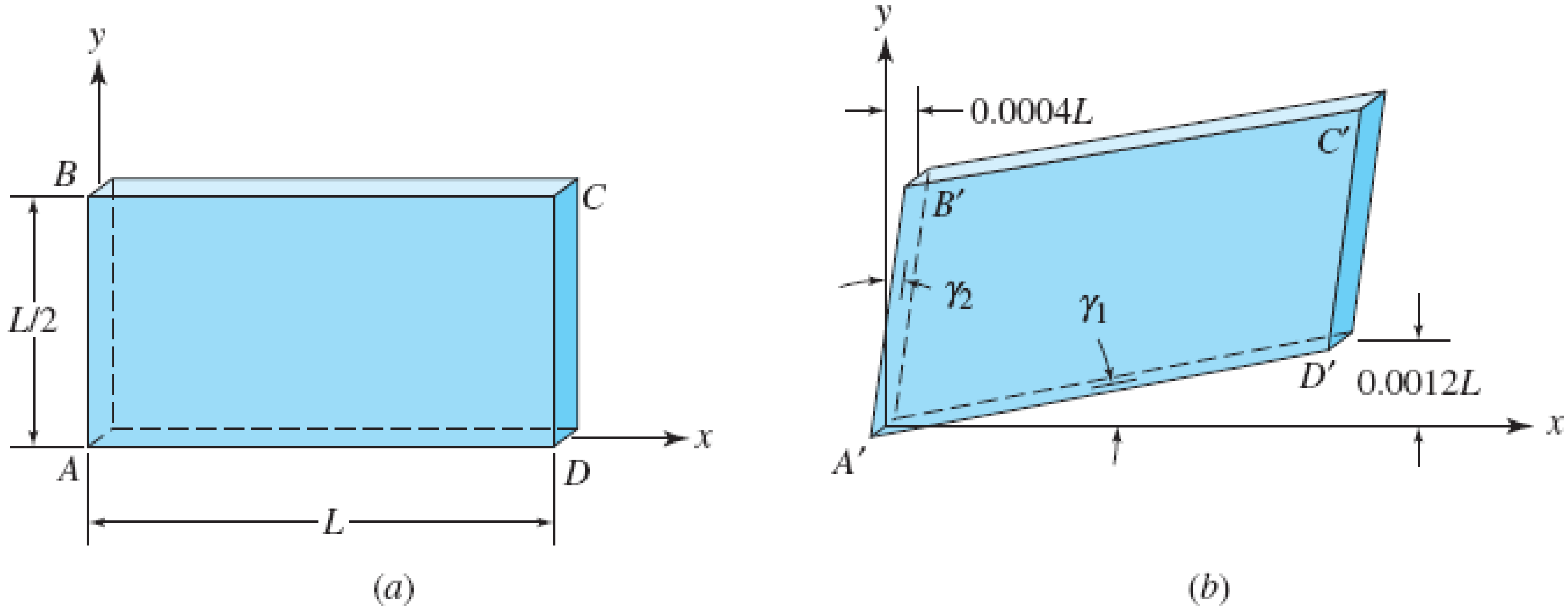


FIGURE 3.5 (a) Initial plate; (b) deformed plate.

Example 3.3

- A thin, triangular plate ABC is uniformly deformed into a shape ABC′, as shown by the dashed lines. (Figure 3.6)

- Find:
  - (a) The normal strain along the centerline OC.
  - (b) The normal strain along the edge AC.
  - (c) The shear strain between the edges AC and BC.

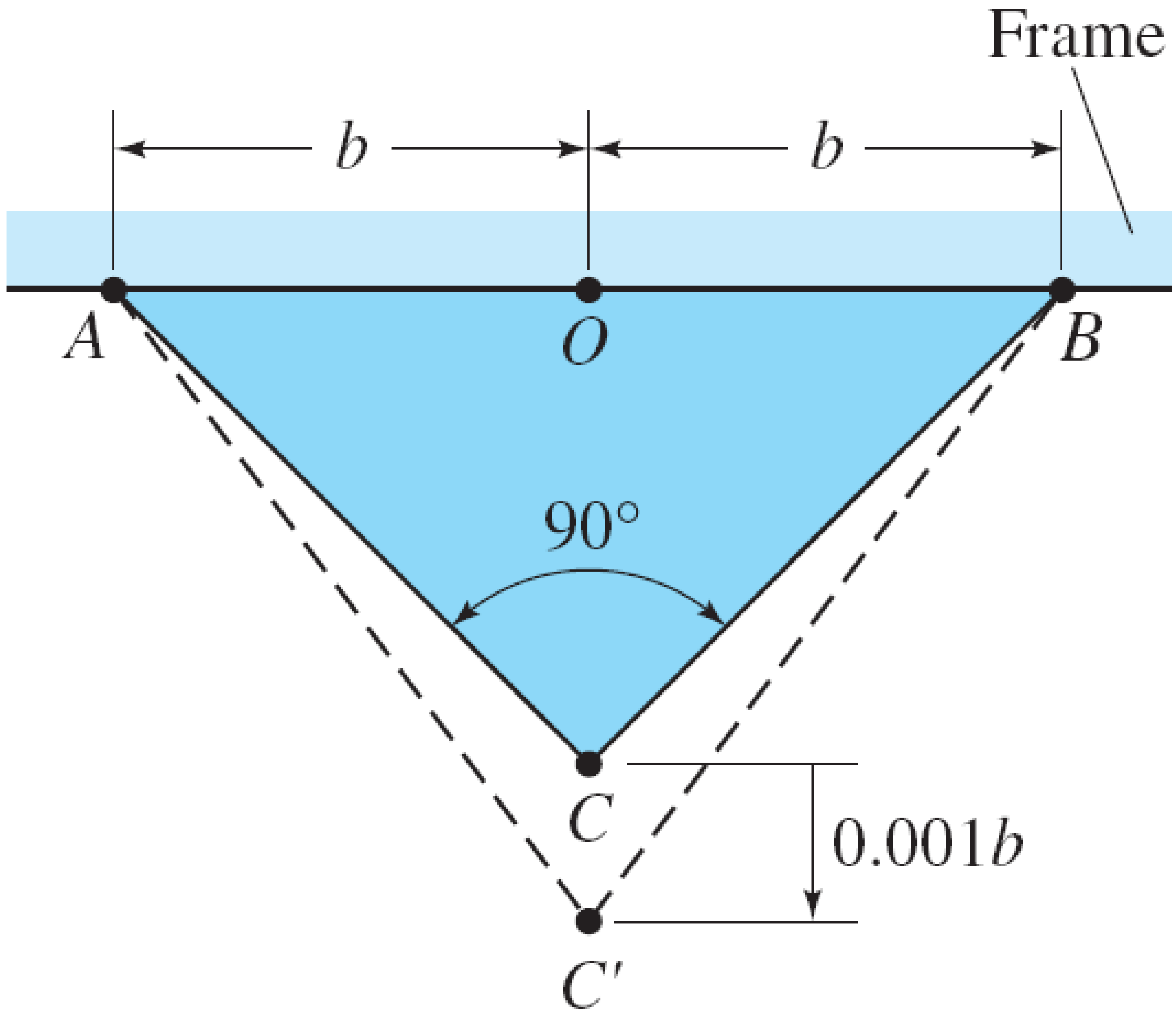


FIGURE 3.6 Deformation of a triangular plate.

COMPONENTS OF STRAIN

$$\varepsilon_x = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{du}{dx}$$

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y}$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

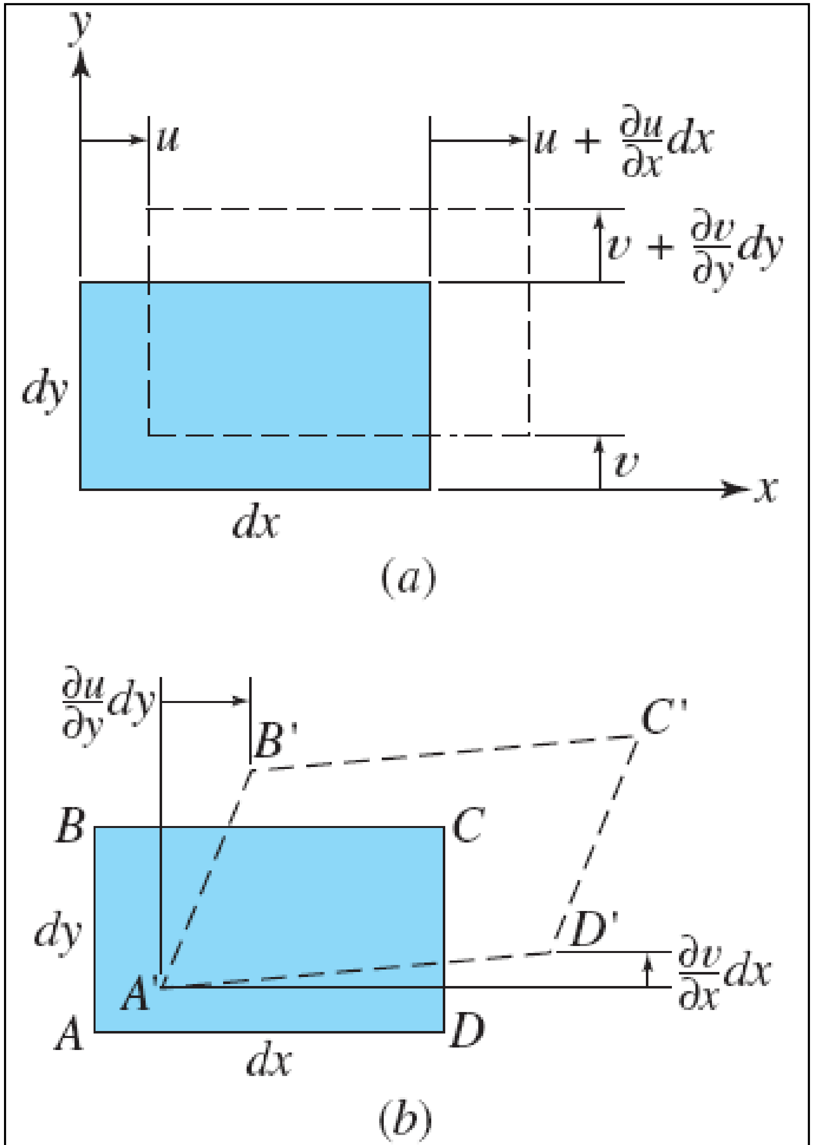


FIGURE 3.7 Deformations of an element: (*a*) linear strain and (*b*) shear strain.

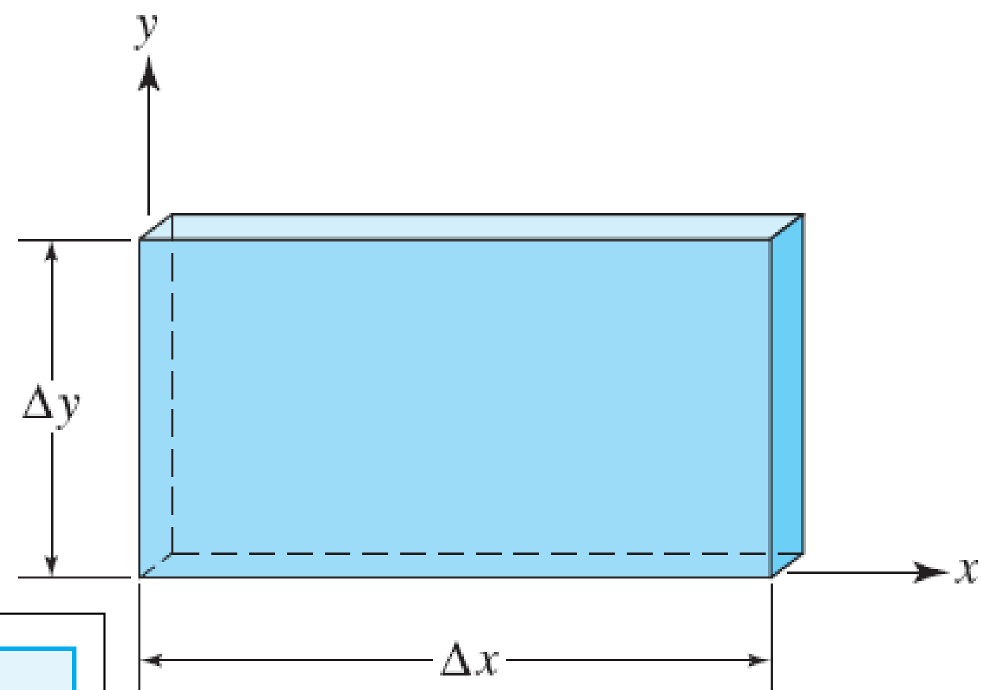


FIGURE 3.8 A rectangular plate of length Δx and width Δy.

$$\varepsilon_x = \frac{\Delta u}{\Delta x} \qquad \varepsilon_y = \frac{\Delta v}{\Delta y}$$

$$\gamma_{xy} = \frac{\Delta u}{\Delta y} + \frac{\Delta v}{\Delta x}$$

EXAMPLE 3.5

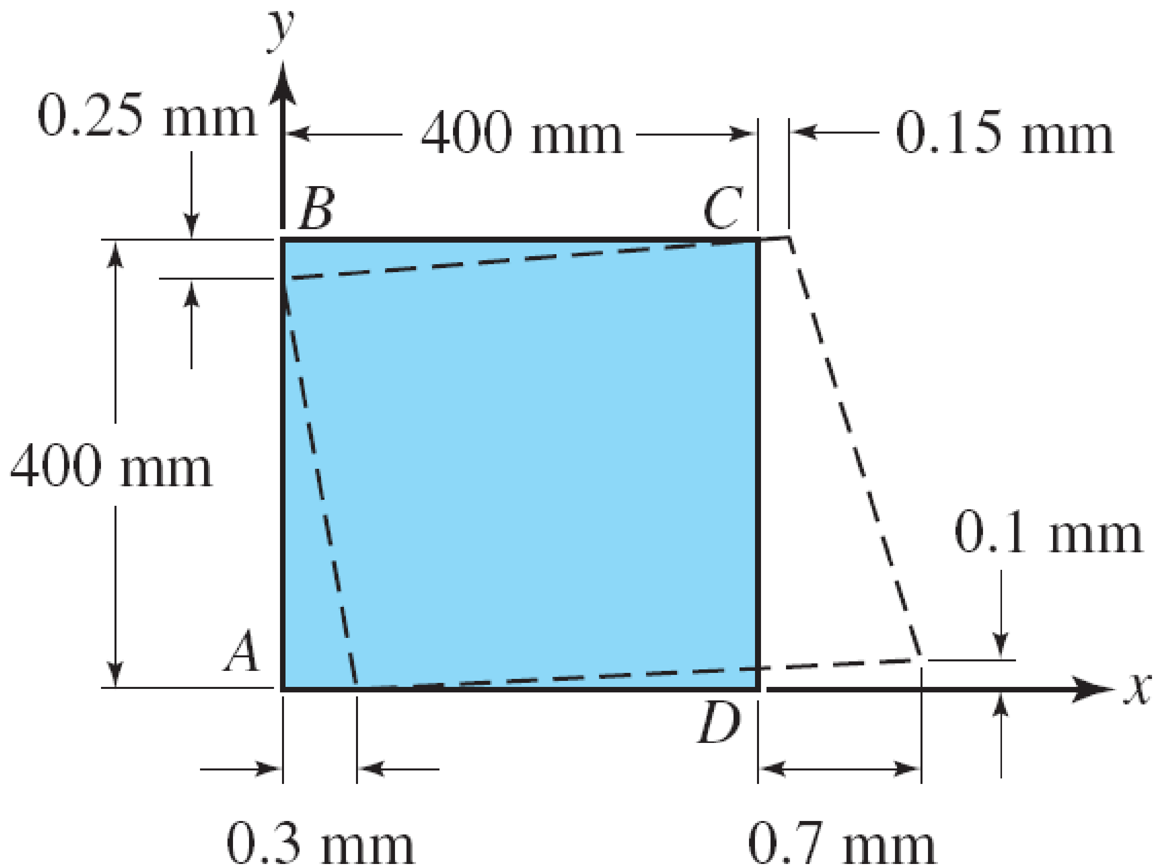


FIGURE 3.9 Deformation of a square plate.

3.2. Material (Mechanical) Properties

Engineering Materials

TABLE 3.1 Typical Engineering Materials

Metallic Materials	
Ferrous Metals	**Nonferrous Metals**
Cast iron	Aluminum
Cast steel	Copper
Plain carbon steel	Lead
Steel alloys	Magnesium
Stainless steel	Nickel
Special steels	Platinum
Structural steel	Silver
Nonmetallic Materials	
Graphite	Plastics
Ceramics	Brick
Glass	Stone
Concrete	Wood

Mechanical properties of materials

- Mechanical properties of engineering materials loaded in tension and compression are determined using a standard testing methods
- Example: ASTM standards, JIS, KS, DIN etc..

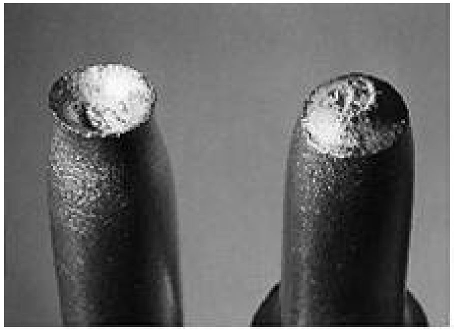
Ductile vs. brittle materials

- ***Ductile materials*** can undergo large inelastic strains prior to fracture.
- For example, structural steel and many alloys of other metals, and nylon, are characterized by their ability to yield at normal temperatures.
- *Percentage elongation* is 5 or more.

Ductile vs. brittle materials

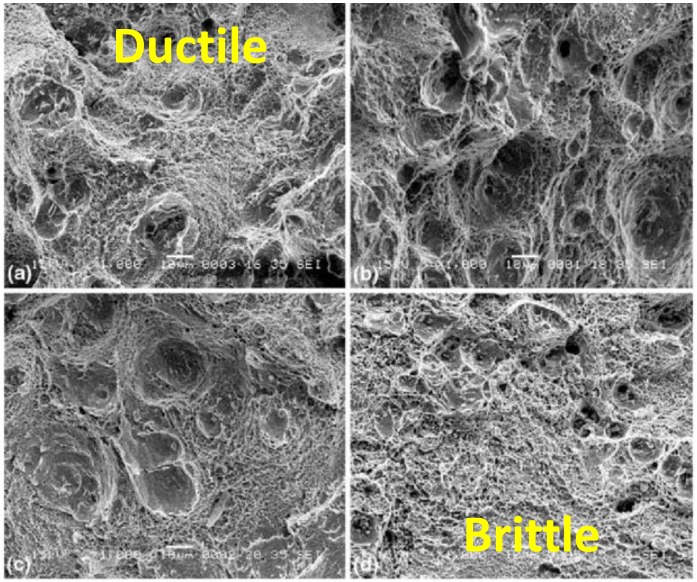
- ***Brittle materials*** (for example, cast iron or concrete) exhibits little deformation before rupture and, as a result, fails suddenly without visible warning.
- Percentage elongation is less than around 5.

Ductile vs. brittle materials



Ductile

Brittle



Homogeneous material

- A ***homogeneous*** solid displays identical properties throughout.

- For example, if a bar made of the same material is divided in to several small pieces (of any shape and size) and the density (weight or mass / volume) of each piece is determined to be same then the material is homegeneous.

Isotropic vs. anisotropic

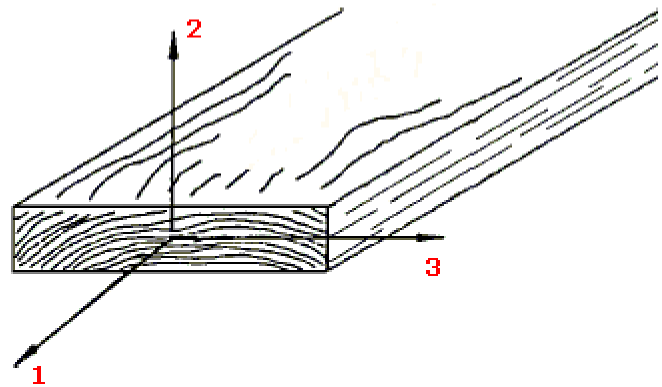
- If the properties of a material are identical in all directions at a point, the material is said to be ***isotropic***
- A nonisotropic, or ***anisotropic***, material displays direction-dependent properties
- An ***orthotropic*** material is a special case of anisotropic material in which the material properties differ in two mutually perpendicular directions

Orthotropic, anisotropic, isotropic

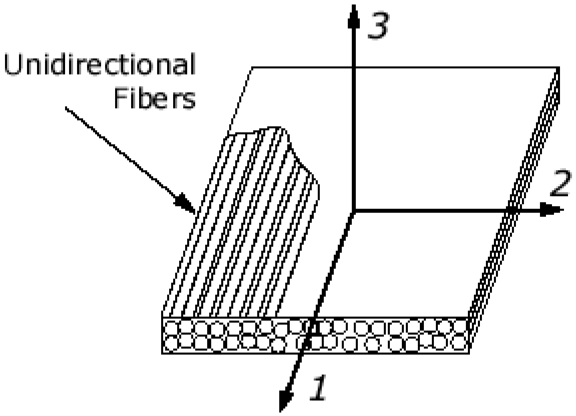
- An ***orthotropic*** material has at least 2 orthogonal planes of symmetry, where material properties are independent of direction within each plane.
- In contrast, a material without any planes of symmetry is fully ***anisotropic***.
- A material with an infinite number of symmetry planes (i.e. every plane is a plane of symmetry) is ***isotropic***.

Orthotropic, anisotropic, isotropic

Isotropic



Transversely isotropic



Orthotropic

Stress strain diagram

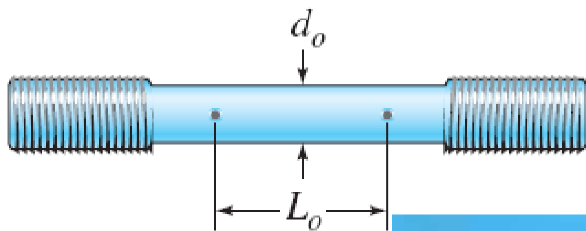
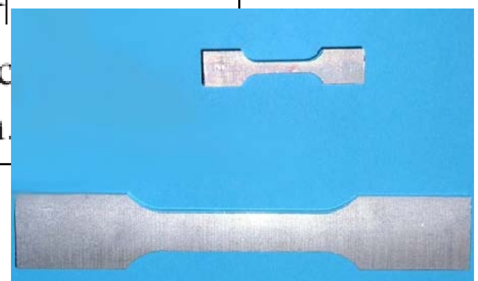
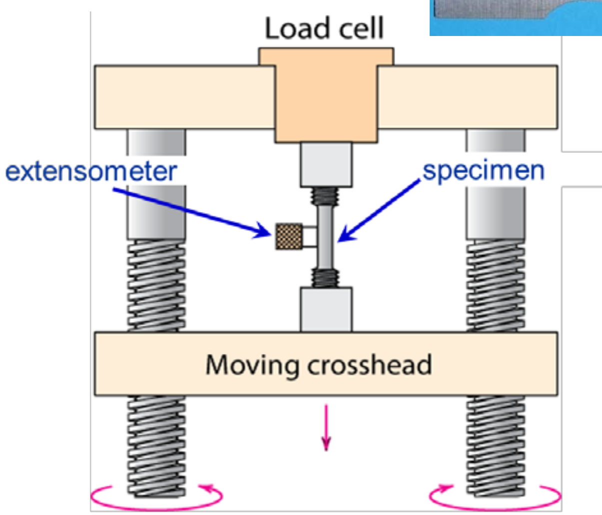


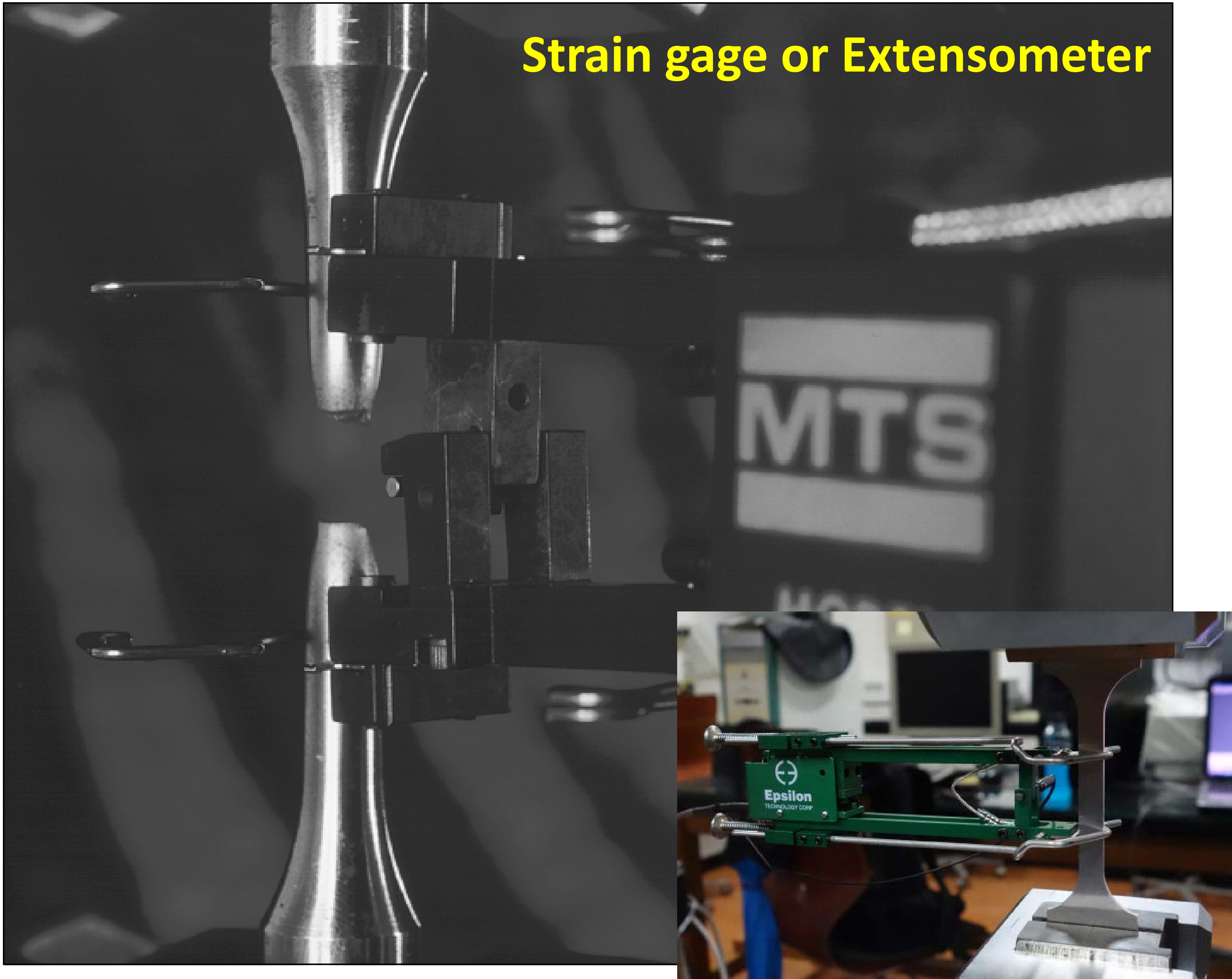
FIGURE 3.10 Typic
tensile-test specimen.

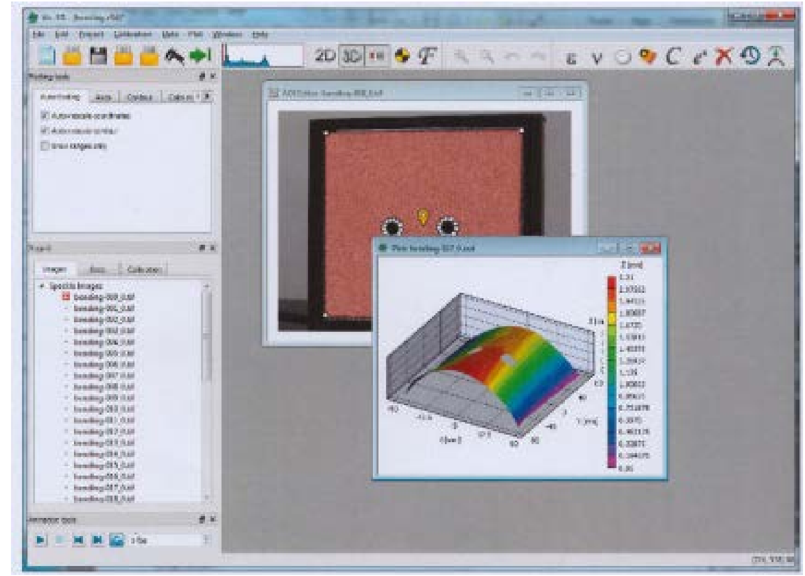




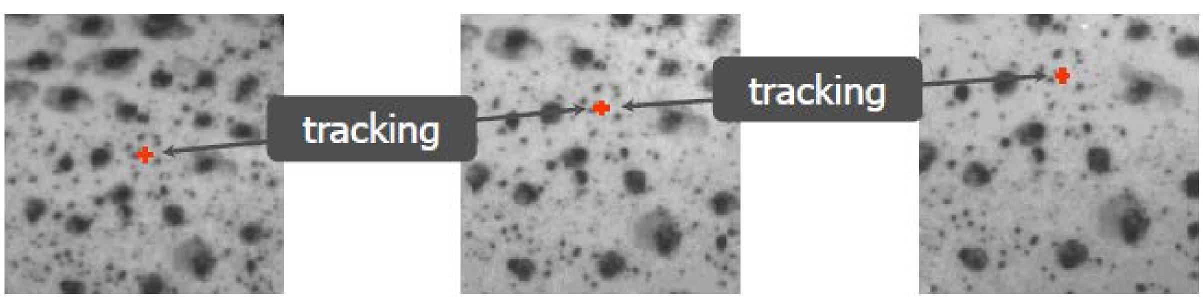
Adapted from Fig. 6.3, *Callister & Rethwisch 8e*. (Fig. 6.3 is taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)

Strain gage or Extensometer



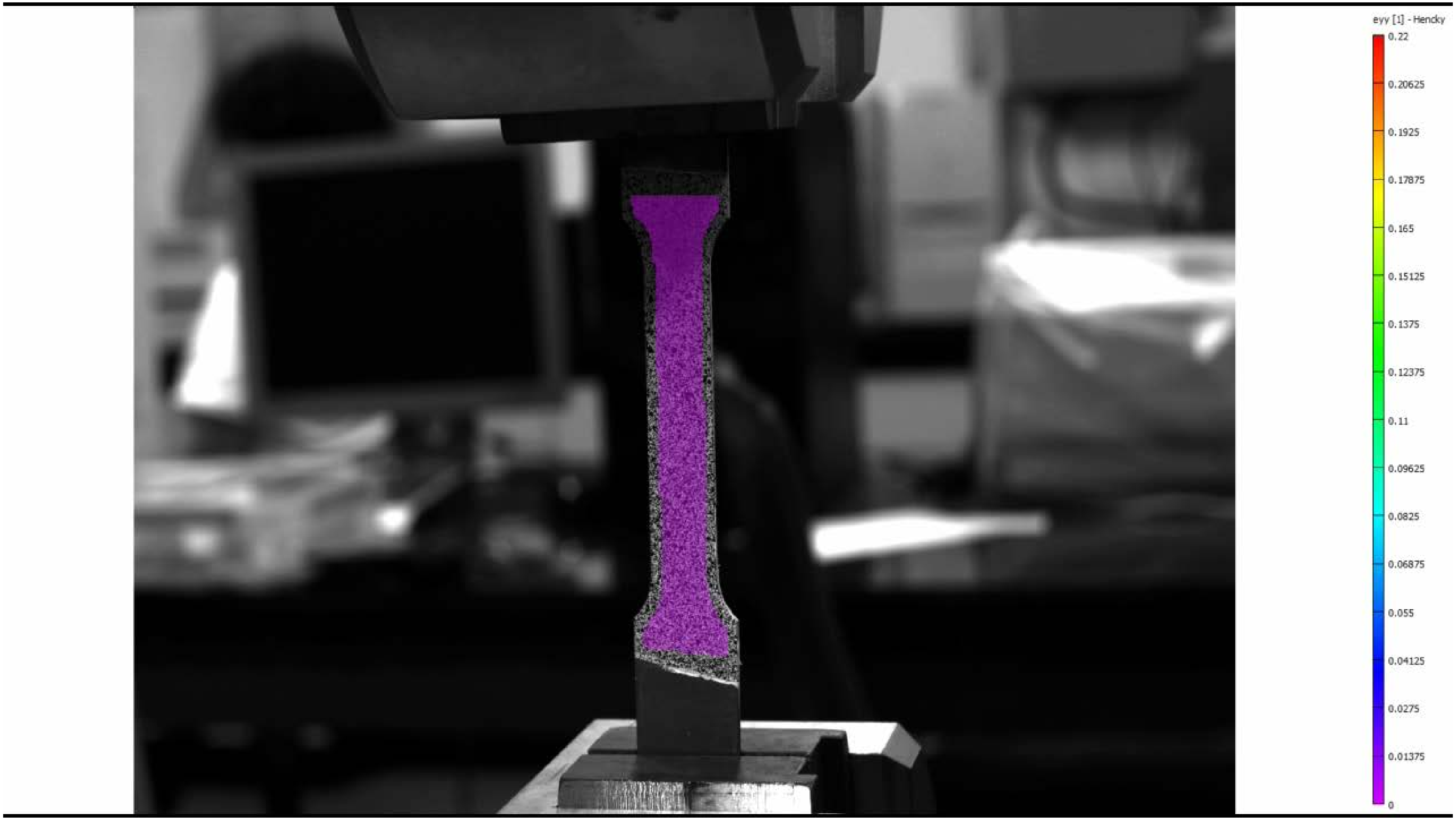


Digital Image Correlation



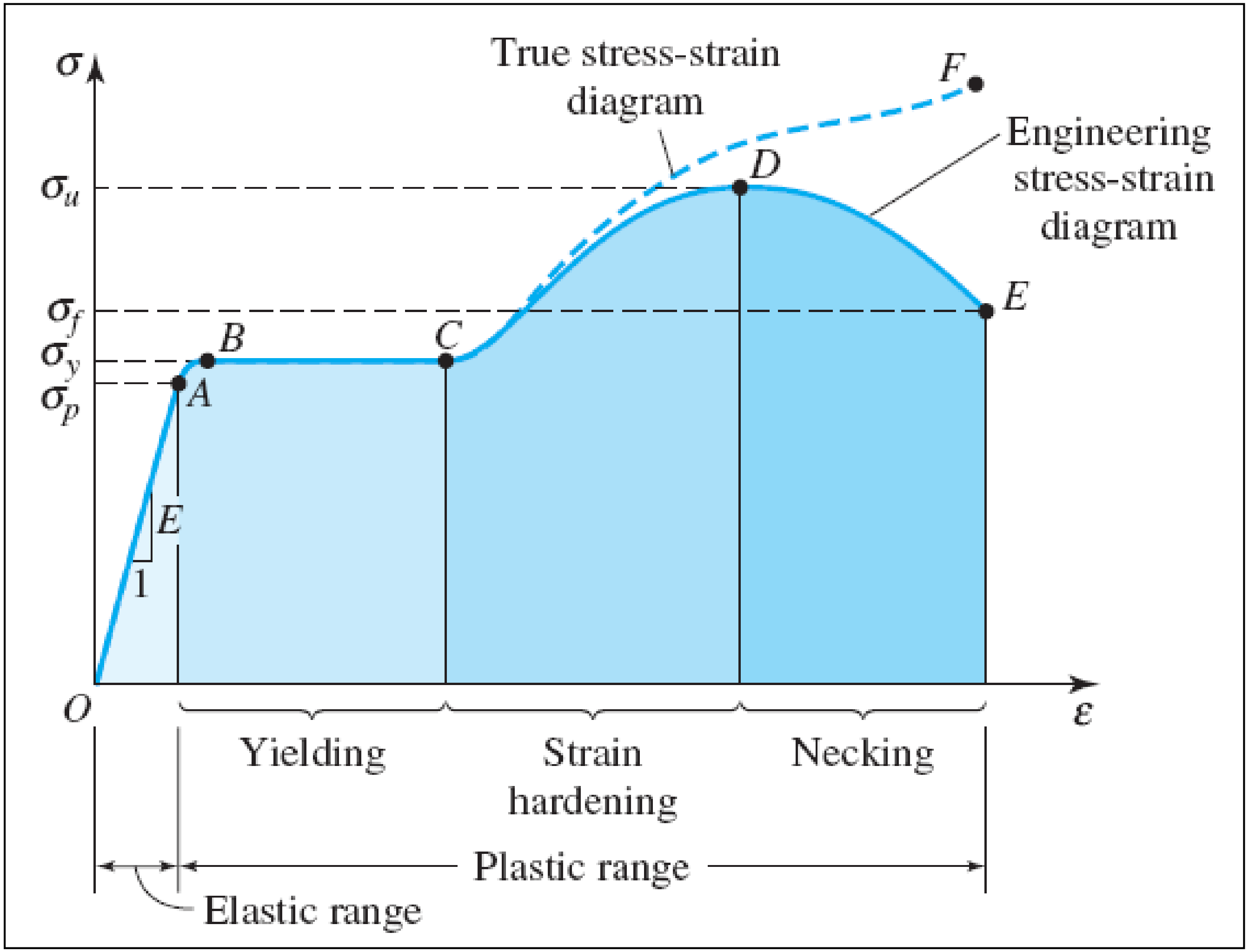
Time t Time t' Time t''

Digital Image Correlation

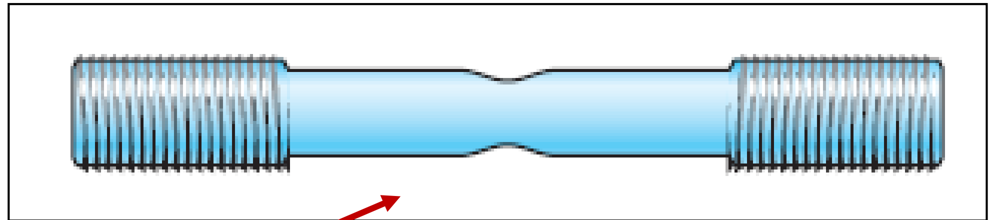


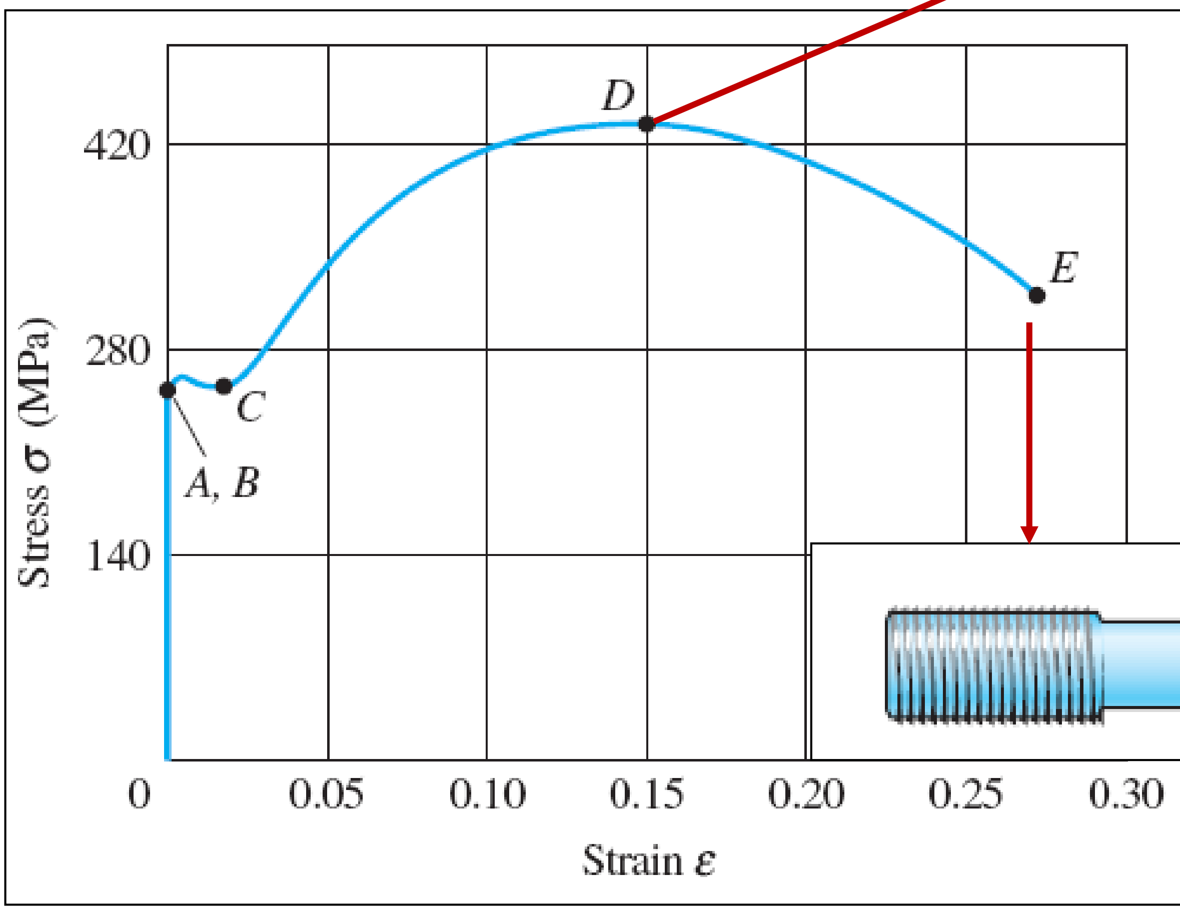
Stress-strain diagram**

Ductile materials

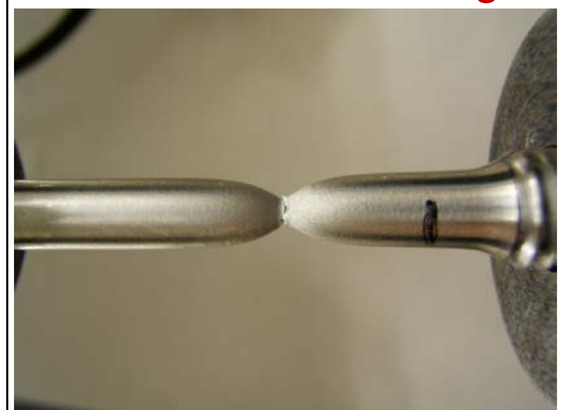


Typical stress-strain diagram

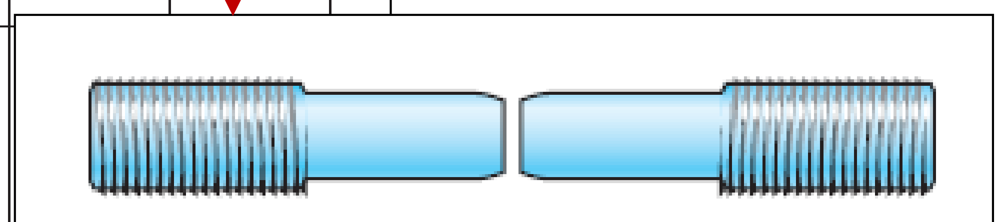




Ultimate tensile strength



At failure



Conventional stress-strain diagram

Measurement of strain or alternative method

$$\text{percent elongation} = \frac{L_f - L_o}{L_o}(100)$$

$$\text{percent reduction in area} = \frac{A_o - A_f}{A_o}(100)$$

How to define yield stress – offset method

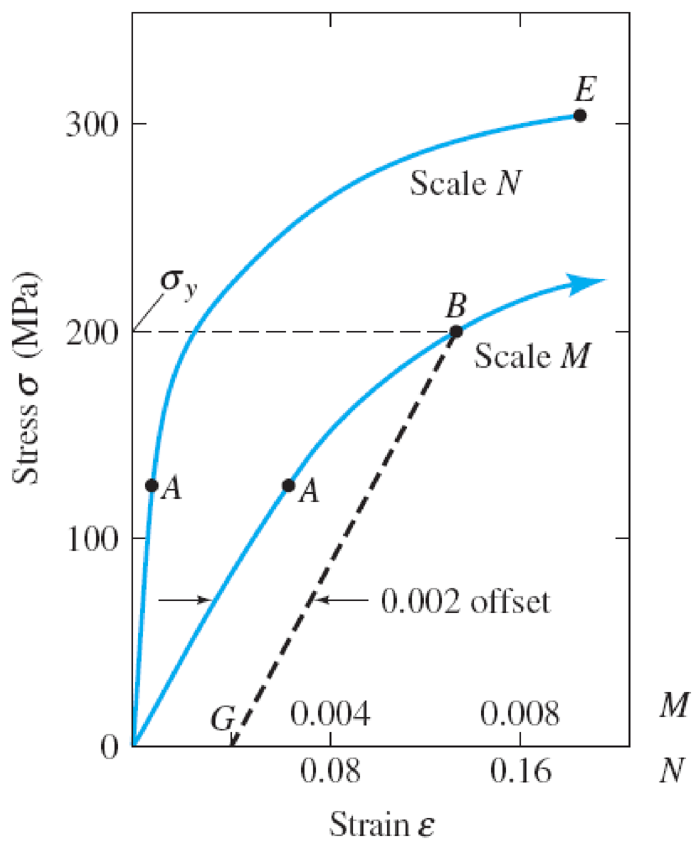
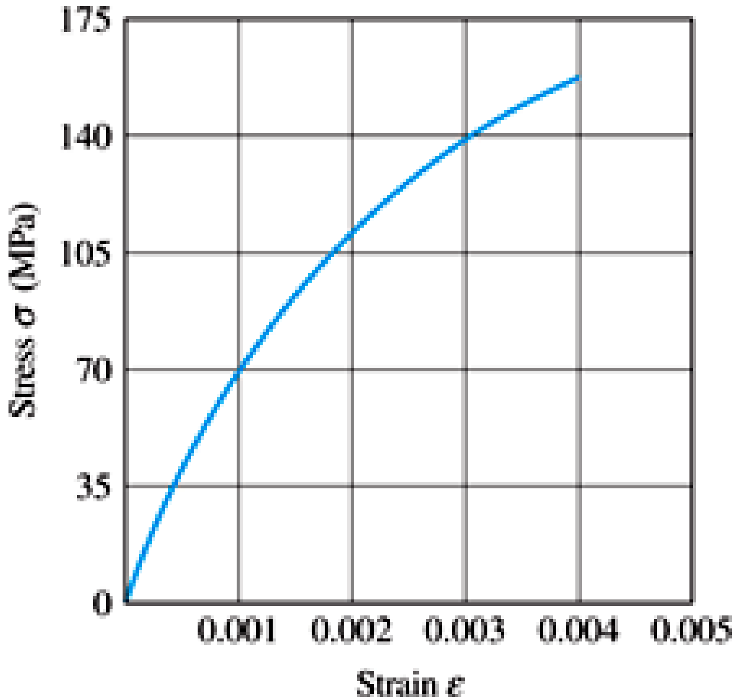


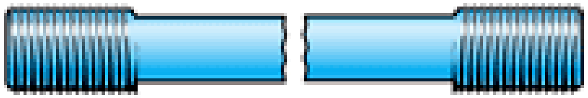
FIGURE 3.16 Determination of yield strength by the offset method of a magnesium alloy in tension.

Stress-strain diagram: brittle materials

Eexhibits no “Necking“



(a)



(b)

What happens in compression?

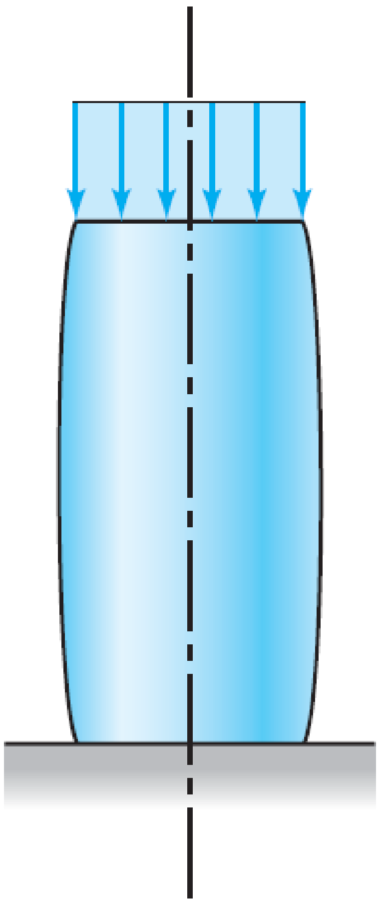


FIGURE 3.18 Gray cast iron in compression.

Mechanical properties in shear

- The properties of a material can also be determined from direct-shear tests or torsion tests (discussed in Chapter 5).

- Shear stress–shear strain diagrams may be obtained from the results of torque (T) versus shear strain (γ) that are analogous to the tension test results.

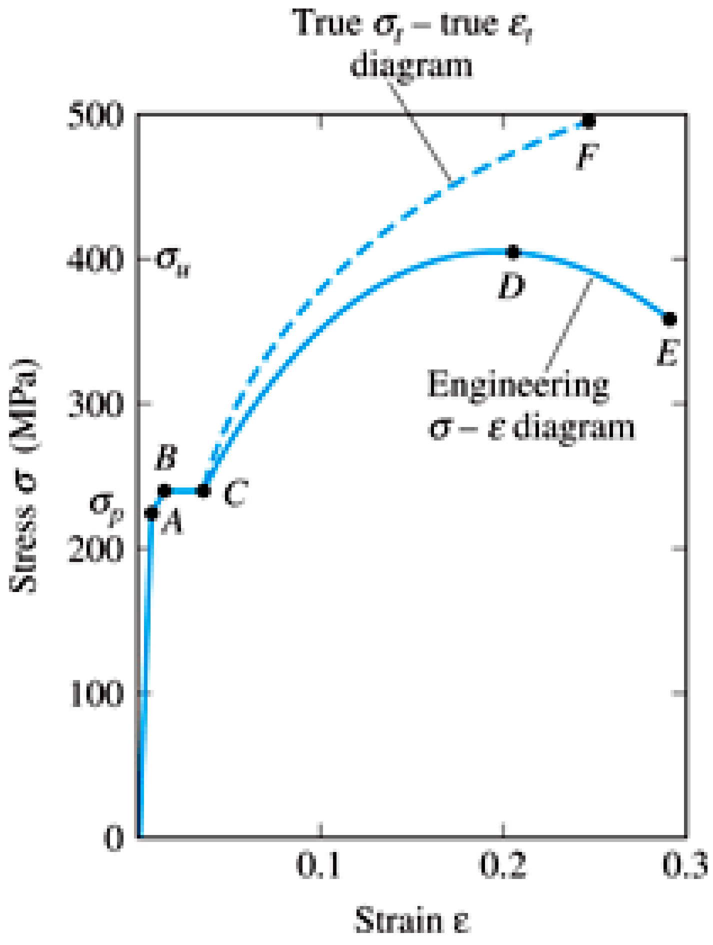
True stress-strain relationship **

True strain

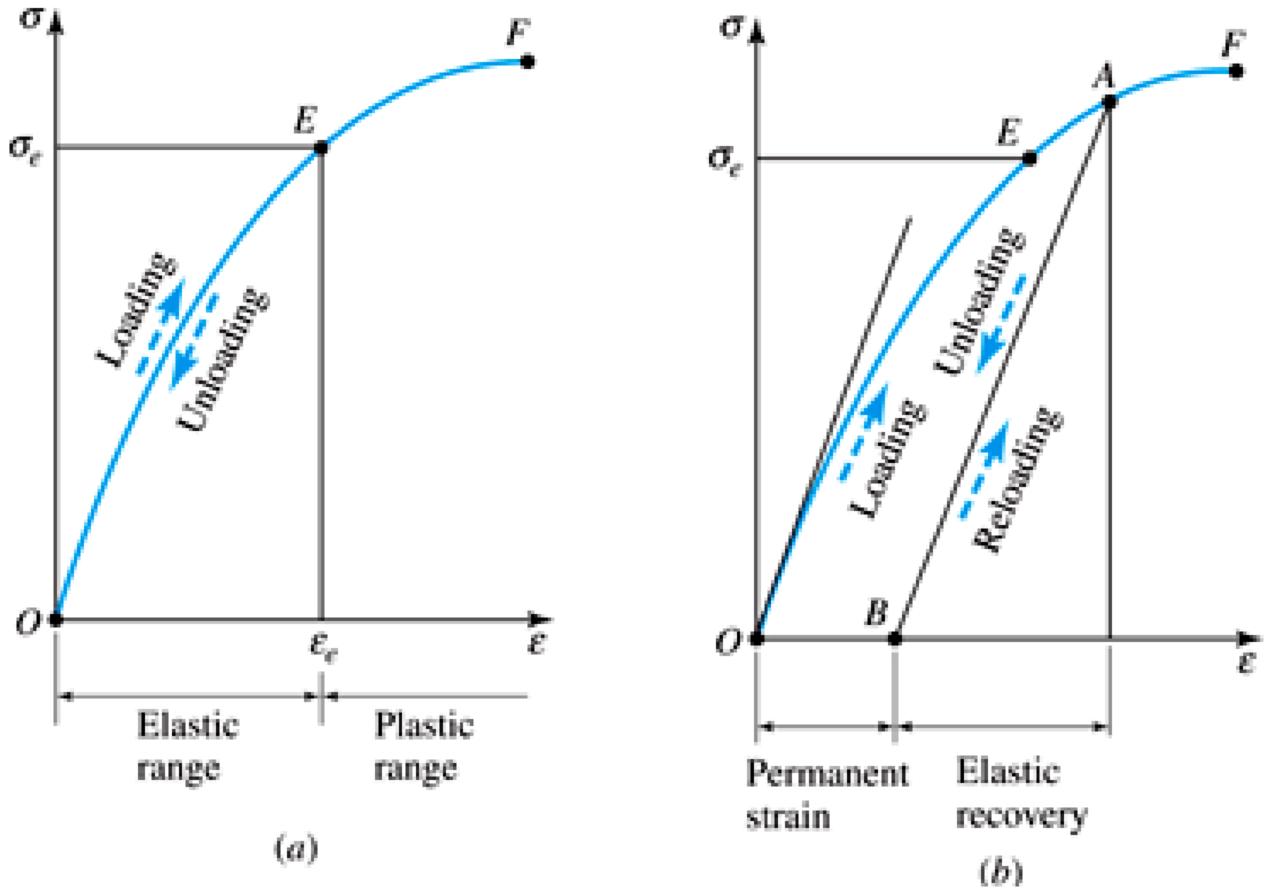
$$\varepsilon_t = \int_{L_o}^{L} \frac{dL}{L} = \ln \frac{L}{L_o} = \ln(1+\varepsilon)$$

True stress

$$\sigma_t = \sigma(1+\varepsilon)$$



Elastic vs. Plastic Behavior

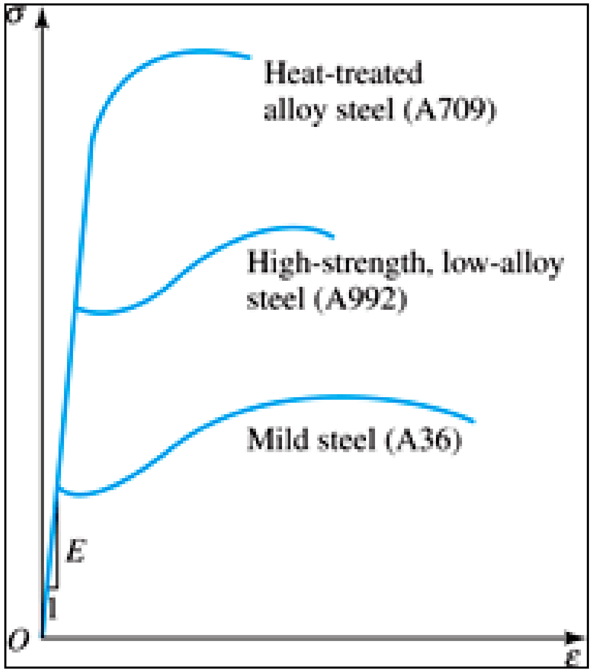


Hooke's Law**

- Simple way to correlate between stress-strain for 1D linear isotropic elastic materials

$$\boxed{\sigma = E\varepsilon}$$

$$\boxed{\tau = G\gamma}$$



- Hookean materials: materials that obey the Hooke's law
- Hookean materials = Linear elastic materials

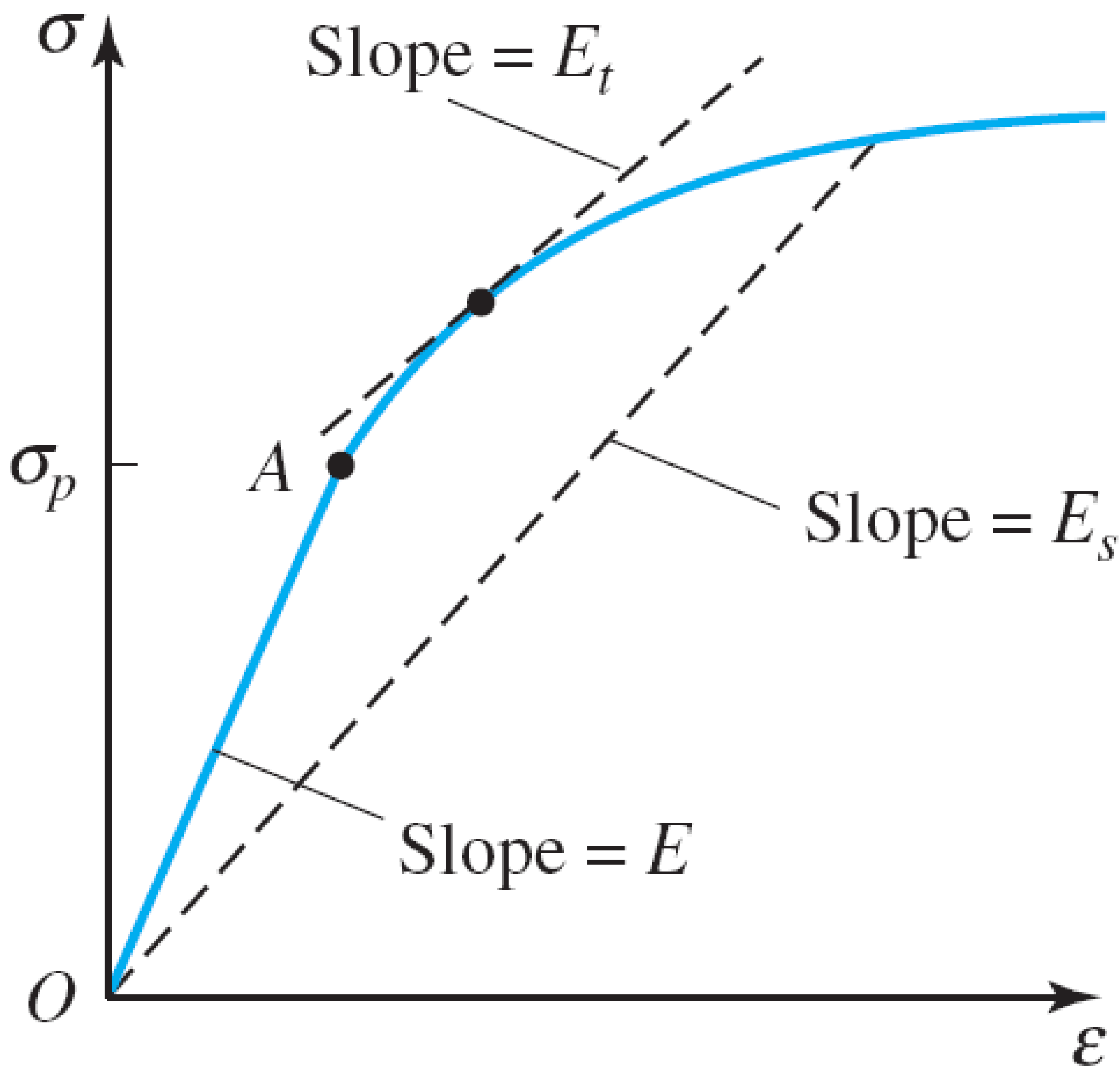


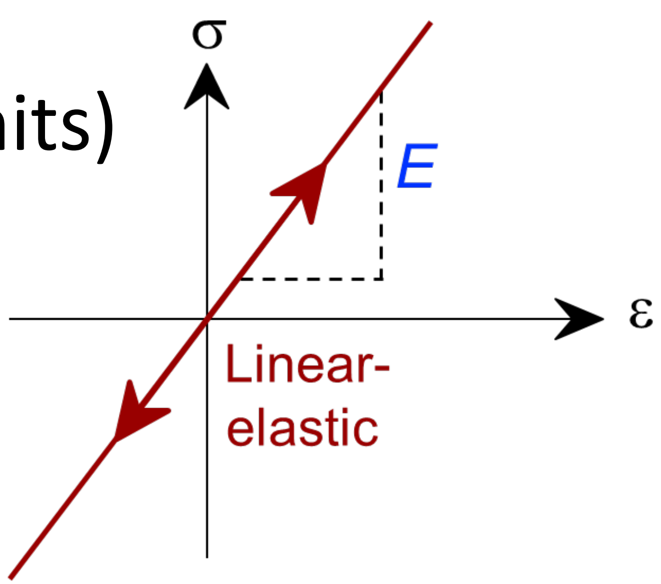
FIGURE 3.24 Various moduli of elasticity.

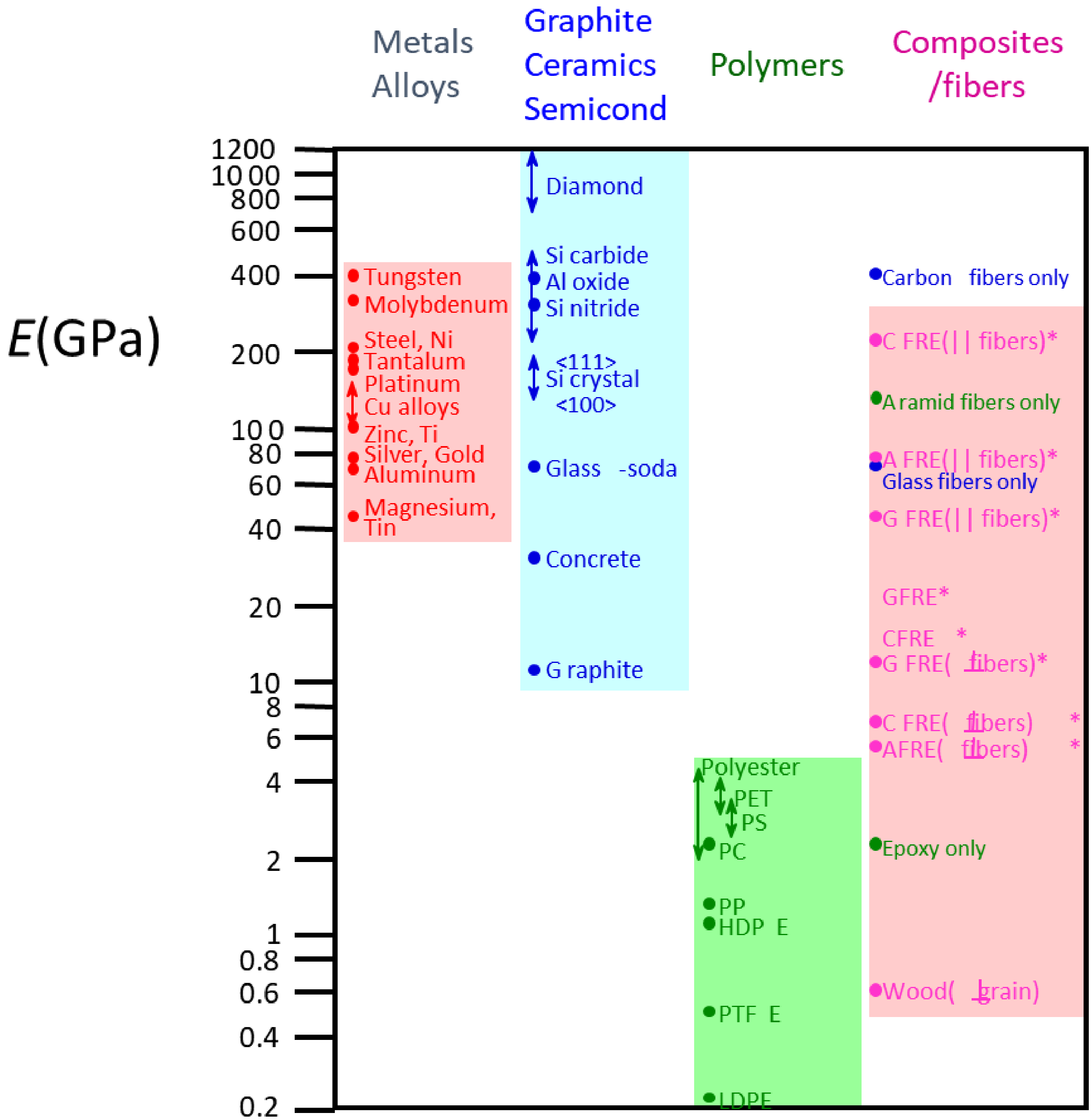
Young's modulus (E)

- Young's modulus or modulus of elasticity is denoted by E.
- The value of E for steel is roughly 3 times that for aluminum.

$E_{steel} \approx 30 \times 10^6$ psi (English units)

$E_{steel} \approx 210$ GPa (SI units)



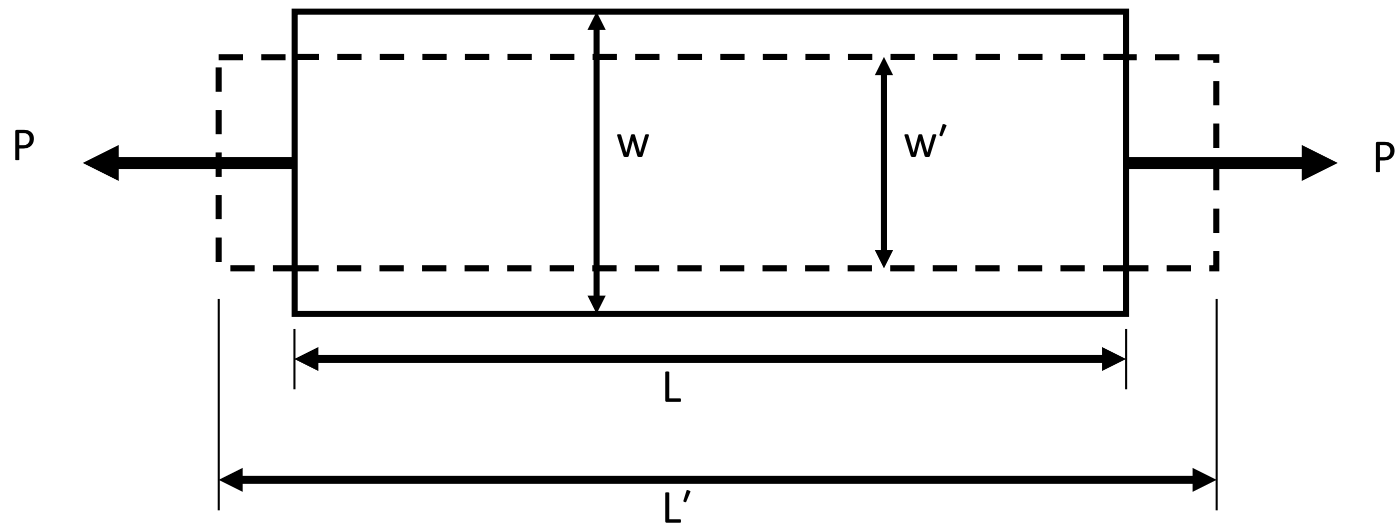


Metals
Alloys
Graphite
Ceramics
Semicond
Polymers
Composites
/fibers
E(GPa)
1200
1000
800
600
400
200
100
80
60
40
20
10
8
6
4
2
1
0.8
0.6
0.4
0.2
Tungsten
Molybdenum
Steel, Ni
Tantalum
Platinum
Cu alloys
Zinc, Ti
Silver, Gold
Aluminum
Magnesium,
Tin
Diamond
Si carbide
Al oxide
Si nitride
<111>
Si crystal
<100>
Glass -soda
Concrete
Graphite
Polyester
PET
PS
PC
PP
HDPE
PTFE
LDPE
Carbon fibers only
C FRE(|| fibers)*
Aramid fibers only
A FRE(|| fibers)*
Glass fibers only
G FRE(|| fibers)*
GFRE*
CFRE *
G FRE(⊥ fibers)*
C FRE(⊥ fibers) *
AFRE(⊥ fibers) *
Epoxy only
Wood(⊥ grain)

Poisson's ration

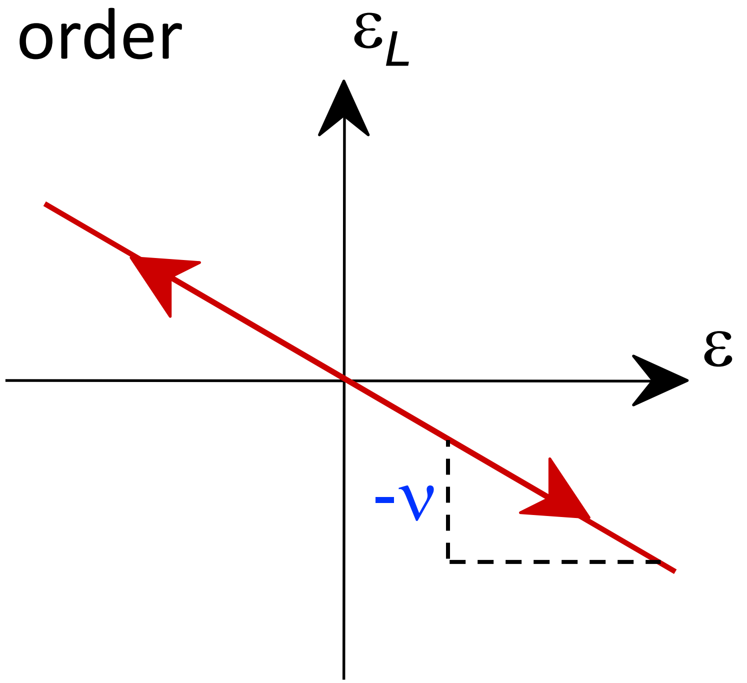
ν = - [lateral strain, ε_w / axial strain, ε_x]

This expression is valid for uniaxial loading



$$\boxed{\nu = -\frac{\text{lateral strain}}{\text{axial strain}}}$$

- Typical values of ν for steel ranges from about 0.25 to 0.3, while for aluminum materials it is of the order of 0.33



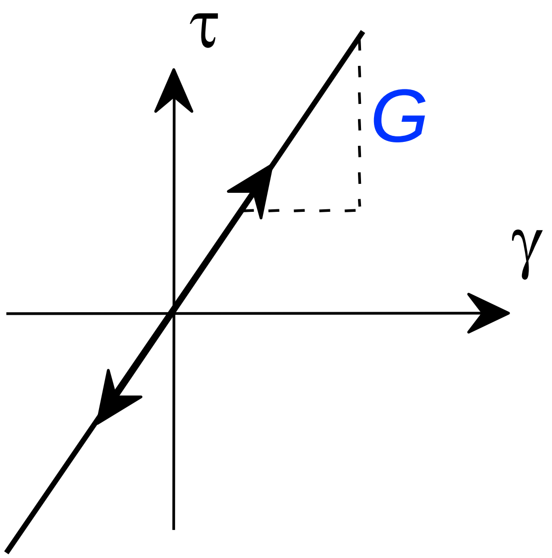
Shear modulus (G)

- Shear modulus or modulus of rigidity is denoted by G. The value of G for steel is a little over 1/3rd the value of E for steel.

$G_{steel} \approx 11.5 \times 10^6$ psi (English units)

$G_{steel} \approx 77$ GPa (SI units)

$$\boxed{G = \frac{E}{2(1+\nu)}}$$



Volumetric strain under tension

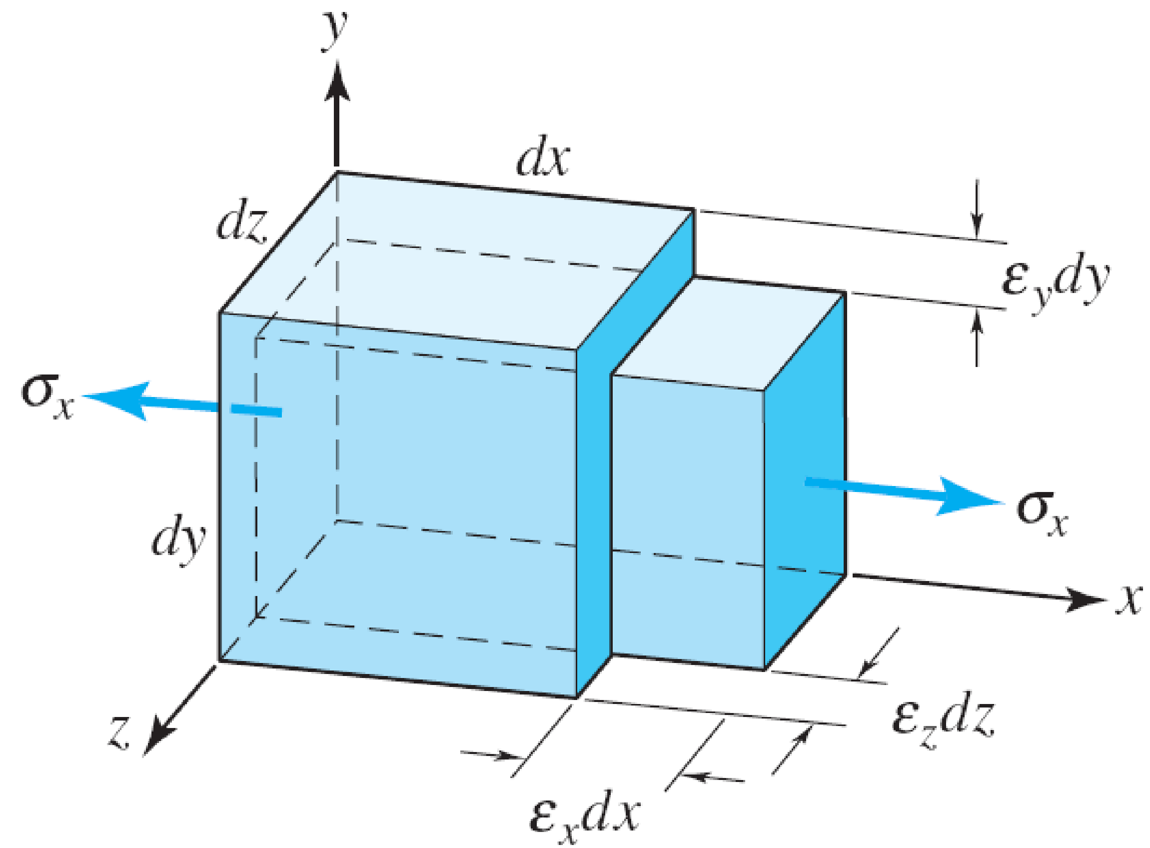


FIGURE 3.27 Lateral contraction of an element in tension.

Volumetric strain under tension

$$\boxed{e = \frac{\Delta V}{V_o}}$$

$$\boxed{e = (1 - 2\nu)\varepsilon_x = \frac{1 - 2\nu}{E}\sigma_x}$$

Generalized Hooke's law

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

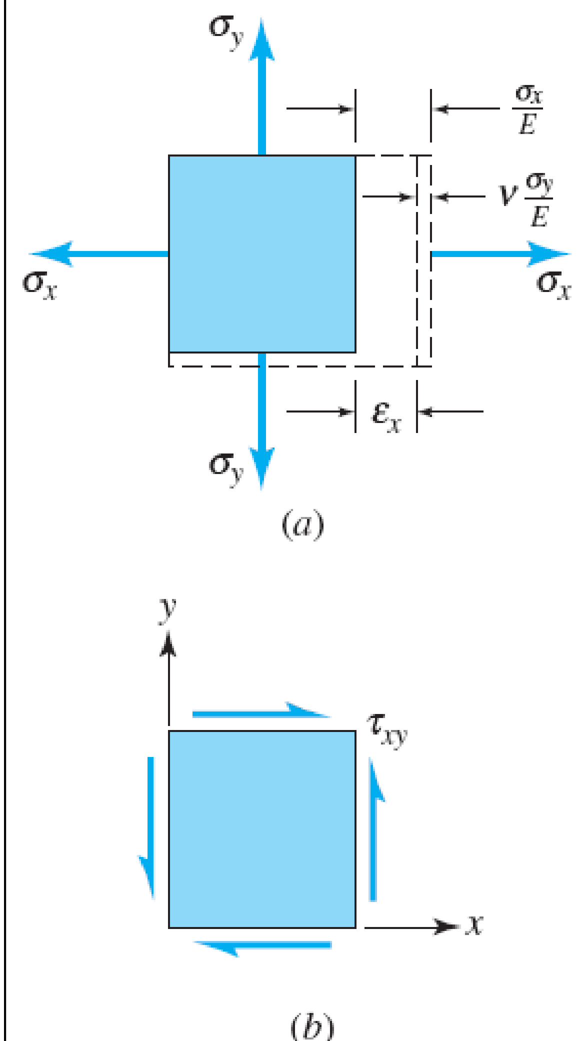


FIGURE 3.29 Element in state of (*a*) biaxial stress and (*b*) pure shear.

Generalized Hooke's law

Three dimensional stress

$$\boxed{G = \frac{E}{2(1+\nu)}}$$

Unit volume change

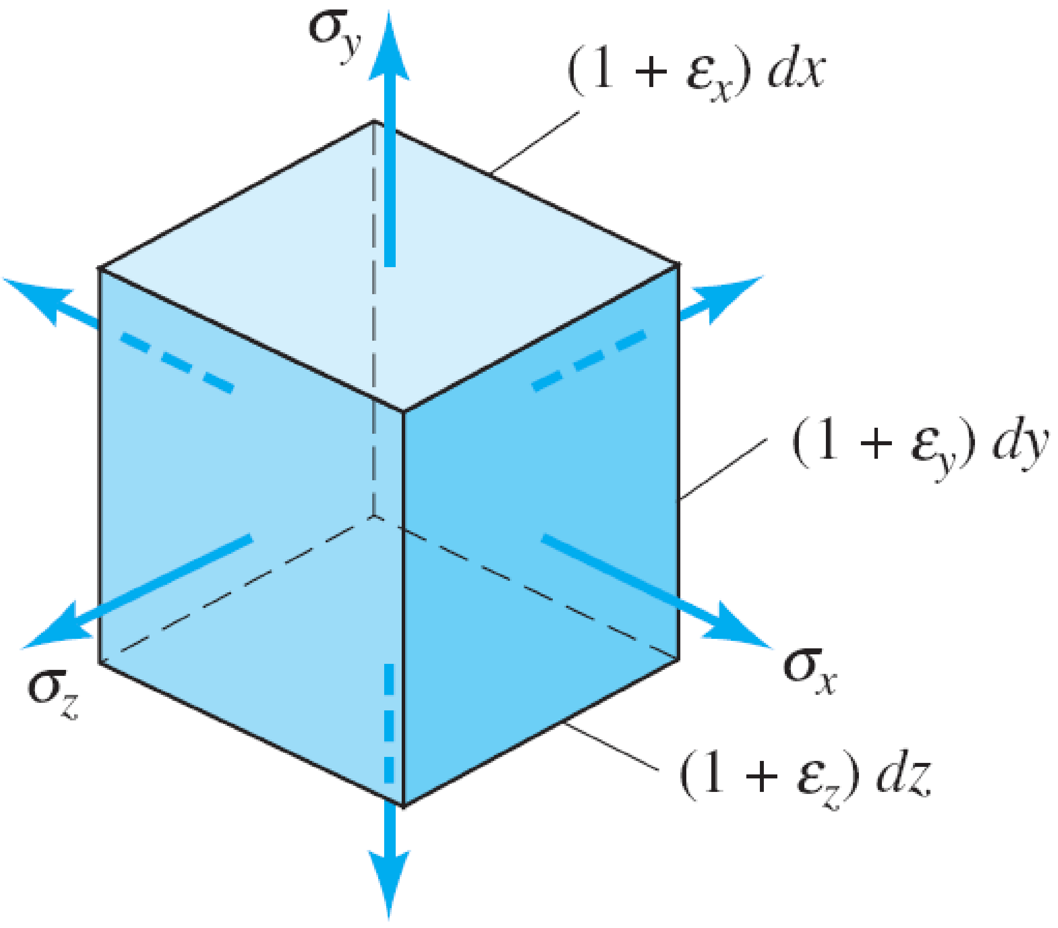


FIGURE 3.30 An element in triaxial stress.

Unit volume change

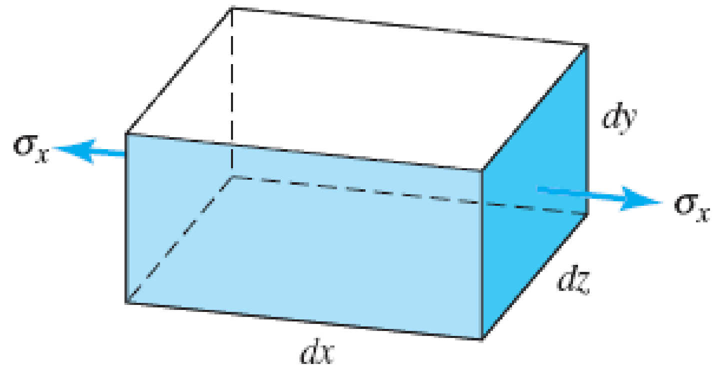
$$\boxed{e = \frac{\Delta V}{V_o} = \varepsilon_x + \varepsilon_y + \varepsilon_z}$$

$$\boxed{e = \frac{1 - 2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)}$$

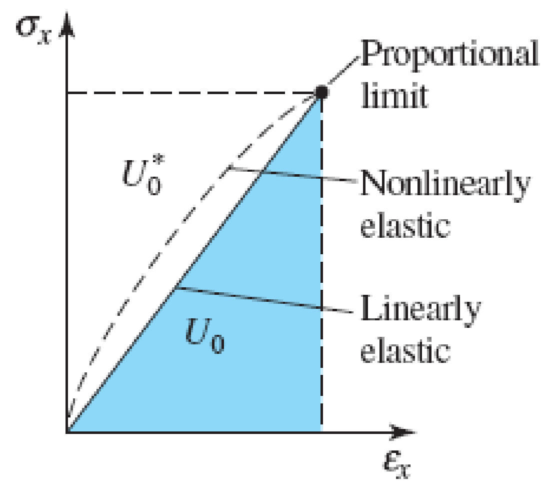
$$\boxed{k = \frac{E}{3(1 - 2\nu)}}$$

Strain energy uniaxial stress

$$U_0 = \frac{E\varepsilon_x^2}{2} \qquad U_0 = \frac{\sigma_x^2}{2E}$$



(a)



(b)

FIGURE 3.33 (a) An element in tension and (b) the stress–strain diagram.

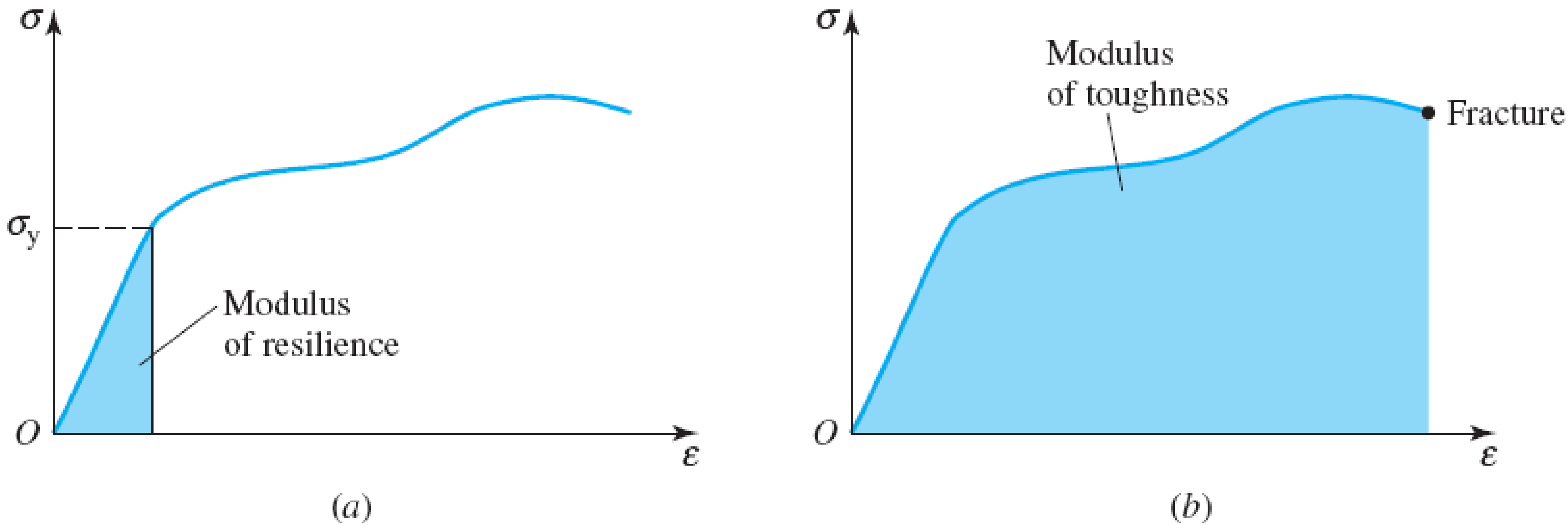


FIGURE 3.34 Typical stress–strain diagram: (*a*) modulus of resilience; (*b*) modulus of toughness.

$$U = \int \frac{\sigma_x^2}{2E}\, dV$$

Strain energy (shear stress)

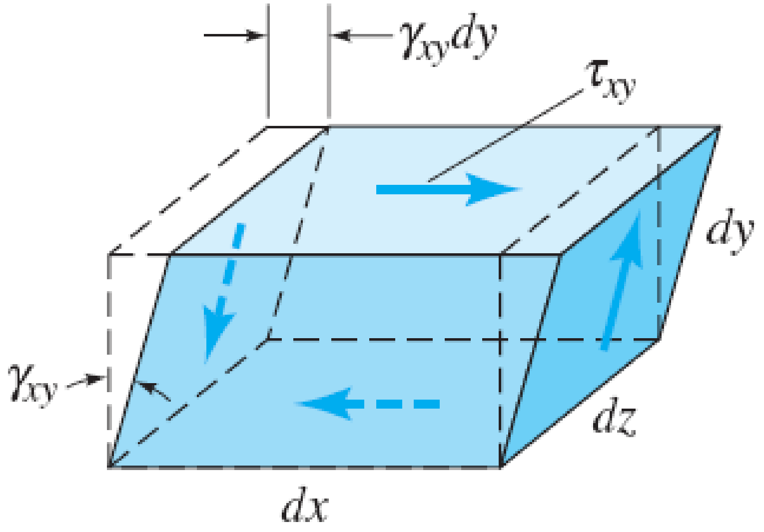


FIGURE 3.35 An element in pure shear stresses.

$$U = \int \frac{\tau_{xy}^2}{2G} \, dV$$