# Chapter 4. Health Reasoning for Diagnosis (Non-Vibration based)

## Prognostics and Health Management (PHM)

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System Health & Risk Management

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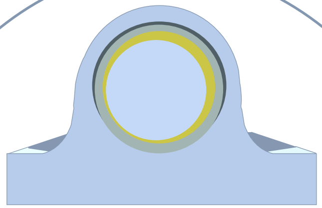
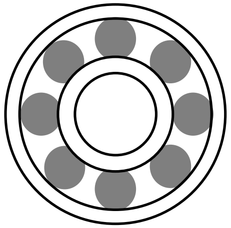
## Vibration Based Application



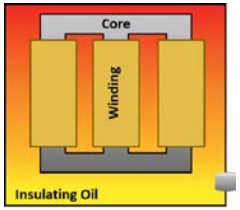
Journal Bearing Rotor



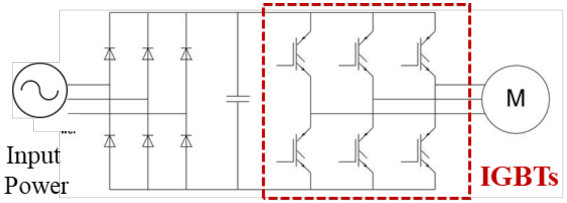
Ball Bearing

Planetary Gear

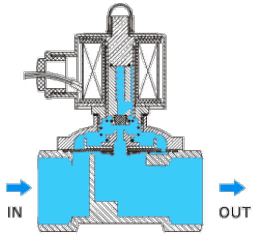
## Non-Vibration Based Application




Power Transformer




Insulated gate bipolar transistors

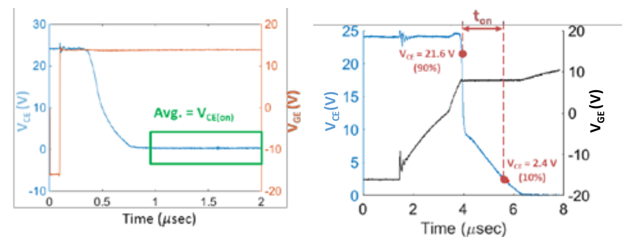



Solenoid Valve

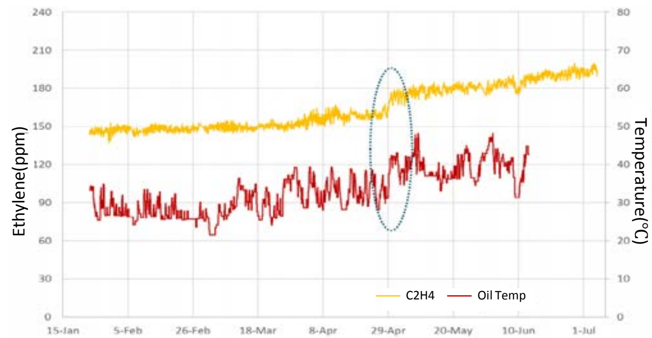
# Non-Vibration Data

- **Non-Vibration Data**
  - Temperature
  - Pressure
  - Current, Voltage
  - Speed, RPM
  - Gas Concentration
  - Humidity

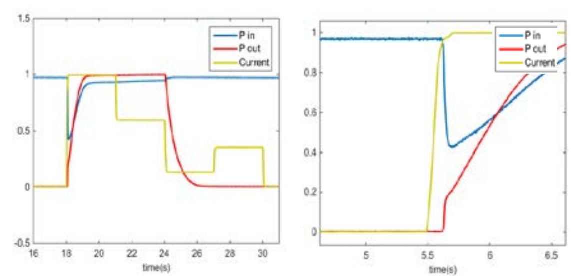
<IGBT>




<Power Transformer>




<Solenoid Valve>




- Mean
- Variance
- Explicit Math
- Correlation Analysis
- Mahalanobis Distance
- PCA
- ⋮

# Feature Engineering

- **Time series transforms – week 4 (vibration based)**
- **Time-independent transforms**
  - Basic mathematical form : difference, ratio, logarithm, power n, etc.
  - Advanced mathematical form : Principal component analysis, etc.
- **Data descriptive statistics**
  - For sensors : RMS, variance, kurtosis, correlation analysis, Mahalanobis distance, etc.
  - For events : count, occurrence rate, duration, time delays, etc.

# Explicit Mathematical Form

| Difference | Ratio | Logarithm | Power |
|---|---|---|---|
| $F(y) = F(x_1 - x_2)$ | $F(y) = F({}^{x_1}/_{x_2})$ | $F(y) = F(\log x)$ | $F(y) = F(x^n)$ |

- Mathematical expression to represent data characteristics better

Example

| Variance | Coefficient of Variation | dB | Standard Deviation |
|---|---|---|---|
| $Var(\mathrm{X}) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_{x_i})^2$ | $\delta_{\mathrm{X}} = \frac{\sigma_{\mathrm{X}}}{\mu_{\mathrm{X}}}$ | $Y_{dB} = 10\log_{10} {}^{x}/_{x_{ref}}$ | $\sigma_{\mathrm{X}} = \sqrt{Var(\mathrm{X})}$ |

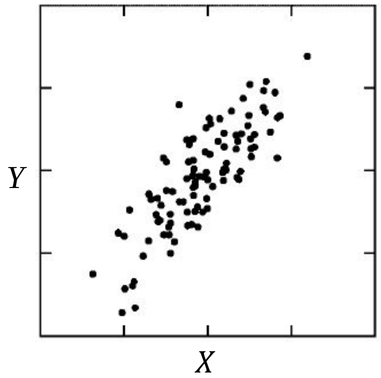
| Application | Health Feature | Mathematical Form | Parameter |
|---|---|---|---|
| Solenoid rubber damage | $HR_R = 10\log {}^{P_{min}}/_{P_{0.99}}$ | Logarithm, Ratio | $P$: Pressure |
| Power generator capacitance | $C = \varepsilon_r \varepsilon_0 {}^{A}/_{t}$ | Ratio | $\varepsilon_r$: Relative static permittivity, $\varepsilon_0$: Electric constant $A$: Area of tester, $t$ : Thickness of insulation |
| IGBT degeneration | $V_{GE(th)} = 1 - A \cdot (N_{ESD})^{\beta} \cdot (V_{ESD} - V_{rated})^{n}$ | Difference, Power | $A$: degradation rate $\beta$, $n$: process-related coefficients |

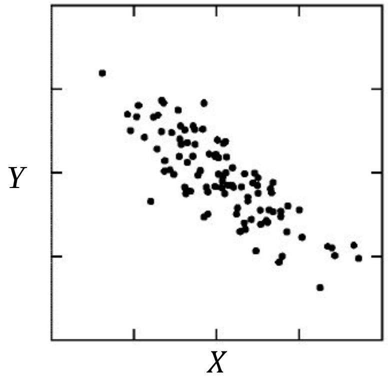
# Correlation Analysis

- **Objective:** To quantify the linear relationship between bivariate serial data
- **Results:** Correlation coefficient quantifying the direction and strength of the linear relationship between the two variables
- **Expectation:** To find the efficient set of signals to analyze in priority
- **Methods:**

$$\rho_{X,Y} = corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}, \qquad |\rho_{X,Y}| \leq 1$$

- **Notice:**
  - Correlation coefficient is dimensionless
  - Invariant under separate changes in location and scale in the two data (e.g., $\rho_{X,Y}=\rho_{X\prime,Y\prime}$, where X′ → aX+b and Y′ → cY+d)
  - Insufficient to detect non-linear relationship → Spearman’s rank, Kendall tau coefficient




$\rho = 1$ : perfect positive correlation

$\rho = -1$: perfect negative correlation

# Correlation Analysis

- **Example:** Pressure data in chemical mechanical polishing (CMP)

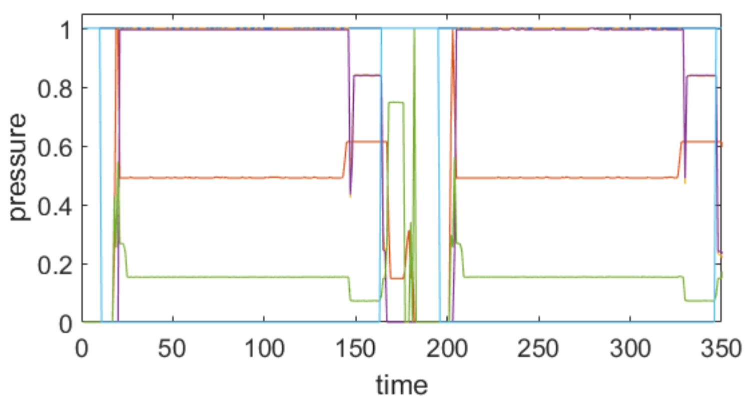
**Correlation matrix**

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1.00 | 0.89 | 0.88 | 0.32 | 0.89 | 0.89 |
| 2 | 0.89 | 1.00 | 1.00 | 0.18 | 0.99 | 0.97 |
| 3 | 0.88 | 1.00 | 1.00 | 0.18 | 0.99 | 0.97 |
| 4 | 0.32 | 0.18 | 0.18 | 1.00 | 0.17 | 0.15 |
| 5 | 0.89 | 0.99 | 0.99 | 0.17 | 1.00 | 0.98 |
| 6 | 0.89 | 0.97 | 0.97 | 0.15 | 0.98 | 1.00 |

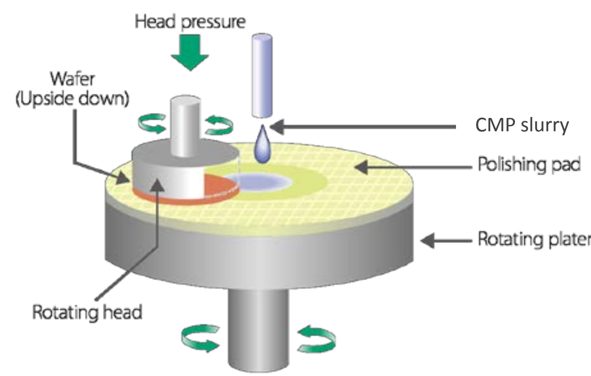
1. Chamber pressure
2. Main outer air bag pressure
3. Center air bag pressure
4. Retainer ring pressure
5. Ripple air bag pressure
6. Edge air bag pressure

→ *High correlation between* **2, 3, 5, 6**

→ *Focus on* **1, 4, 5** *for analyzing*




**Raw pressure data (Normalized)**




**Schematic of CMP process**

# Principal Component Analysis (PCA)

- **Objective**: For a given integer $k$, the PCA computes the $k$-principal components of the real valued data.
- **Results**: To find best low dimensional space that conveys maximum useful information
- **Expectation**: To extract new features with reduced dimension
- **Methods**: To minimize the sum of squares of distance to the line = To maximize the sum of squares of the projections on that line (variance of the projected data points)

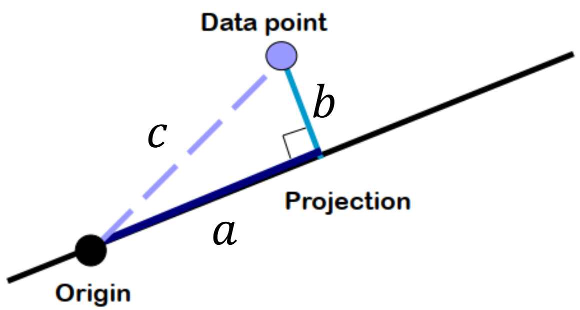
$$\max(\mathbf{X}^T\mathbf{B}^T\mathbf{B}\mathbf{X}) \text{ , subject to } \mathbf{X}^T\mathbf{X} = 1$$

where **X**: $n \times k$ matrix for the $k$ direction vectors (so that $\mathbf{X}^T\mathbf{X} = 1$)

**B**: $m \times n$ matrix of the coordinates of the $m$ data points (after the mean centering)

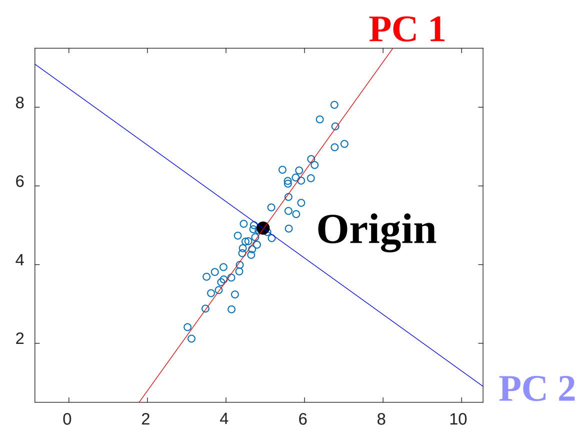
**BX**: the coordinates of the projection of m data points into the $k$ direction vectors **X**.

$\mathbf{X}^T\mathbf{B}^T\mathbf{B}\mathbf{X}$: the variance of the projected data points



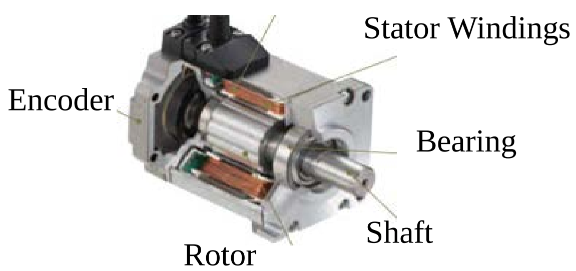

$$a^2 + b^2 = c^2 \ (const.)$$

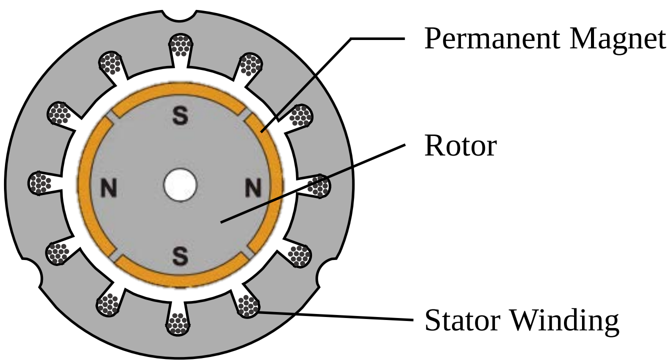
$$\therefore \min\left(\sum b^2\right) = \max\left(\sum a^2\right)$$

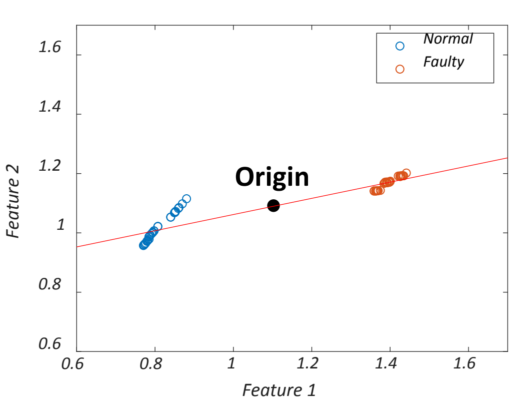


# Principal Component Analysis (PCA)

- **Example :** Motor current signals
  - Failure mode: Stator winding short
  - Feature 1: Root mean square (RMS)
  - Feature 2: Magnitude of fundamental frequency



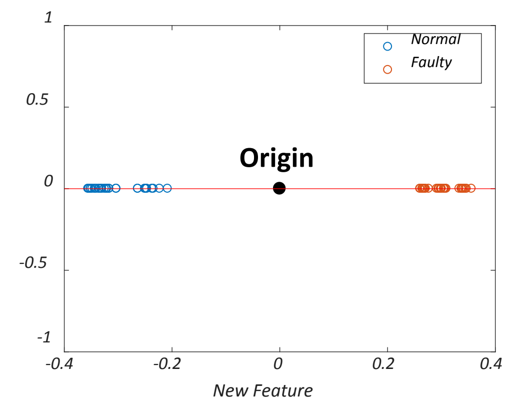




Two features

**X** = (0.96,0.26)

Projection




One features

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# Mahalanobis Distance (MD)

## Formula

Single variable

$$z = \frac{x-\mu}{\sigma}$$

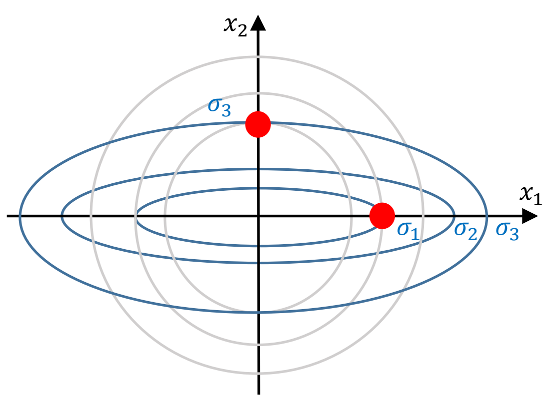
Multi-variables

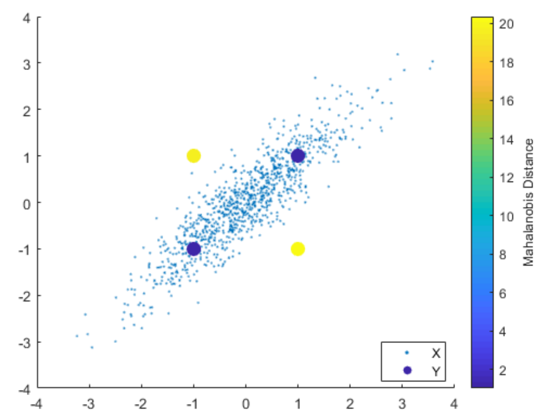
$$\mathrm{D} = \sqrt{z^T z} = \sqrt{(x-\mu)^T \Sigma^{-1} (x-\mu)}$$

where $\mu$ : *mean*, $\sigma$ : *standard deviation*, $\Sigma$ : *covariance matrix*

## Properties

- A measure of the distance between a point P and a distribution D. It is a multi-dimensional generalization of the idea of measuring how many standard deviations away P is from the mean of D.
- All variables are re-scaled to have unit variance. The MD corresponds to standard Euclidean distance in the transformed space.
- The MD is thus unitless and scale-invariant.
- Dataset must be homogeneous; otherwise results may not be reliable.

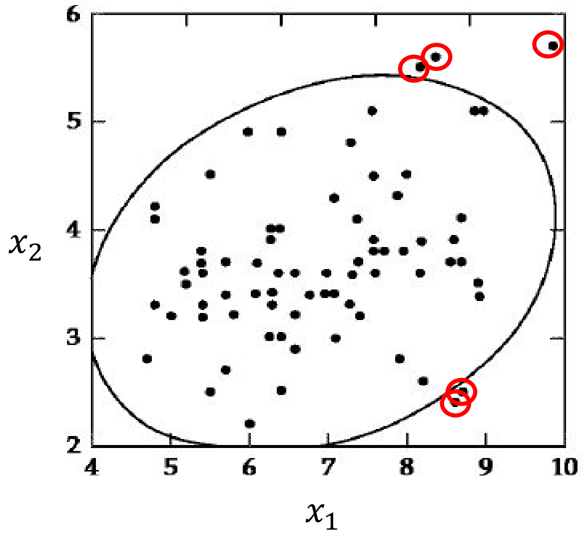


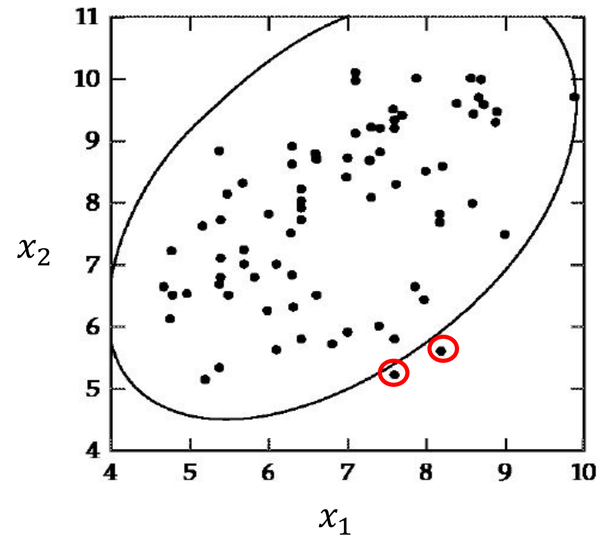



# Mahalanobis Distance

## Applications

- Cluster analysis and classification techniques
  - Calculated covariance matrix of each class about input dataset, belonging to the class which makes Mahalanobis distance minimum
  - Used as a weighted distance among the vectors, purposed to measure similarity of the vectors
- Detecting Outliers
  - A point that has a greater Mahalanobis distance compared to the rest of the sample points defined to be outlier

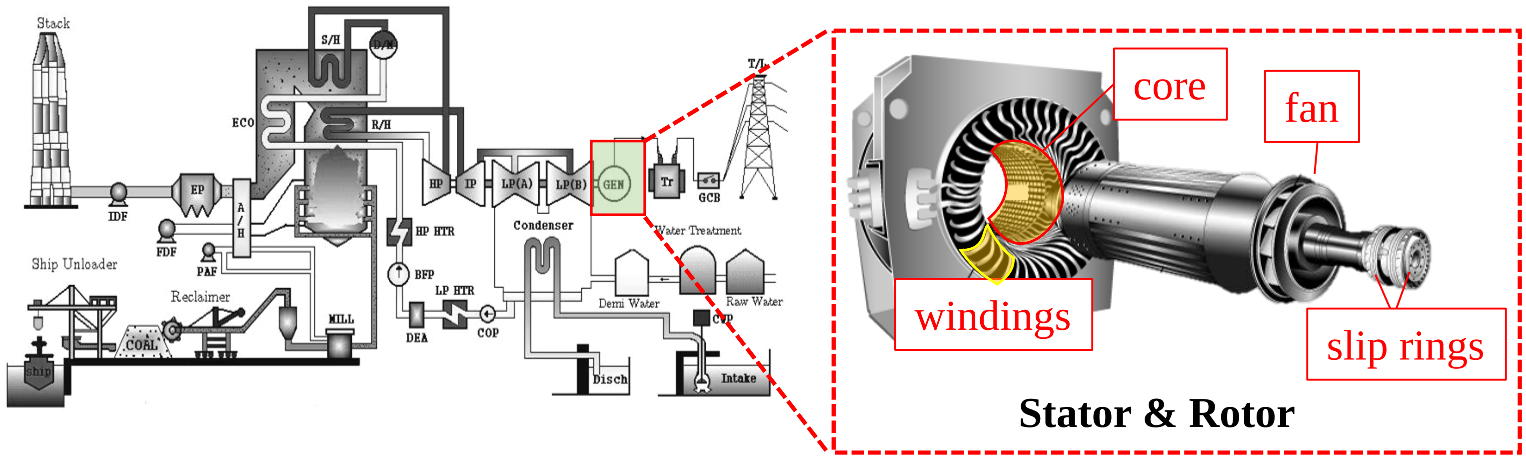


# Case Study : Power Generator in Power Plant System

| | |
|---|---|
| Objective | Health diagnosis and prognosis for power generator against water absorption |
| Target Products | Stator windings |
| Failure Modes | stator winding crack, water absorption |
| Used Signal | Insulation capacity |

- **Power Generator in Power Plant System**

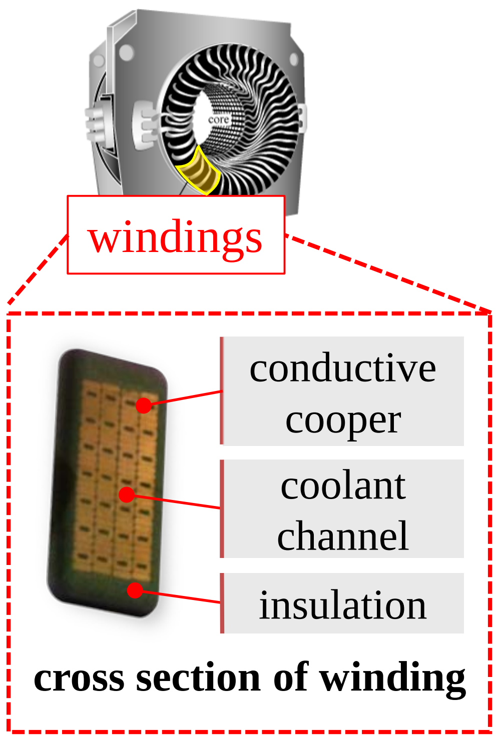



Stator & Rotor

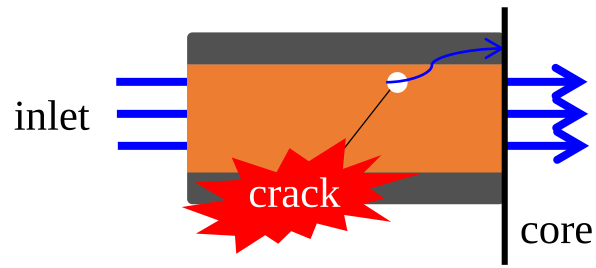
# Case Study : Power Generator in Power Plant System

**Failure Physics**

- Windings in Generator Stator
- **Coolant water** + **Electricity** = **Burn & Failure**
- **Water penetration** of the ground wall insulation
- Evaluating the amount of water absorption




**cross section of winding**




**Side of a winding**

# Case Study : Power Generator in Power Plant System

## Measurement

- Measuring the **capacitance** of the ground wall insulation
- Estimating the extent of water penetration **indirectly**; **WET** insulation ⇨ **HIGH** capacitance
- **Non-destructive** to the stator bar
- 43 windings with 10 measurements (turbine/collector ends and top/bottom) for each winding

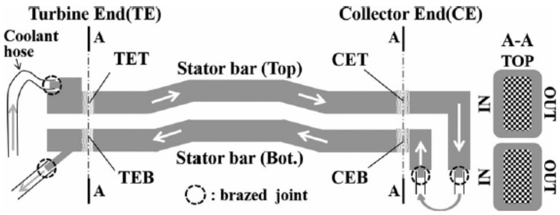
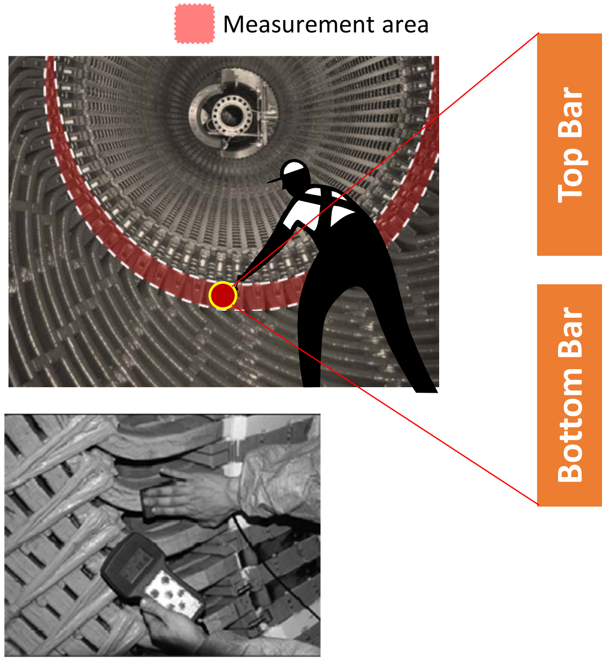



Fig. 4. Structure diagram of a water-cooled power generator with two-path cooling system and four brazed locations.




*Output*

$$\boxed{C} = \varepsilon_r \varepsilon_0 \frac{A}{t}$$

- **Capacitance [pF]**

$\varepsilon_r$: Relative static permittivity
$\varepsilon_{water}$ : 80.4
$\varepsilon_{mica}$ : 5.6 - 6.0
$\varepsilon_0$: Electric constant (~ 8.854x10-12 F/m)
$A$: Area of tester
$t$ : Thickness of insulation

- **Irregular measurement every overhaul**

Once every 2 or 3 years

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# Case Study : Power Generator in Power Plant System

**Result**

- Mahalanobis Distance (MD): a relative health measure with statistical correlation
- Health grade system: three health class classification
- RUL Prediction: trend modeling based on fick's second law

$$\mathrm{MD}(\mathbf{X}_i) = \sqrt{(\mathbf{X}_i - \boldsymbol{\mu})^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} (\mathbf{X}_i - \boldsymbol{\mu})}$$

$$\text{where, } \mathbf{X}_i = \left(X_{1,i}, \cdots, X_{N,i}\right)^{\mathrm{T}} \text{and } \boldsymbol{\mu} = (\mu_1, \cdots, \mu_N)^{\mathrm{T}}$$

$X_{n,i}$ : raw capacitance data at the $n^{\mathrm{th}}$ measurement location of the $i^{\mathrm{th}}$ winding unit
$\mu_n$ : mean of the capacitance data at the $n^{\mathrm{th}}$ measurement location
$\boldsymbol{\Sigma}$ : covariance matrix

| | CET | | | CEB | | TET | | | TEB | |
|---|---|---|---|---|---|---|---|---|---|---|
| Correlation Matrix | TOP ($X_1$) | OUT ($X_2$) | IN ($X_3$) | OUT ($X_4$) | IN ($X_5$) | TOP ($X_6$) | OUT ($X_7$) | IN ($X_8$) | OUT ($X_9$) | IN ($X_{10}$) |
| CET | TOP ($X_1$) | 1 | | | | | | | | |
| | OUT ($X_2$) | 0.4761 | 1 | | | | | | | |
| | IN ($X_3$) | 0.4194 | 0.5503 | 1 | | | | | | |
| CEB | OUT ($X_4$) | 0.0849 | 0.1572 | 0.1354 | 1 | | | | | |
| | IN ($X_5$) | −0.039 | 0.1686 | 0.0765 | 0.3445 | 1 | | | | |
| TET | TOP ($X_6$) | 0.3341 | 0.1553 | 0.1868 | 0.0343 | −0.052 | 1 | | | |
| | OUT ($X_7$) | 0.1972 | 0.2506 | 0.2729 | 0.0879 | 0.0171 | 0.4377 | 1 | | |
| | IN ($X_8$) | 0.2295 | 0.1423 | 0.3296 | 0.0082 | 0.0457 | 0.4269 | 0.4900 | 1 | |
| TEB | OUT ($X_9$) | 0.0438 | −0.128 | −0.097 | 0.0186 | −0.114 | 0.0887 | −0.010 | −0.003 | 1 |
| | IN ($X_{10}$) | 0.0354 | −0.040 | −0.004 | 0.0457 | 0.0870 | −0.048 | 0.1084 | 0.0215 | 0.3385 |

# Case Study : Power Generator in Power Plant System

## Result

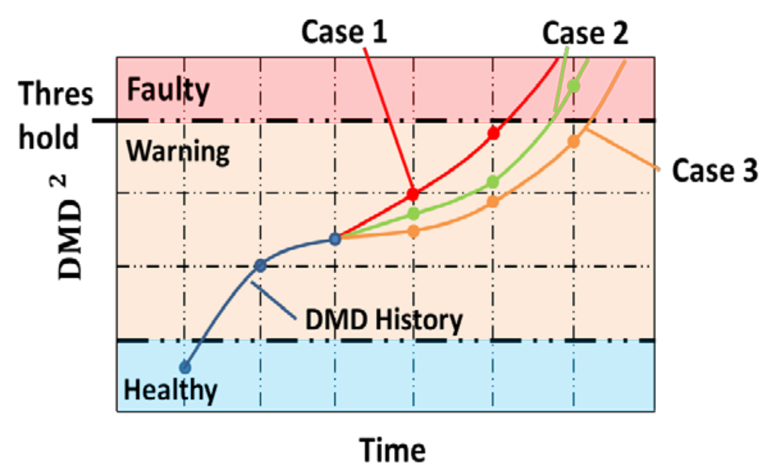
- Mahalanobis Distance (MD): a relative health measure with statistical correlation
- Health grade system: three health class classification
- RUL Prediction: trend modeling based on fick's second law

$$\mathrm{MD}(\mathbf{X}_i) = \sqrt{(\mathbf{X}_i - \boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{X}_i - \boldsymbol{\mu})}$$

$$\text{where, } \mathbf{X}_i = \left(X_{1,i}, \cdots, X_{N,i}\right)^{\mathrm{T}} \text{and } \boldsymbol{\mu} = (\mu_1, \cdots, \mu_N)^{\mathrm{T}}$$

$X_{n,i}$ : raw capacitance data at the $n^{\text{th}}$ measurement location of the $i^{\text{th}}$ winding unit
$\mu_n$ : mean of the capacitance data at the $n^{\text{th}}$ measurement location
$\boldsymbol{\Sigma}$ : covariance matrix

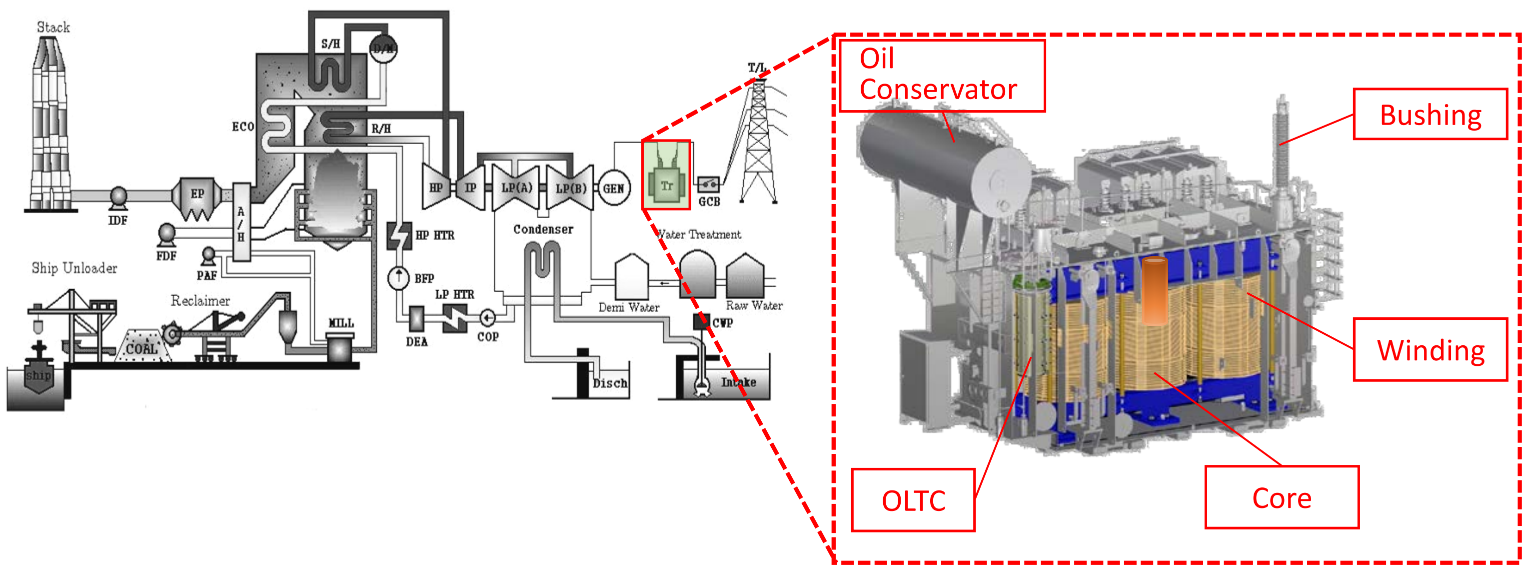
| | TOP group | BOTTOM group |
|---|---|---|
| Range | DMD² ≥25 | DMD² ≥16.67 |
| Meaning | Faulty (Water absorption) | |
| Range | 15≤ DMD² <25 | 10≤ DMD² <16.67 |
| Meaning | Warning | |
| Range | DMD² <15 | DMD² <10 |
| Meaning | Healthy | |



# Case Study : Power Transformer in Power Plant System

| | |
|---|---|
| Objective | Development of robust diagnosis algorithm for power transformer |
| Target Products | Diagnosis algorithm using Dissolved Gas Analysis (DGA) |
| Failure Modes | Arc, corona, spark, overheating ($T_1$<300℃, 300℃<$T_2$<700℃, 700℃<$T_3$) |
| Used Signal | Dissolved gas from oil-filled power transformer |

- **Power Transformer in Power Plant System**



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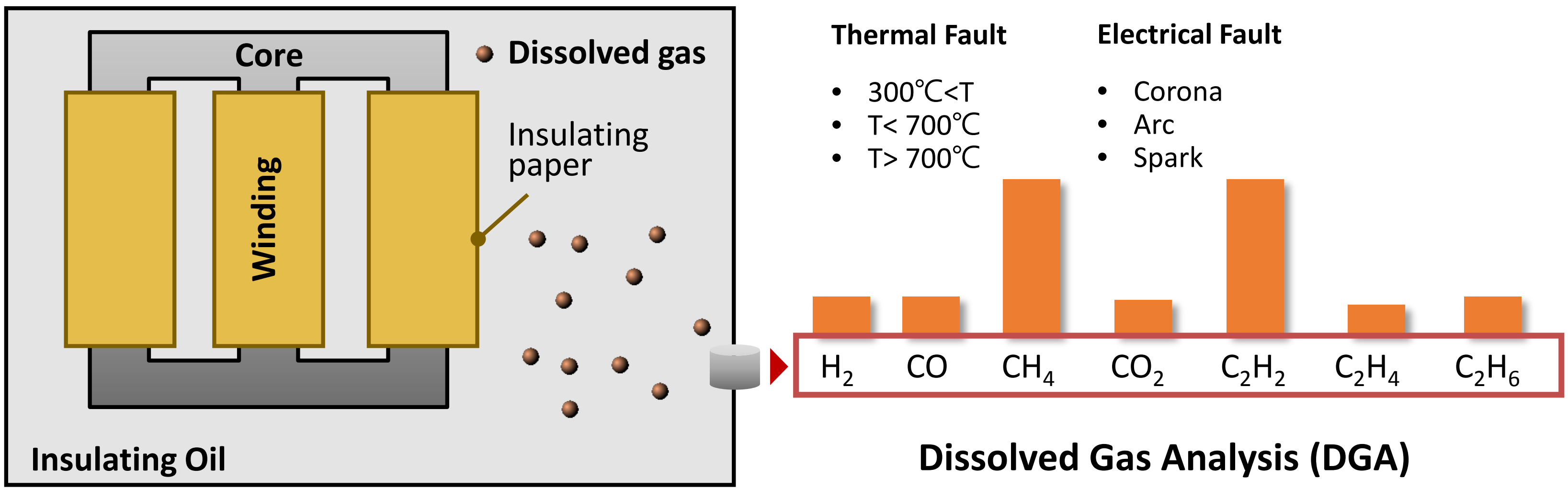
# Case Study : Power Transformer in Power Plant System

**Failure Physics of Electrical & Thermal Fault**

- Degradation of transformer winding insulation (paper, oil)
- **Degraded** insulating paper/oil+ **electricity** = **hot spot & electric discharge**
- **Insulation degradation of bushing**
- Evaluating the amount of hydrocarbon gas

**Measurement**

- Measuring the **dissolved gas** from insulating oil
- Obtain gas concentration by chromatography
- Acquire dissolved gas w/o shutting down power transformer



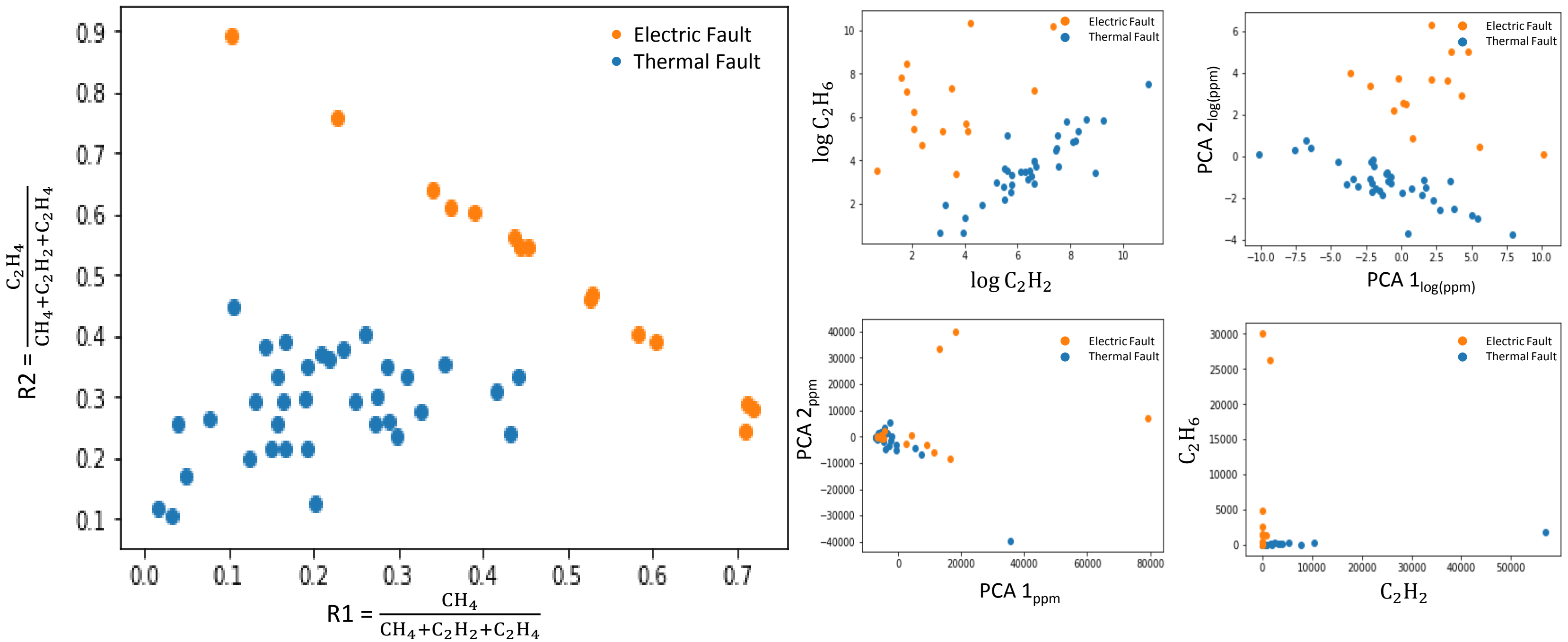

**Dissolved Gas Analysis (DGA)**

# Case Study : Power Transformer in Power Plant System

**Result**

- Dissolved Gas Analysis (DGA) : the study of dissolved gases to diagnose incipient fault
- Health feature : DGA concentration and ratio

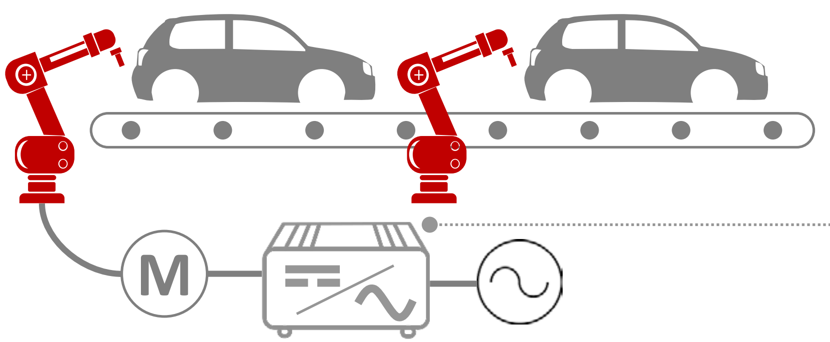
<Comparison of Duval, PCA, log and ppm features for electrical and thermal fault>

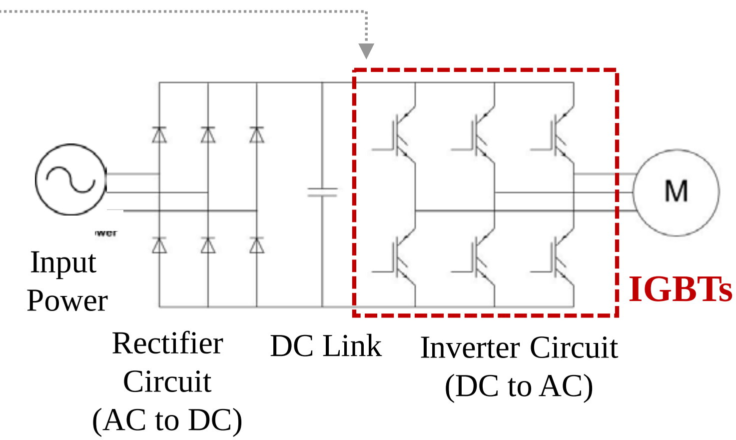


# Case Study: IGBT (Insulated gate bipolar transistors) in Inverter System

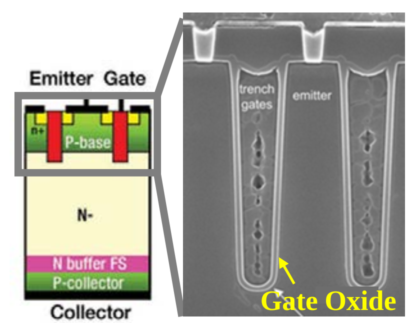
| | |
|---|---|
| Objective | Development of the failure precursors for IGBTs in inverter system |
| Target Products | Trench gate and filed stop IGBTs |
| Failure Modes | Gate oxide degradation |
| Used Signal | Collector-Emitter voltage($V_{CES}$) |

- **Inverter in Product Line**




- **Fault Injections : Electrostatic discharge (ESD)**
  - To obtain failure data in short time
  - ESD: To emulate predominant failures in the field




ESD simulator

# Case Study : IGBT in Inverter System

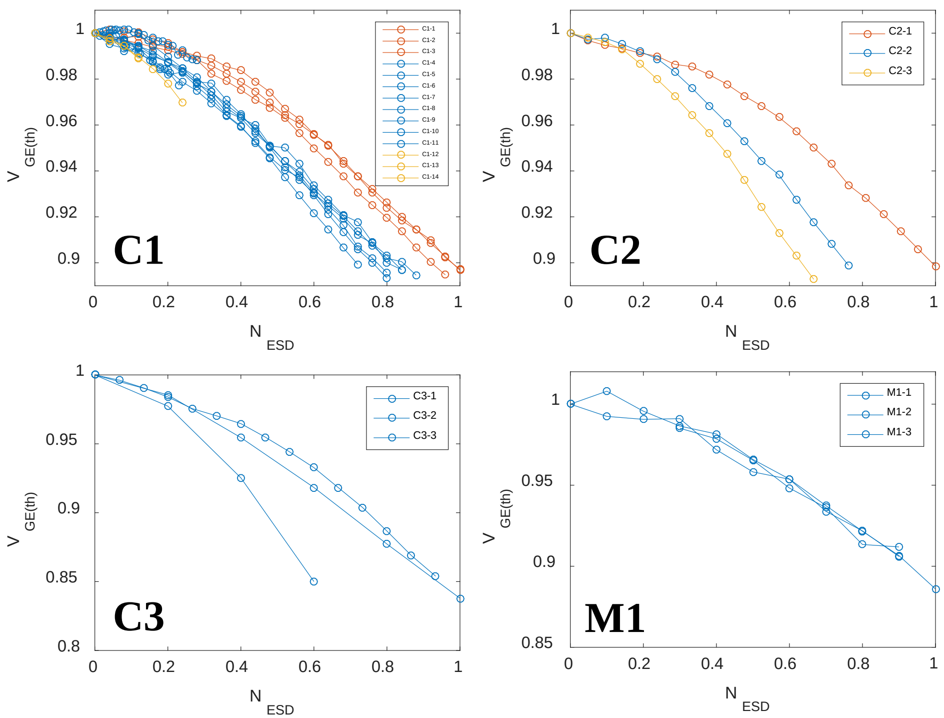
## Results

- Three components (C) and one modules (M)
- Failure Precursor : Gate-Emitter threshold voltage ($V_{GE(th)}$)

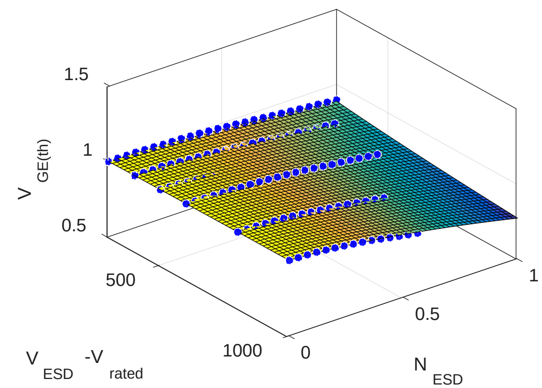
### <Experimental Results>

$N_{ESD}$ : The number of ESD injections
All the values are normalized.

Stress level 1
Stress level 2
Stress level 3




### <Proposed Degradation Model>




$$V_{GE(th)} = 1 - A \cdot (N_{ESD})^{\beta} \cdot (V_{ESD} - V_{rated})^{n}$$

where

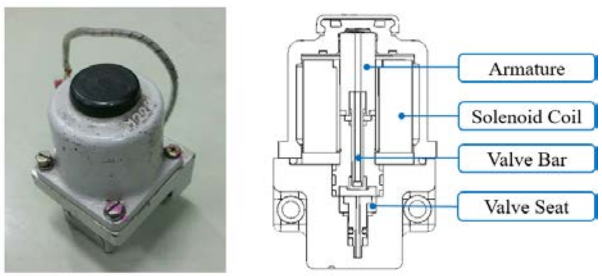
$A$: degradation rate

$\beta$, $n$: process-related coefficients

# Case Study : Solenoid Valve Diagnosis for Railway Braking System

| | |
|---|---|
| Objective | Development of CBM Model for Solenoid Valve in Railway Braking System |
| Target Products | Service braking valves in urban railway vehicles |
| Failure Modes | Burnout of coils, rubber damage, debris accumulation, |
| Used Signal | Current, input and output pressure |

- **Solenoid Valve System**




**Solenoid Valve**

**Testbed (SHRM)**

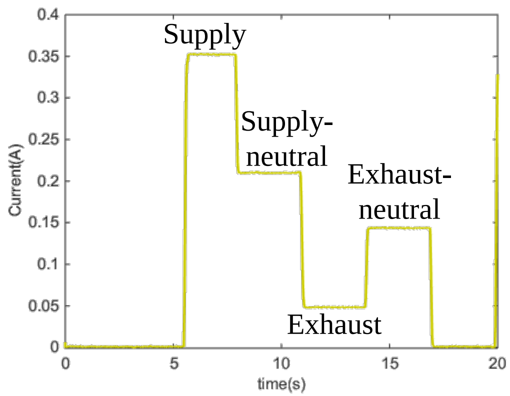
- **FMECA on Solenoid Valve**

| Item | Function | Failure Mode | Failure Effect | Failure Cause |
|---|---|---|---|---|
| Solenoid coil | Controlling valve bar | **Burnout of coils** | Abnormal valve bar behavior | Interruption of coils |
| Valve seat | Controlling air flow | **Rubber damage** | Valve bar jamming | Fatigue failure by valve bar impact |
| | | **Debris accumulation** | Air leakage | Impurities in air from other components |

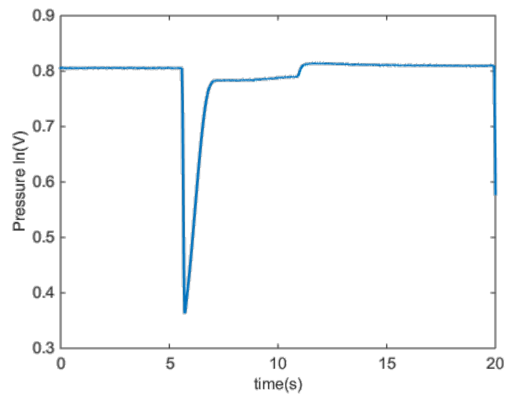
# Case Study : Solenoid Valve Diagnosis for Railway Braking System

## Data Acquisition from Testbed

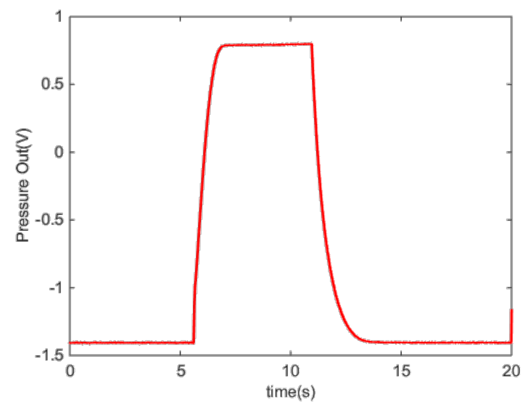
Current ($I_S$)




Input pressure ($P_{in}$)

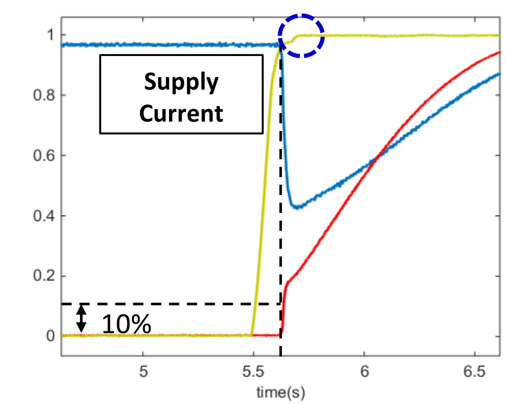



Output pressure ($P_{out}$)



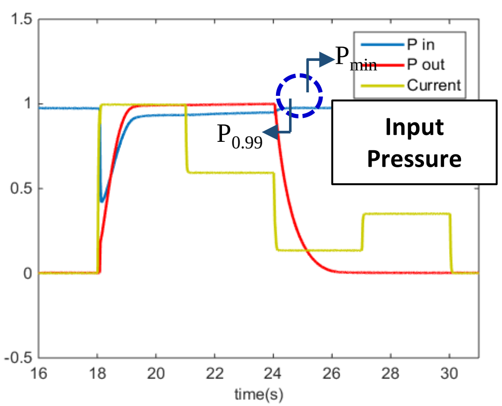

## Health Reasoning of Solenoid Valve

Burnout of coils



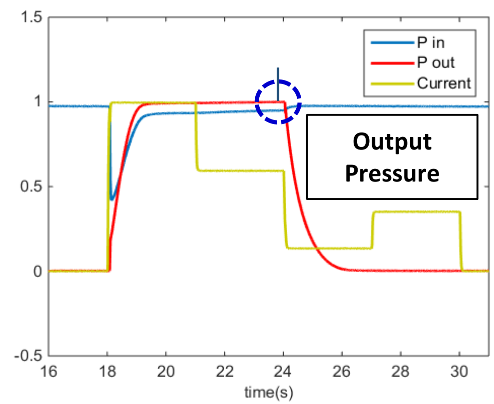

$HR_B = Max(I_S)$

Rubber damage




$HR_R = 10\log(P_{min}/P_{0.99})$

Debris accumulation




$HR_D = Max(P_{out})$

# THANK YOU
# FOR LISTENING

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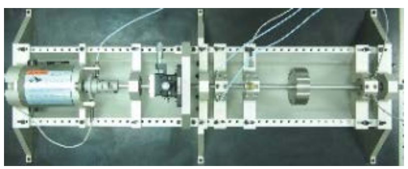
[6] R. O. Duda, P. E. Hart, and D. G. Stork, Pattern classification, Vol. 2, New York: Wiley, 1973.

[7] B. C. Jeon, et al., "Datum unit optimization for robustness of a journal bearing diagnosis system," *IJPEM*, 16.11, (2015): 2411-2425.

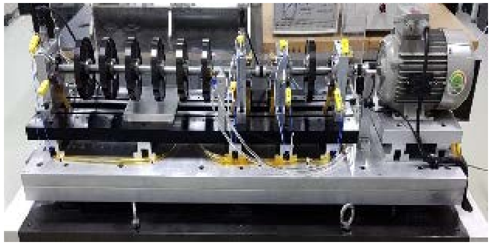
# Health Reasoning for Vibration Signals from Journal Bearing System

| | |
|---|---|
| Objective | Development of robust diagnosis algorithm for journal bearing rotor system |
| Target Products | Diagnosis algorithm using vibration signals |
| Failure Modes | Unbalance, rubbing, misalignment, oil whirl |
| Used Signal | Vibration signals from gap sensors |

- **Journal Bearing Rotor System**

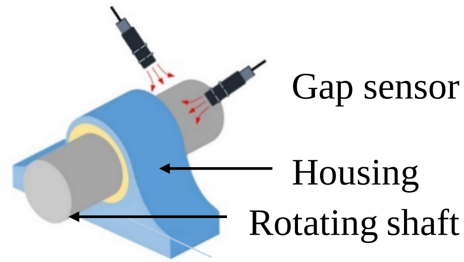


RK4 (Bently Nevada, GE)



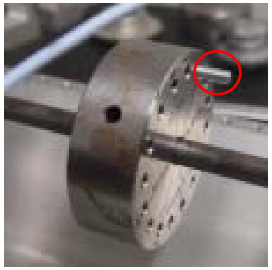
Rotor Simulator (SHRM)

Journal bearing

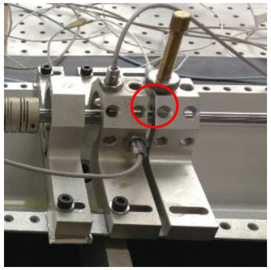



- **Fault Modes**

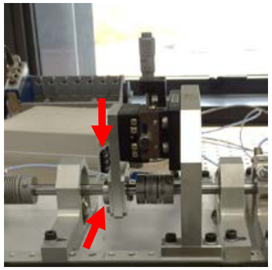
Unbalance



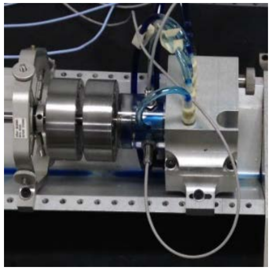
Rubbing



Misalignment



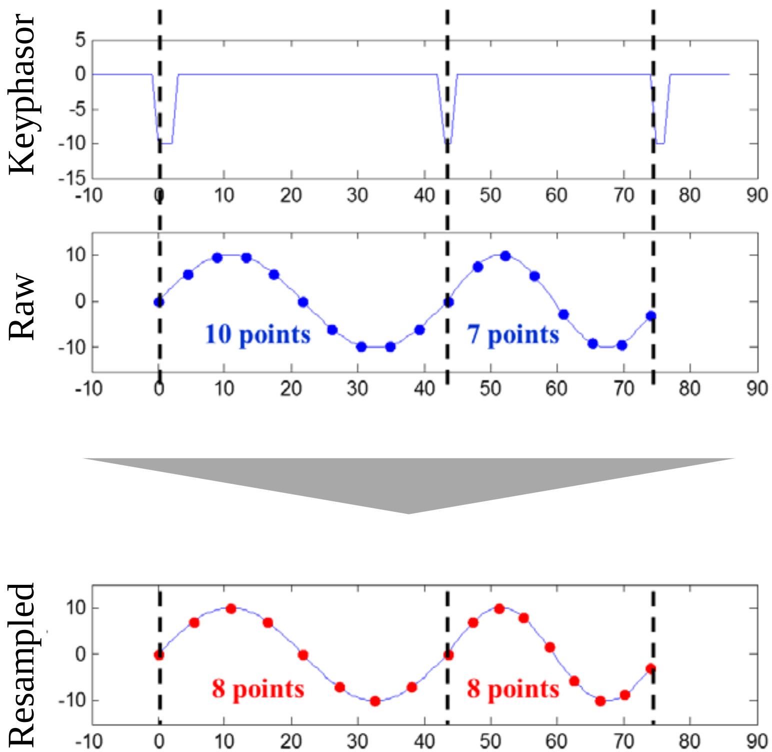
Oil Whirl



# Health Reasoning for Vibration Signals from Journal Bearing System

## Health Reasoning for Vibration Signals

<Keyphasor Signal Based Resampling>




Phase & Time Sync

<Feature Extraction>

- **Time Features**
  - Kinetic energy related → Mean, max, RMS
  - Data statistics related → Skewness, kurtosis
  - Waveform related → Crest / impulse / shape factor
- **Frequency Features**
  - Data statistics related → Mean, RMS, Variance (Frequency)
  - Component ratio to 1x → Sub- & Super-harmonics / 1x

Time & Freq. Features

Separability Evaluation

# Kullback-Leibler divergence

## Background

- Originated from Information Theory
  - Goal of the Information theory : Quantify how much information is in data
  - Entropy : lower bound on the number of bits needed to transmit the state of a random variable
  - Entropy formula

$$H = -\sum_{i=1}^{N} p(x_i) \cdot \log p(x_i)$$

where,
$p(x)$: Probability density function

- If uncertain distribute function $p$ is approximated to designed distribution $q$, the expectation value of the difference between two probability distribution is KL divergence

## Formula

- Slight modification of a formula for entropy (KL divergence or relative entropy)

$$D_{KL}(p \parallel q) = \sum_{i=1}^{N} p(x_i) \cdot \log \frac{p(x_i)}{q(x_i)}$$

where,
$p(x)$: True distribution
$q(x)$: Approximated distribution

# Kullback-Leibler divergence

## Kullback-Leibler Formula

$$D_{KL}(p \parallel q) = \sum_{i=1}^{N} p(x_i) \cdot \log \frac{p(x_i)}{q(x_i)} \qquad D_{KL}(p \parallel q) = -\int p(x) \ln \left\{ \frac{q(x)}{p(x)} \right\} dx$$

## Properties

- Typically P represent the distribution of the data, while Q represents a theoretical expectation of the data
- Way to covering whole distribution (Non-local)
  - Useful for non Gaussian distribution
- KL divergence provides a measure of the similarity of two known distribution
  - Minimize the KL diverse value is same as to build MLE(Maximum likelihood estimation)

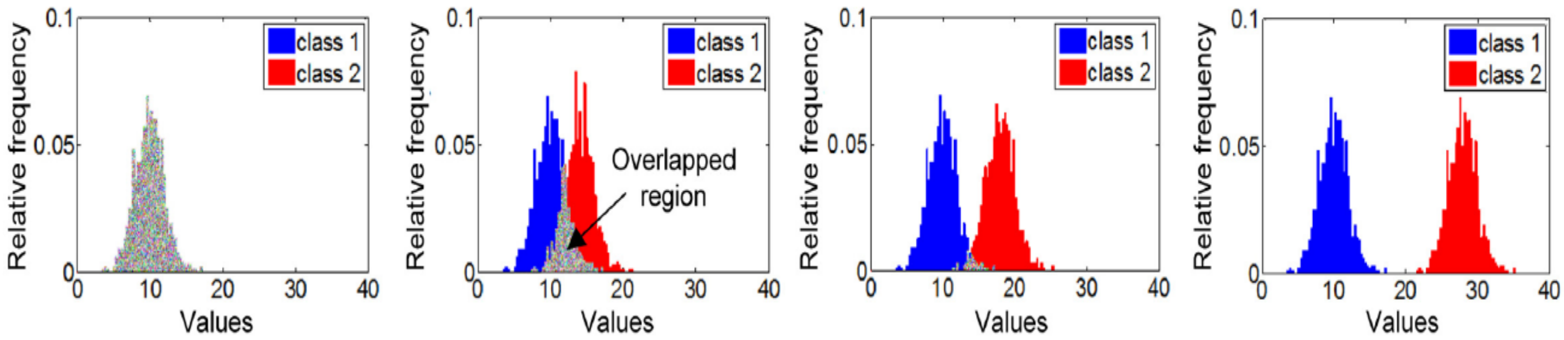
## Distinction (Divergence ≠ Distance)

- KL divergence signify a distance between two distribution, However it isn't mean a distance because it cannot satisfy the symmetric

# Kullback-Leibler divergence

## Application

- Class Separation




0 6.589 20.278 23.692

KL Divergence

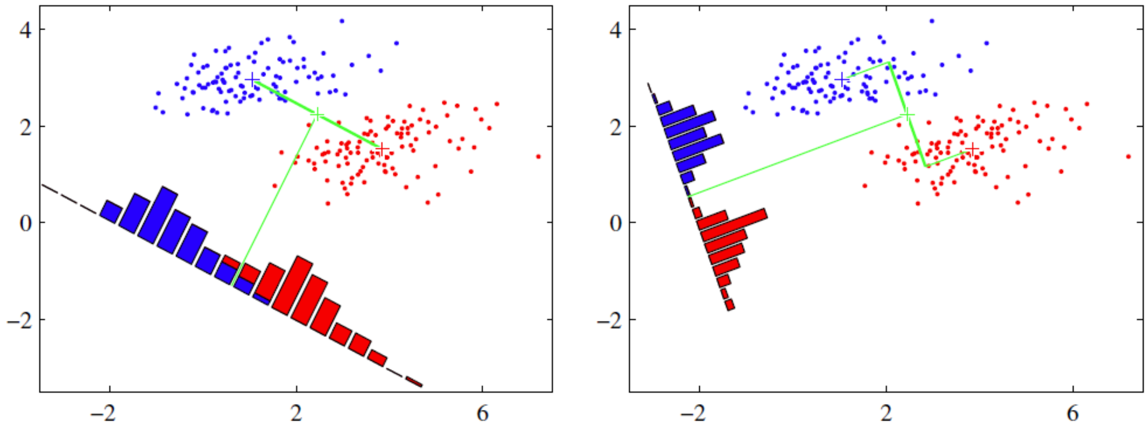
# Fisher Discriminant Analysis

## Fisher Discriminant Ratio(FDR)

$$FDR = J(w) = \frac{(m_2 - m_1)^2}{s_1^2 + s_2^2} \qquad m_k = \frac{1}{N_k}\sum_{n \in C_k} x_n \qquad s_k = \sum_{n \in C_k} (y_n - m_k)^2$$

$where\ m_1$ : mean of $X_1$
$m_2$ : mean of $X_2$
$\mu_1$ : standard deviation of $X_1$
$\mu_2$ : standard deviation of $X_2$

- Separable ability for two class data
- High FDR value means it can distinguish an abnormal condition from another condition



# Fisher Discriminant Analysis

## Fisher Linear discriminant

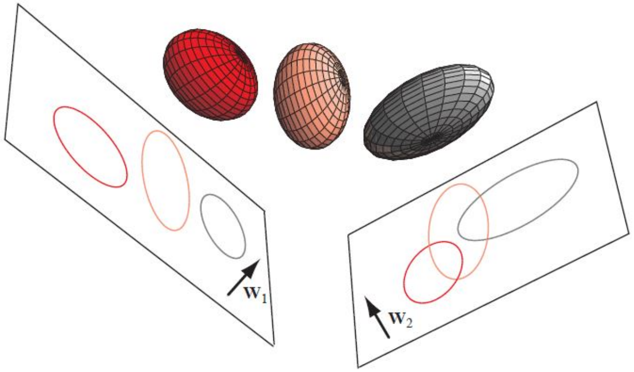
$y = w^T x$ $\quad$ $project\ input\ vector\ X\ to\ 1 - D(y)$

$$J(w) = \frac{w^T S_B w}{w^T S_w w}$$

$$S_B = (\mu_2 - \mu_1)(\mu_2 - \mu_1)^T$$
$$= between - class\ covariance$$

$$S_W = \sum_{n \in C_1} (x_n - m_1)(x_n - m_1)^T + \sum_{n \in C_2} (x_n - m_1)(x_n - m_2)^T$$
$$= within - class\ covariance$$

- FDA algorithm is to find a hyper-plane (projection vector *w*), where projected data on to this plane maximizes the function (FDR)
- Maximize projected class mean & Minimize projected class variance
- FDA is used for a classification scheme



# Probability of Separation

## Probability of Separation (PoS)

- Two-class separability measure
  - Based on load-strength interference (probability of failure)

$$P_f = \int_{-\infty}^{\infty} f_L(s) F_S(s) dx$$

- Probability of failure described by the load-strength interference is used to formulate non-separable region of the two classes

## Non-separable region

$$P_{NS} = \int_{-\infty}^{\infty} f_{c1}(x) F_{c2}(x) dx$$

For $\tilde{x}_{c1} \leq \tilde{x}_{c2}$

$0 \leq P_{NS} \leq 0.5$

where
$f_{c1}(s)$ : probability density function(PDF) of class 1
$F_{c2}(s)$ : cumulative distribution function(CDF) of class 2

## PoS Formula

$$PoS = (e^{(1-2\times P_{NS})} - 1)/(e-1)$$

$0 \leq PoS \leq 1$

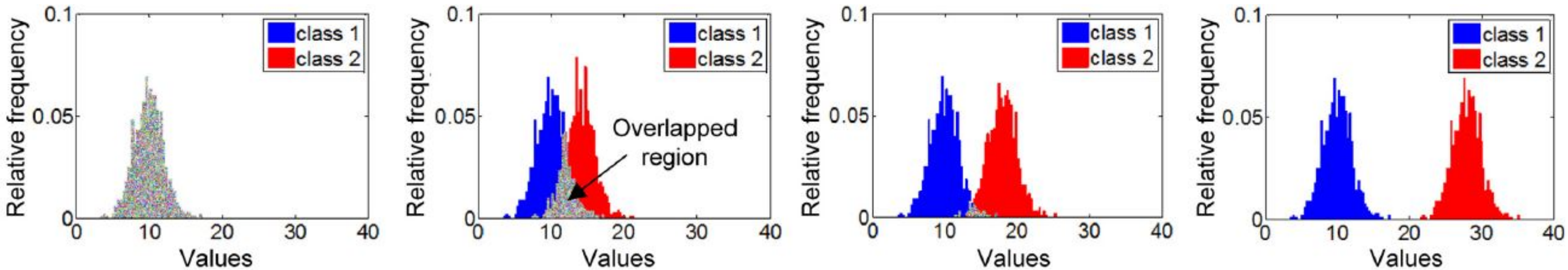
# Probability of Separation

## Probability of Separation (PoS)

$$PoS = (e^{(1-2\times P_{NS})} - 1)/(e - 1)$$

- "0" : two different classes overlaps perfectly
- "1" : two different classes not overlapped at all
- Bounded and normalized value

## Comparative study among KLD, FDR, PoS




| | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| KLD | 0 | 6.589 | 20.278 | 23.692 |
| FDR | 0 | 2.004 | 8.017 | 40.584 |
| PoS | 0 | 0.752 | 0.989 | 1 |