445.204

Introduction to Mechanics of Materials

(재료역학개론)

Myoung-Gyu Lee, 이명규

Tel. 880-1711; Email: myounglee@snu.ac.kr

TA: Seong-Hwan Choi, 최성환

Lab: Materials Mechanics lab.

Office: 30-521

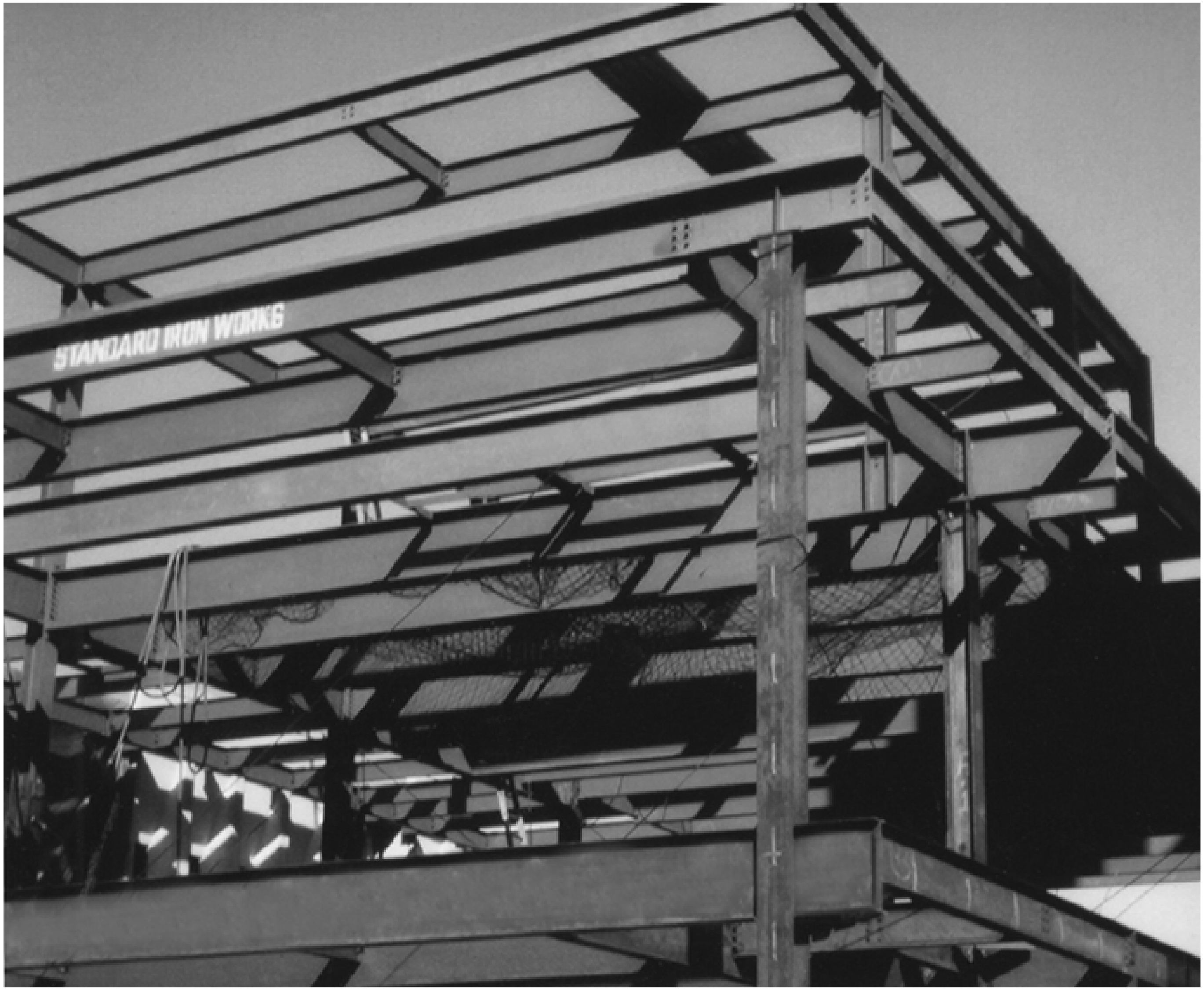
Email: cgr1986@snu.ac.kr

Chapter 4

Axially loaded members

Axially loaded members in structures

Axially loaded members in structures



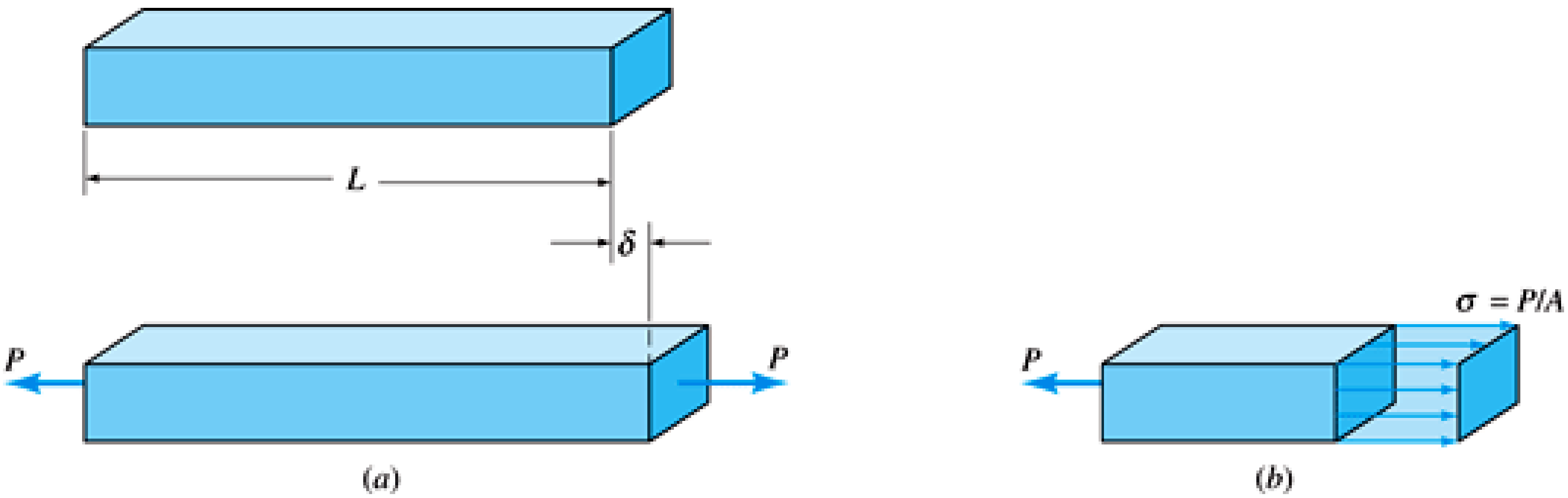
Outline

- Deformation of axially loaded members
- Statically indeterminate problem
- Method of superposition
- Thermal deformation and stress
- Transformation of stress: stresses on Inclined planes
- Saint-Venant's Principle
- Stress Concentrations

Introduction – what we learn…

- Concepts of axial stress and deformation
- Deflections based on linear strain or small deformation assumption
- Example of superposition principle
- Stress developed without external force, but developed by temperature change
- Non-uniform stress distributions due to circular hole and fillet

Deformation in axially loaded members



(a) Elongation of the prismatic bar

(b) Freebody diagram & average (normal) stress

Review – Engineering stress & strain

$\varepsilon = \delta/L$ – *eng. Strain (normal)*

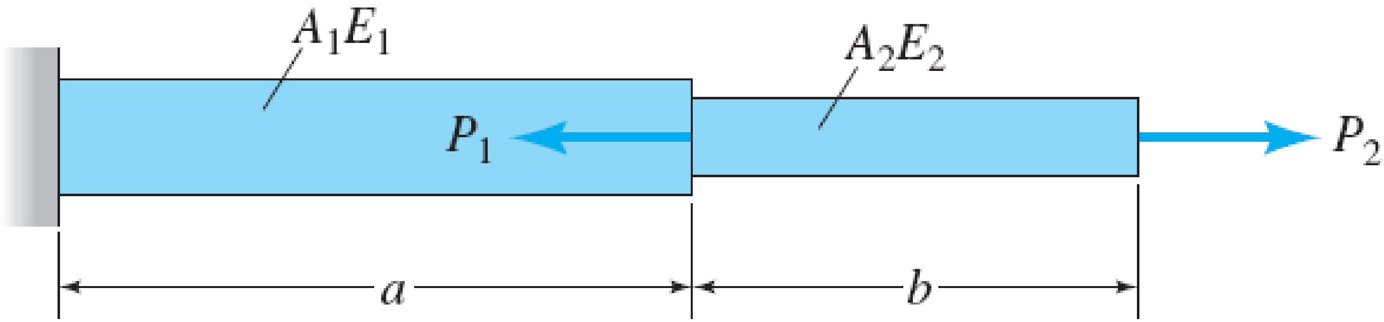
$\sigma = P/A_0$ – *eng. Stress (normal)*

$\sigma = E\varepsilon$ – *Hooke‘s law*

$\boxed{\delta = PL/AE \text{ – Deformation vs. Load \& Geometry}}$

„ AE : axial rigidity“

Stepped bar with multiple loadings



$$\delta = \sum_{i=1}^{n} \frac{P_i L_i}{A_i E_i}$$

Flexibility vs. stiffness

- Stiffness, K

$$k = \frac{AE}{L}$$

- Flexibility, f

  (Compliance)

$$f = L/AE$$

c.f.

$$\delta = \sum_{i=1}^{n} \frac{P_i L_i}{A_i E_i}$$

Non-uniform bars

$$\delta = \int_0^L \frac{P_x\,dx}{A_x E}$$

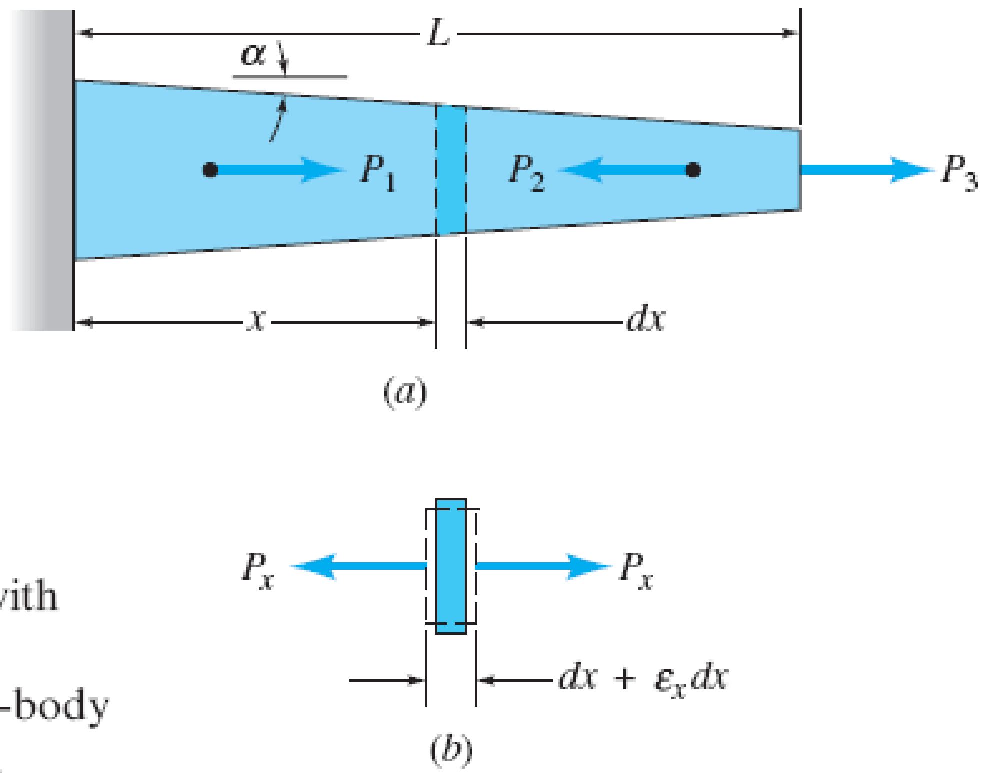


FIGURE 4.3 (*a*) Bar with variable cross section and axial load; (*b*) free-body diagram of an element.

$$P_x = P(x) \qquad A_x = A(x)$$

Example of a stepped bar

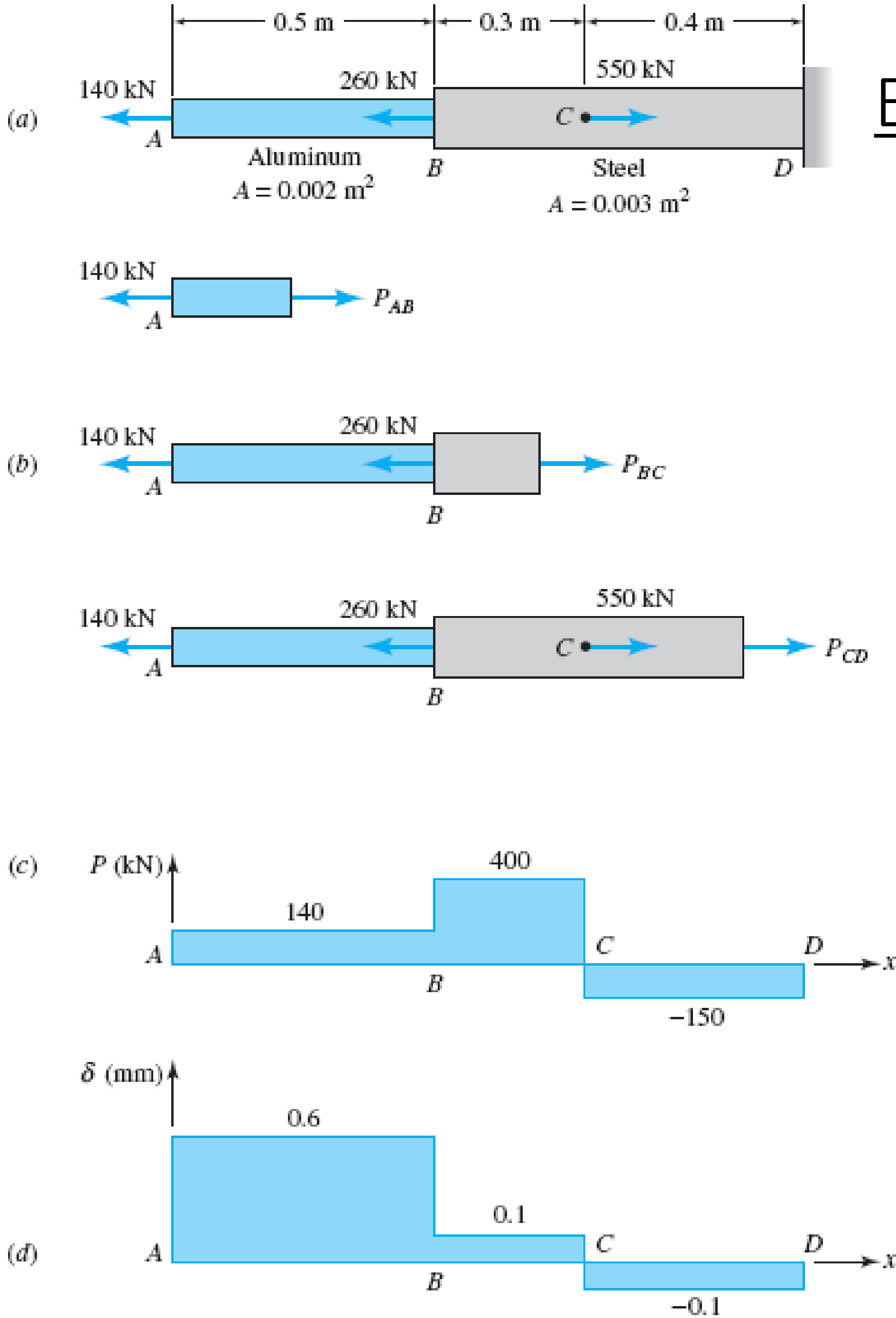


FIGURE 4.4 (*a*) A bar consisting of two segments; (*b*) three typical parts; (*c*) axial force diagram; (*d*) displacement diagram.

A tapered cone

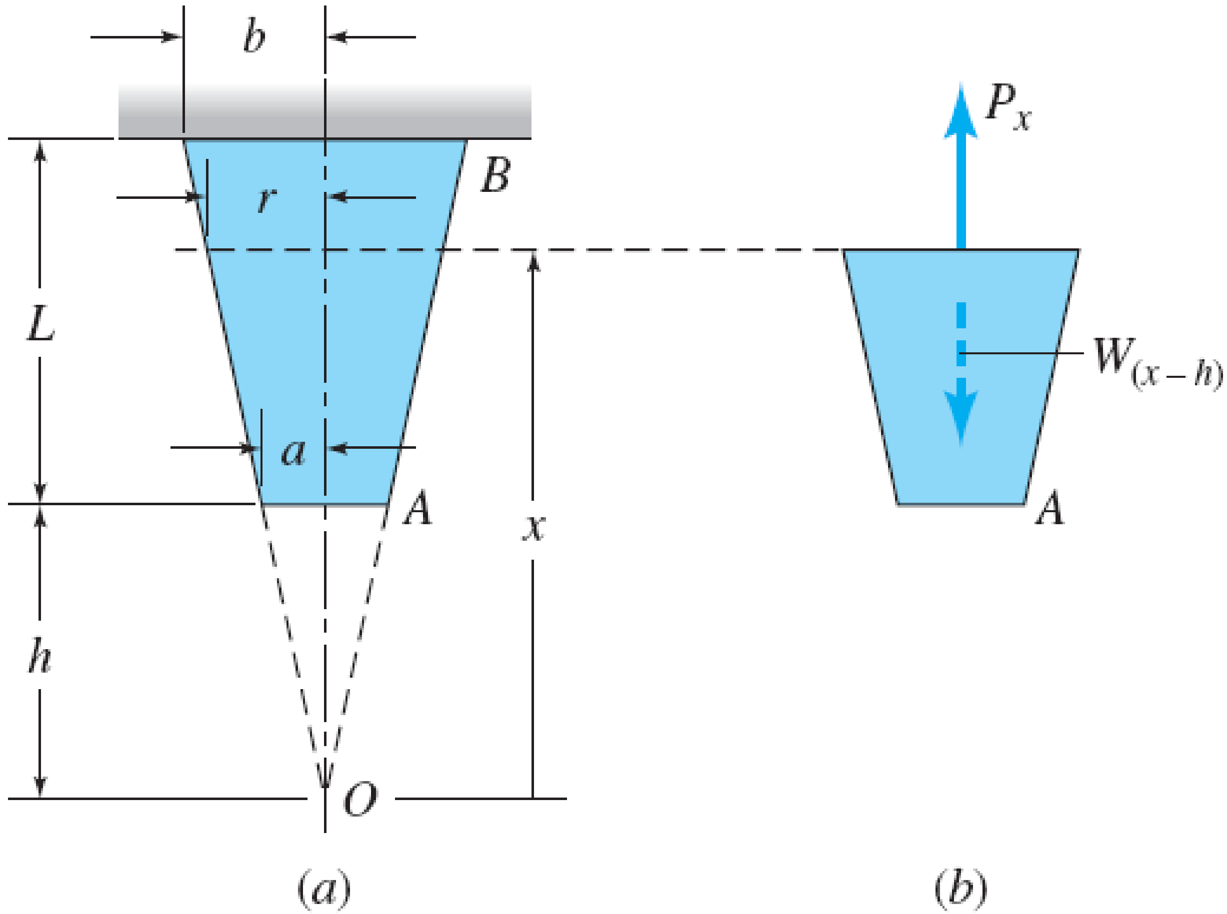


FIGURE 4.6 (*a*) A truncated cone; (*b*) free-body diagram of a part.

Statically determinate vs. indeterminate problems

- Statically determinate: Equations of equilibrium are enough to solve for the unknowns in a loaded structure
- Statically in-determinate: Equations of equilibrium alone are not enough to solve for the unknowns in a loaded structure

Statically indeterminate structures

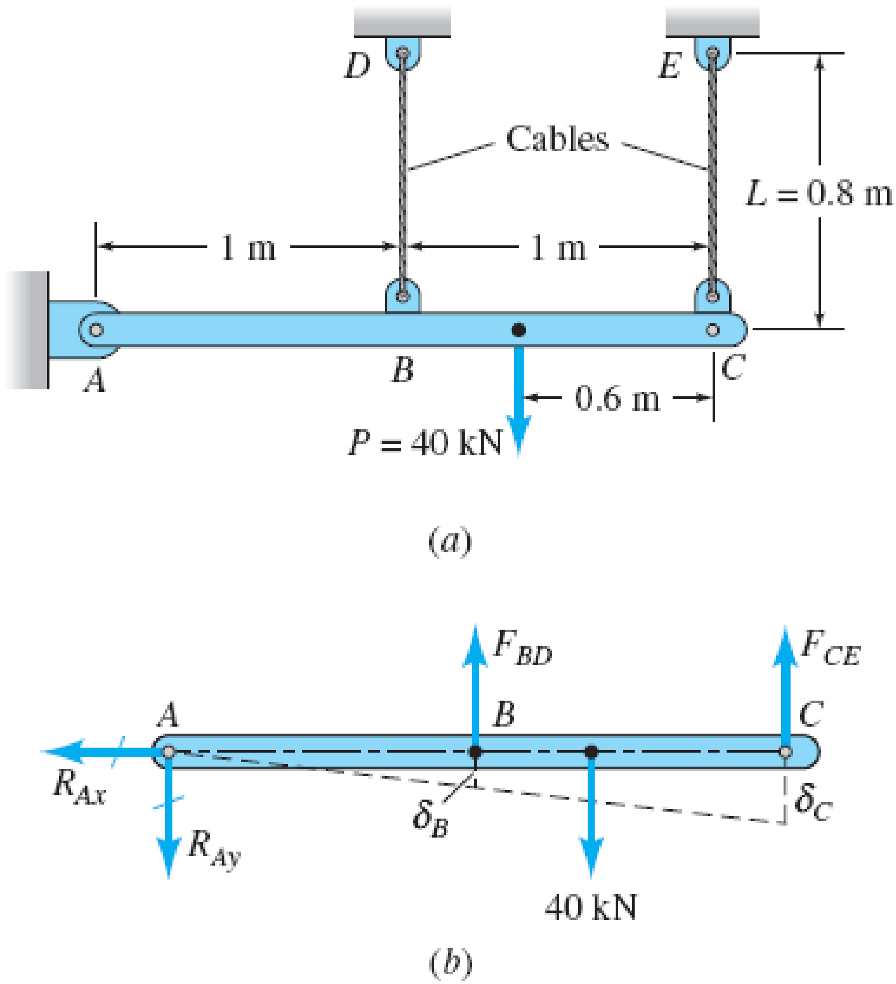


FIGURE 4.8 (*a*) Assembly of a bar and cables; (*b*) free-body and centerline displacement diagrams of bar *AC*.

Stepped bar fixed at both ends

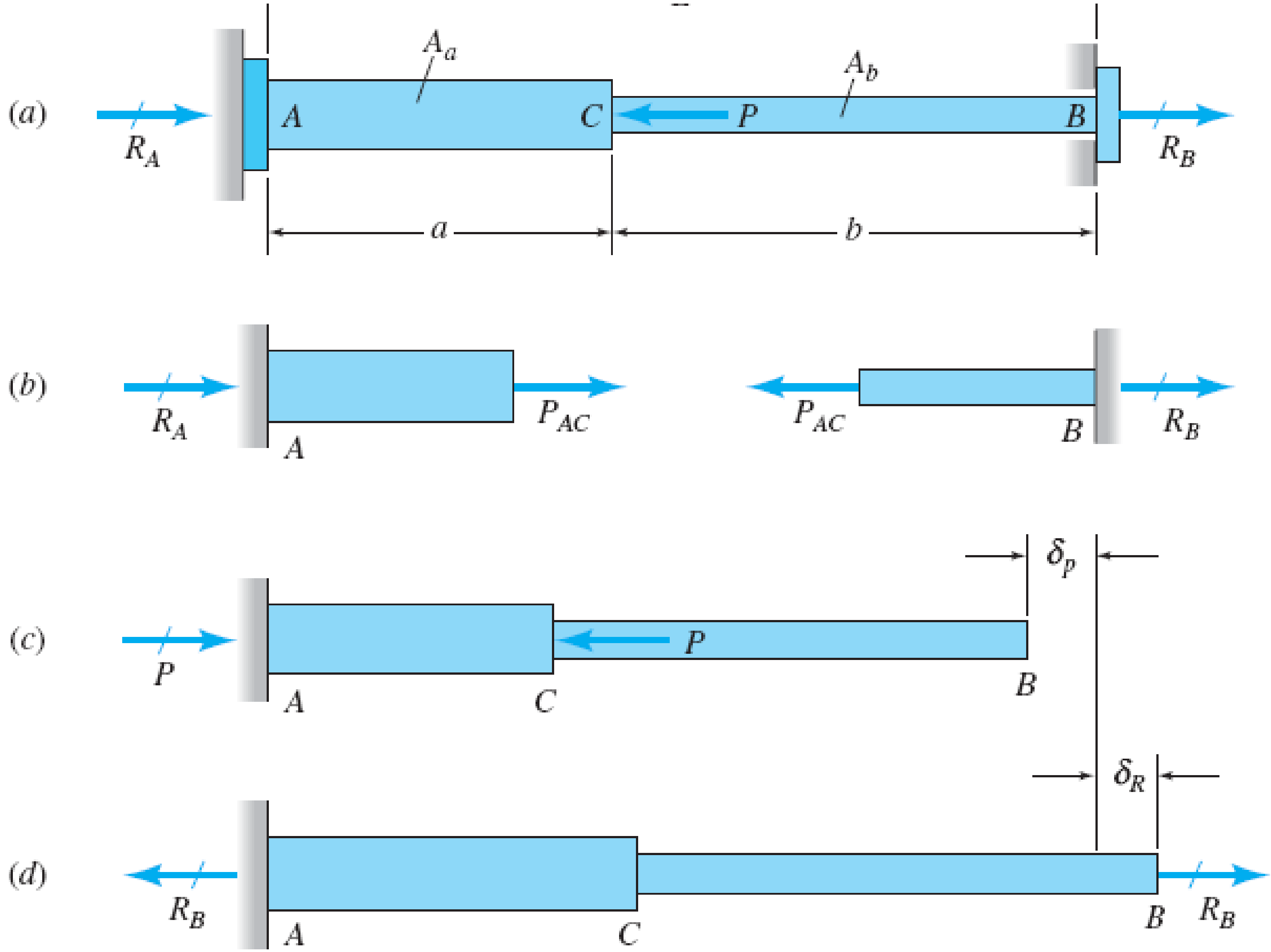


FIGURE 4.10 (*a*) A bar consists of two segments under an axial load P; (*b*) free-body diagrams of the left and right parts; (*c*) contraction due to load P; (*d*) expansion due to reaction R_B.

Solution by a superposition method

$$\delta = \sum_{i=1}^{n} \frac{P_i L_i}{A_i E_i}$$

$$\delta_P + \delta_R = 0$$

Applying Eq. (4.1) and taking the elongations to be positive, we have

$$-\frac{Pa}{A_a E} + \left(\frac{R_B a}{A_a E} + \frac{R_B b}{A_b E}\right) = 0 \tag{a}$$

from which

$$R_B = \frac{P}{1 + (bA_a/aA_b)}$$

This result is the same as that given by Eq. (4.10). The remaining reaction can be obtained from the condition of statics: $R_A + R_B = P$.

The **method of superposition** employed above may be summarized as follows:

1. One of the unknown reactions is designated as *redundant* and released from the member by removing the support.
2. The remaining member, which is rendered statically determinate, is loaded by the actual load (P) and the redundant (R_B) itself. Note that the redundant is considered to be an *unknown load*.
3. The expressions for the displacements due to these loads are obtained and substituted into the equation of geometric compatibility to calculate the redundant reaction. The other unknown reaction is found by applying statics.

1. One of the unknown reactions is designated as *redundant* and released from the member by removing the support.
2. The remaining member, which is rendered statically determinate, is loaded by the actual load (P) and the redundant (R_B) itself. Note that the redundant is considered to be an *unknown load*.
3. The expressions for the displacements due to these loads are obtained and substituted into the equation of geometric compatibility to calculate the redundant reaction. The other unknown reaction is found by applying statics.

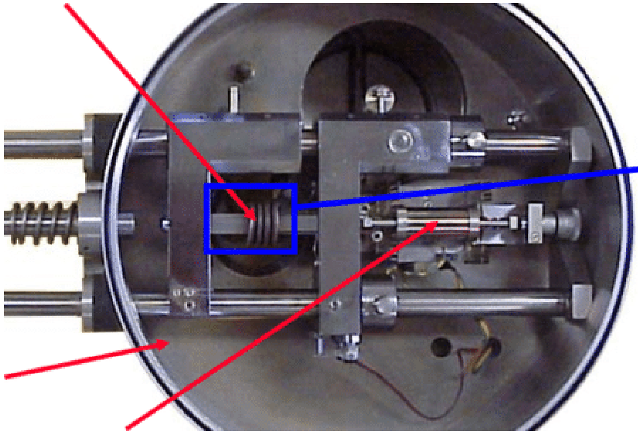
Thermal deformation & stress

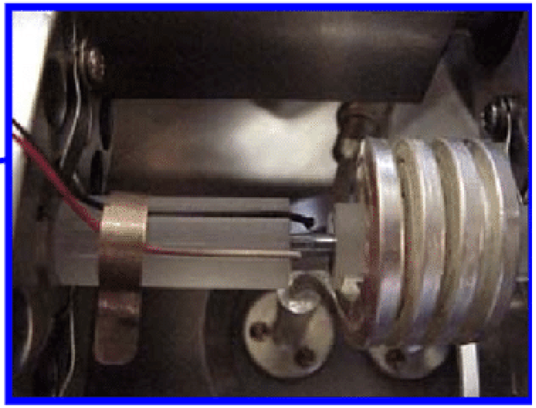
$$\boxed{\delta_t = \alpha(\Delta T)L}$$

$$\boxed{\varepsilon_t = \alpha\,\Delta T}$$

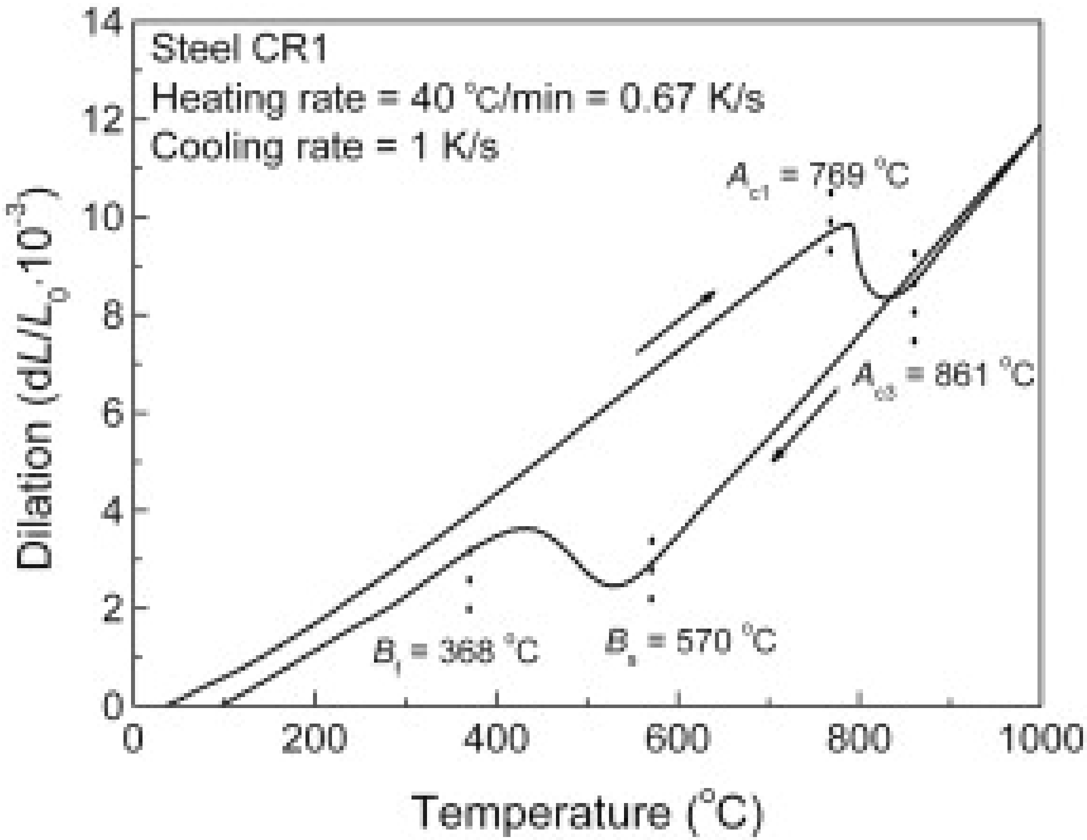
α: Thermal expansion coefficient

Dilatometry





Ref. Phase transformations and microstructure-mechanical properties relations in complex phase high strength steels



Example

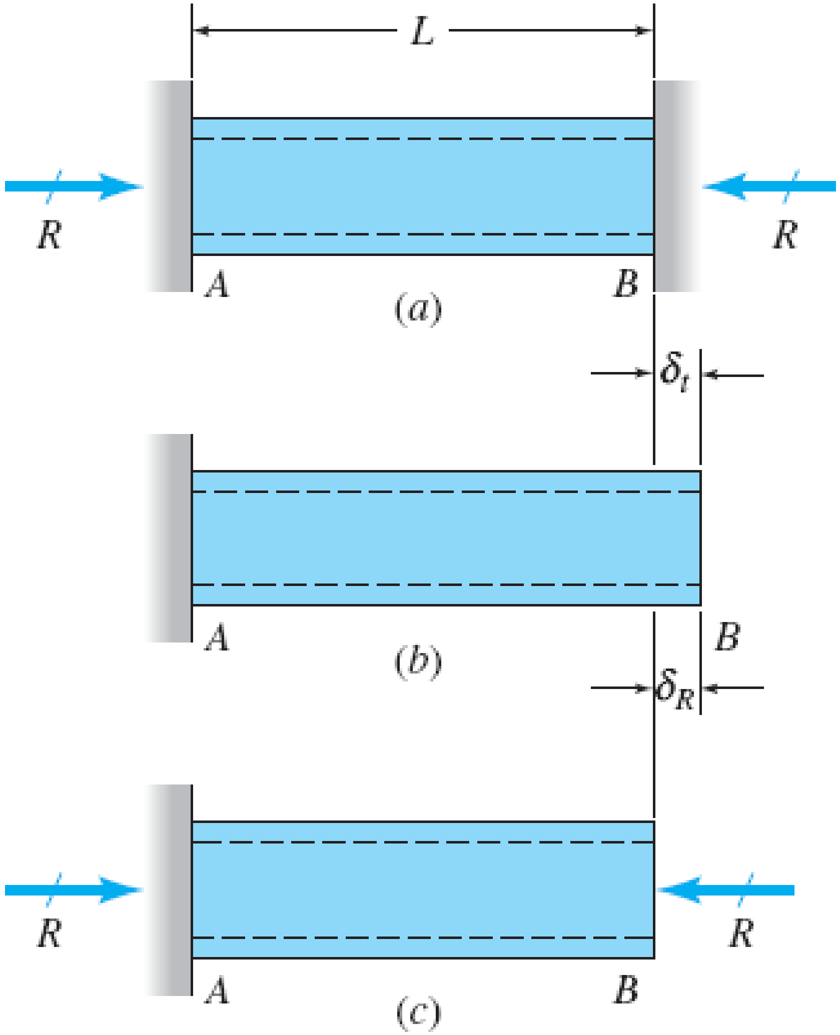
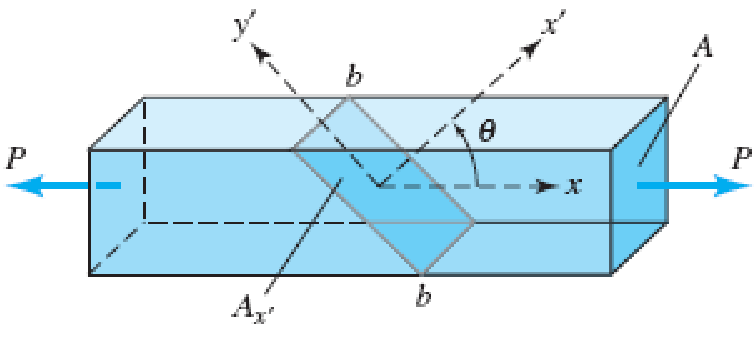


FIGURE 4.11 (*a*) Tube with restrained ends; (*b*) thermal expansion; (*c*) contraction due to reaction *R*.

Transformation of stress tensor:

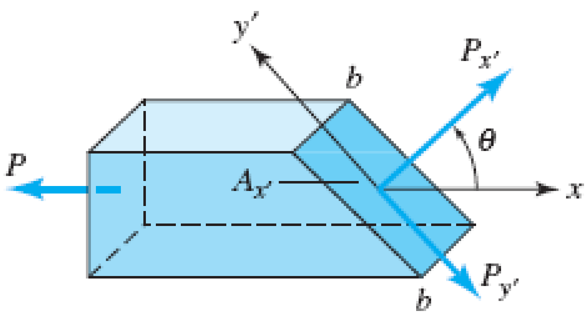
Stress components with different coordinate system



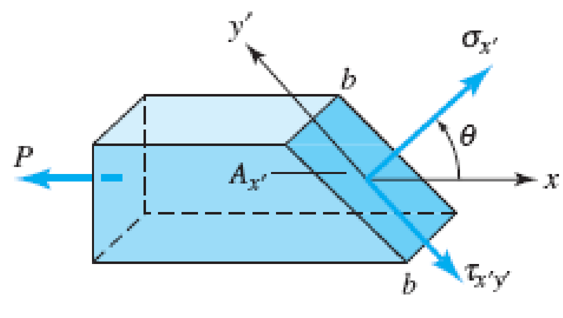
(a)

$$\boxed{\sigma_{x'} = \frac{P_{x'}}{A_{x'}} = \sigma_x \cos^2\theta}$$

$$\boxed{\tau_{x'y'} = -\frac{P_{y'}}{A_{x'}} = -\sigma_x \sin\theta\cos\theta}$$



(b)



(c)

FIGURE 4.13 Prismatic bar in tension with forces and stresses on inclined planes.

Maximum shear plane

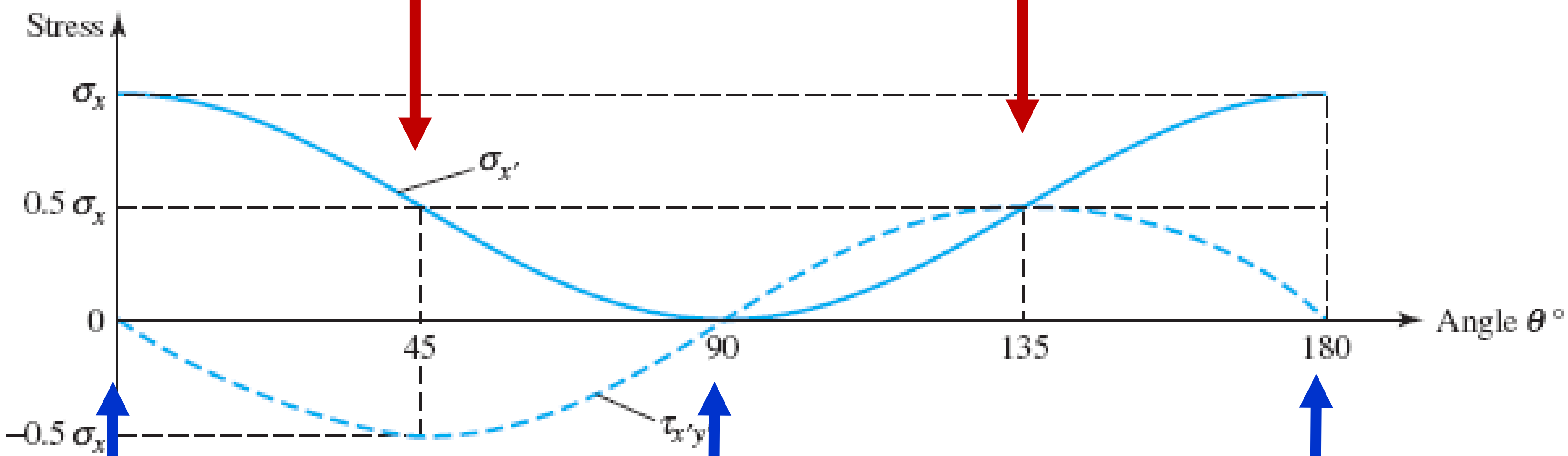


FIGURE 4.14 Variation of stress with the inclined section in the bar shown in Fig. 4.13a.

No shear stress : principal stress plane

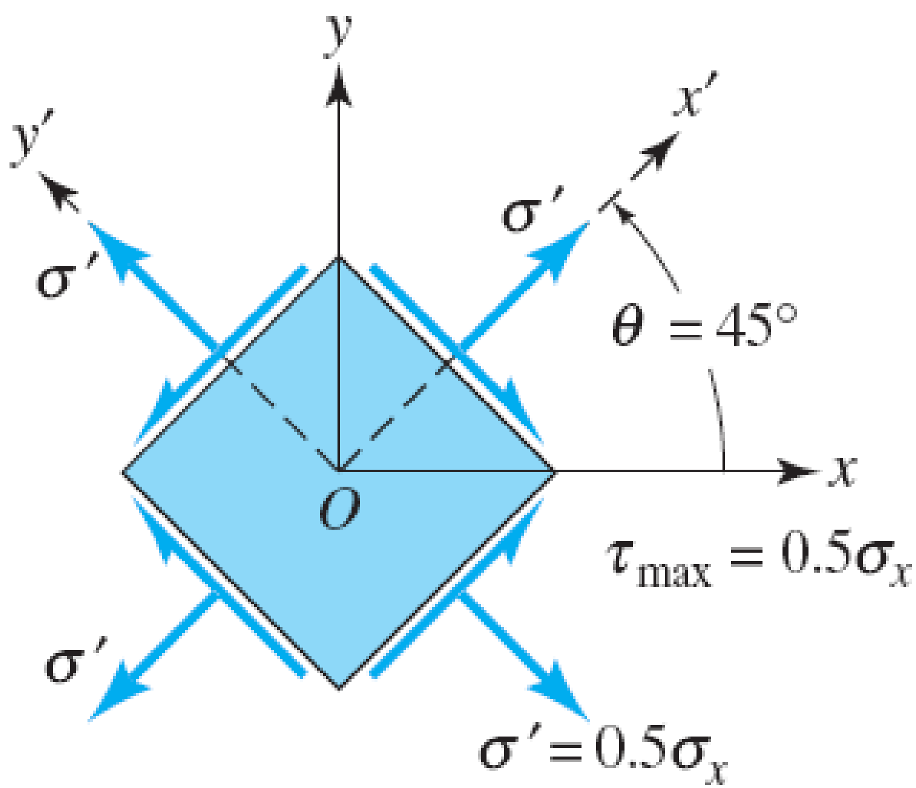


FIGURE 4.15 Planes of maximum shear stress for a bar shown in Fig. 4.13*a*.

Saint-Venant's principle

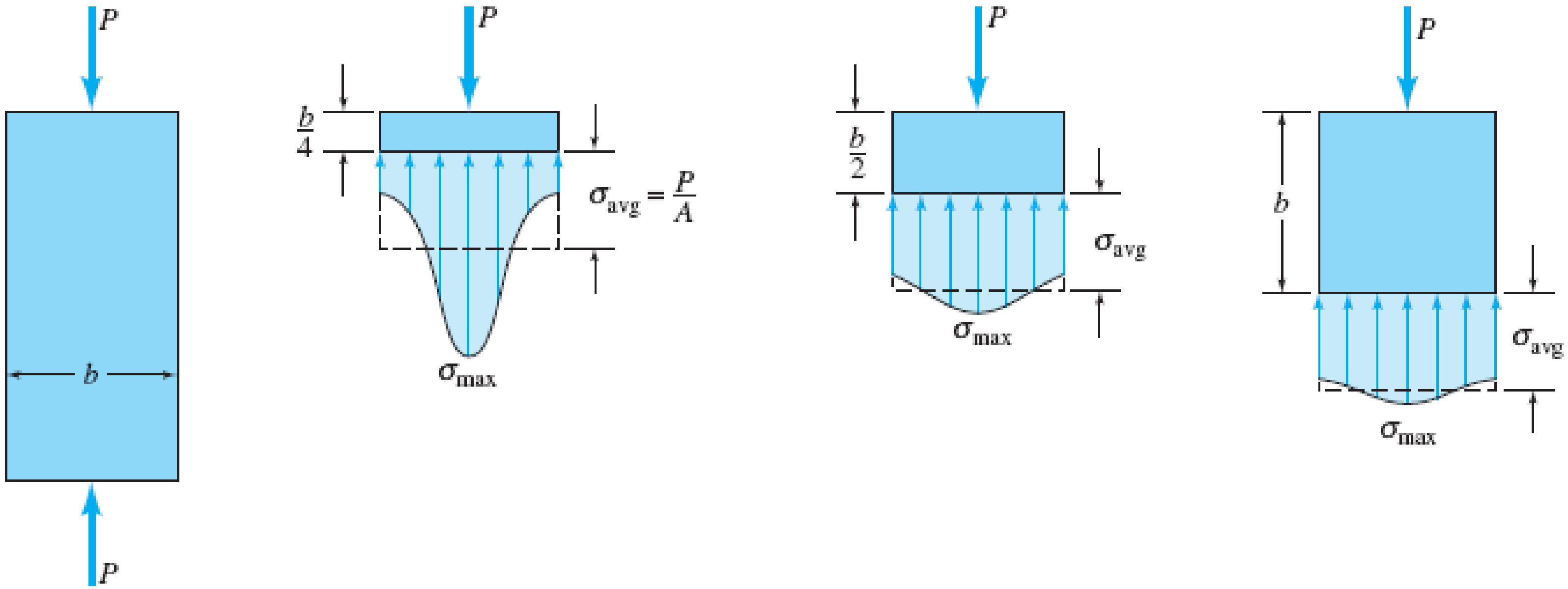


FIGURE 4.17 Stress distribution due to a concentrated load in a rectangular elastic bar confirming Saint-Venant's principle.

Stress concentration

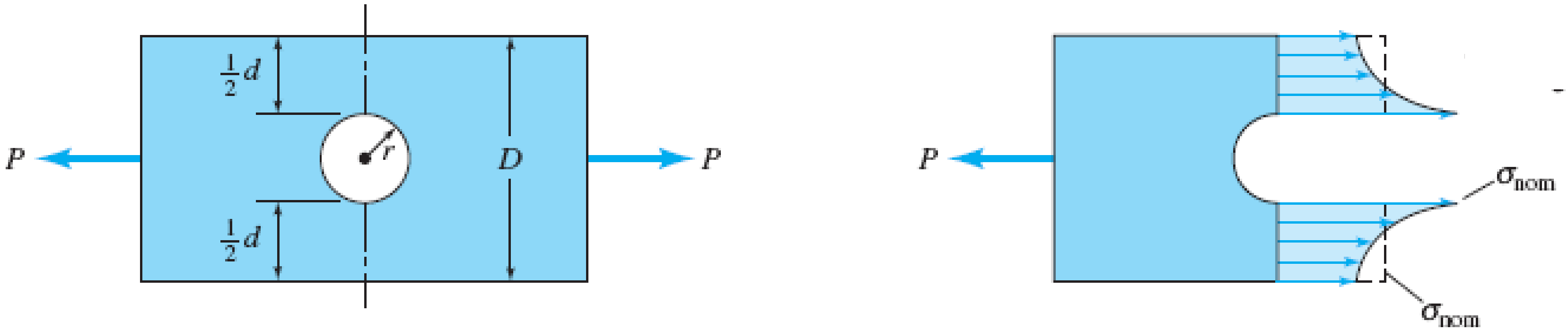


FIGURE 4.18 Stress distribution in an axially loaded flat bar with a circular hole.

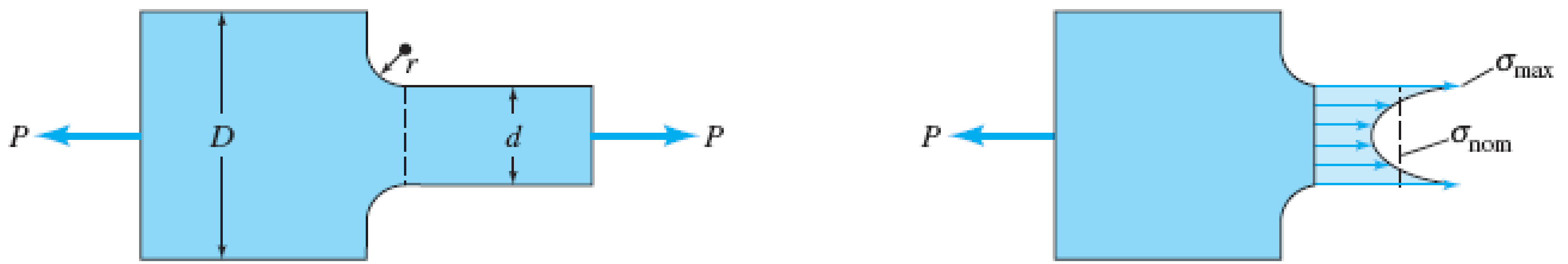


FIGURE 4.19 Stress distribution in an axially loaded flat bar with fillets.

$$\boxed{\sigma_{max} = K\sigma_{nom} = K\frac{P}{A}}$$

Stress concentration

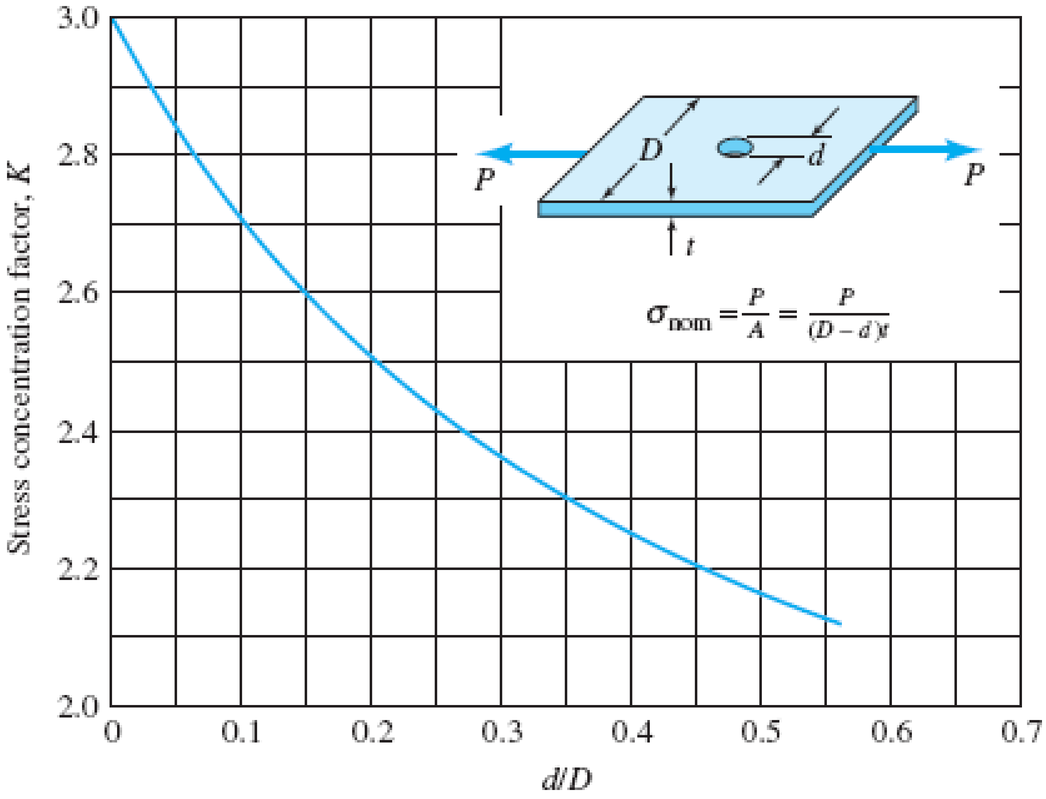


FIGURE 4.20 Stress concentration factor K for a flat bar with transverse circular hole in axial tension (Refs. 4.4 and 4.5).

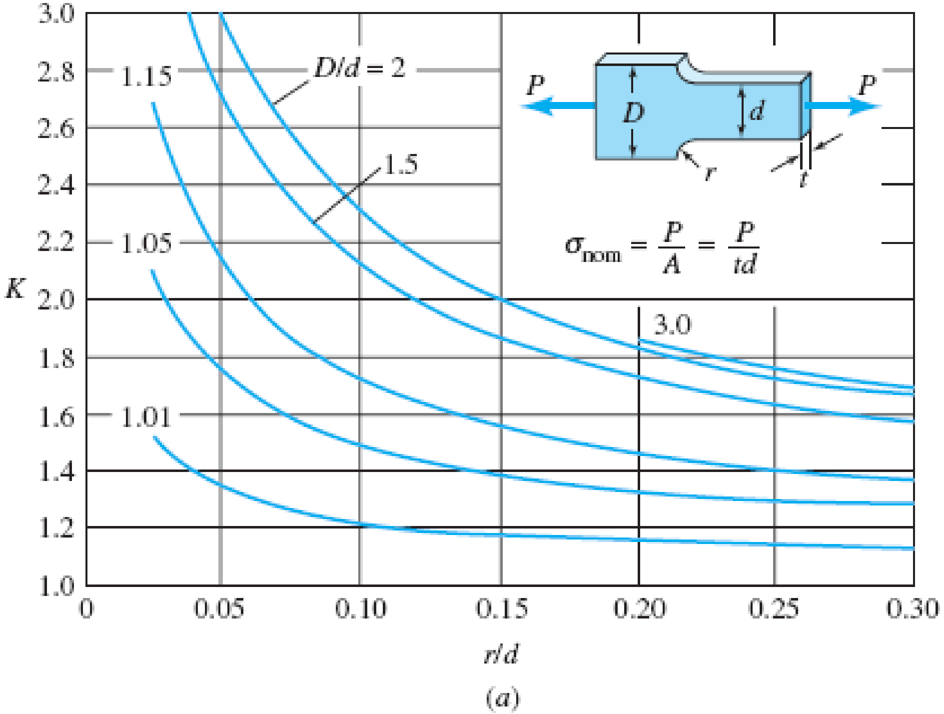
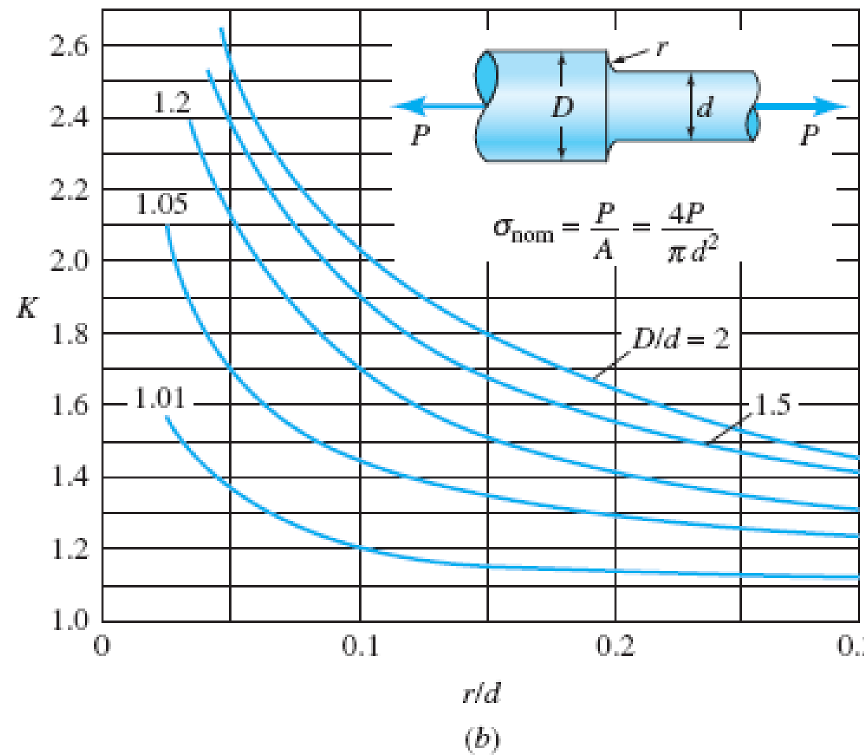


FIGURE 4.21 Stress concentration factors K for bars in axial tension (Refs. 4.4 and 4.6): (*a*) flat bars with fillets; (*b*) round bars with shoulder fillets.



KEY CHAPTER EQUATIONS*

Deformation of prismatic bar	$\delta = PL/AE$
Multiple prismatic bar	$\delta = \sum_{i=1}^{n} P_i L_i / A_i E_i$
Nonprismatic bar	$\delta = \int_o^L P_x \, dx / A_x E$
Thermal strain	$\varepsilon_t = \alpha \, \Delta T$
Thermal deformation	$\delta_t = \alpha (\Delta T) L$
Normal and shearing stresses	$\sigma_{x'} = \sigma_x \cos^2 \theta$ $\tau_{x'y'} = \sigma_x \sin \theta \cos \theta$
Maximum normal stress	$\sigma_{\max} = K \sigma_{\text{nom}} = K \dfrac{P}{A}$