4. Stress-strain relationships and material properties

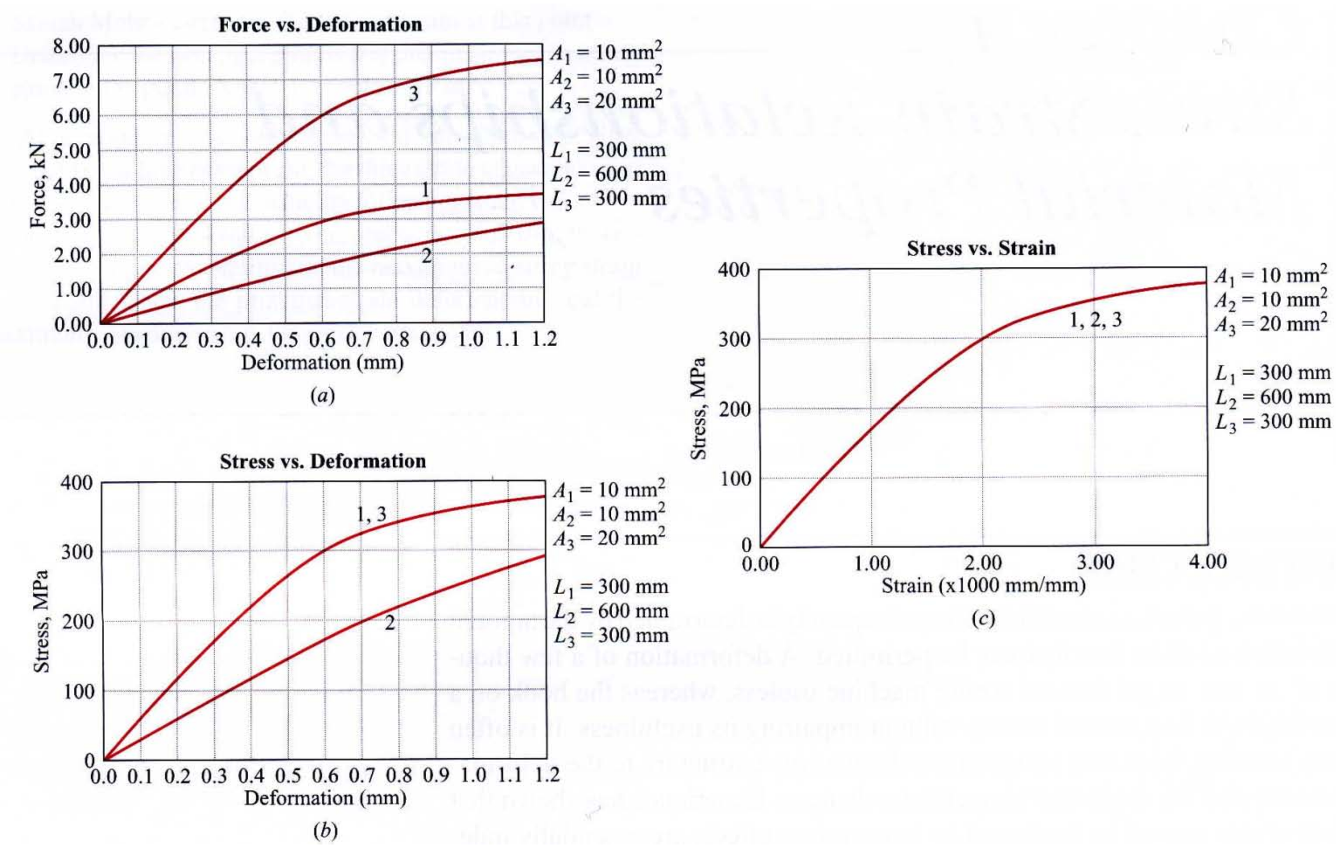
4.1 Introduction

- Deformations

  - due to load and temperature are independent of each other.

4.2 Stress-strain diagrams

- Stress-strain relationship is independent of the size and shape of the member and depends on the type of material.



4.2 Stress-strain diagrams

- True stress: Stress obtained by dividing the load by the actual area

$$\sigma_{true} = \frac{F}{A(F)}$$

- True strain: The sum of all the instantaneous engineering strains

$$\varepsilon_{true} = \int_{l_0}^{l_f} \frac{dl}{l} = \ln \frac{l_f}{l_0} = \ln \frac{l_0 + \Delta l}{l_0} = \ln\left(1+\varepsilon\right)$$

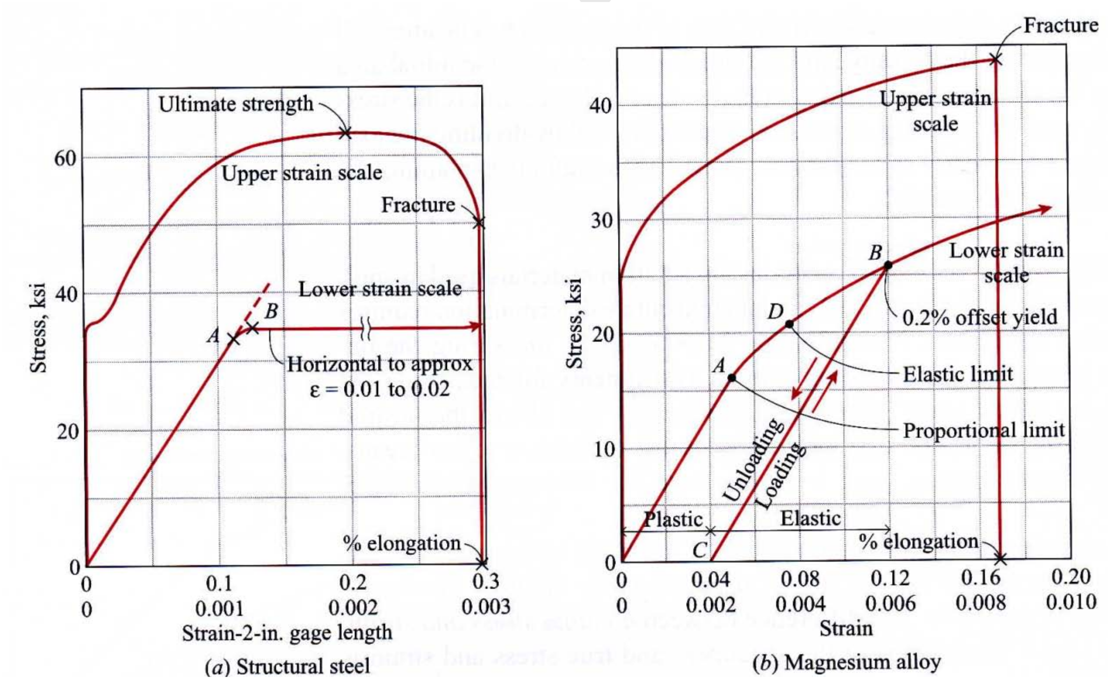
- Hooke’s law and modulus of elasticity (Young’s modulus)

$$\sigma = E\varepsilon$$

- Shear modulus (modulus of rigidity)

$$\tau = G\gamma$$

4.2 Stress-strain diagrams



(*a*) Structural steel (*b*) Magnesium alloy

- Elastic limit (D): the max. stress for which the material acts elastically

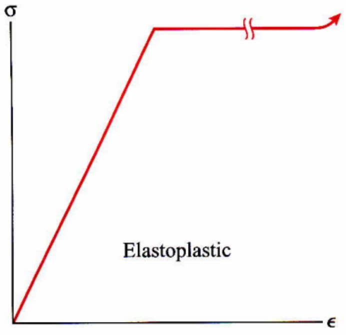
- Yield point: the stress at which there is an appreciable increase in strain with no increase in stress

- Yield strength (B) : the stress that will induce a specified permanent strain (0.2%)

- Ultimate strength: the max. stress developed in a material before rupture

4.2 Stress-strain diagrams

- Elastoplastic materials ⟶



- Ductility: the capacity for plastic deformation in tension or shear

- Creep limit: the max. stress for which the plastic strain will not exceed a specified amount during a specified time interval at a specified temperature

- Poisson's ratio: the ratio of the lateral or perpendicular strain to the longitudinal or axial strain

$$\nu = -\frac{\varepsilon_l}{\varepsilon_a}$$

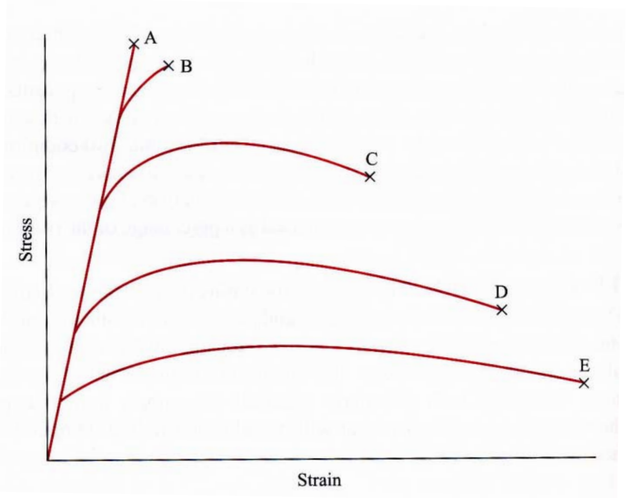
$$G = \frac{E}{2(1+\nu)}$$

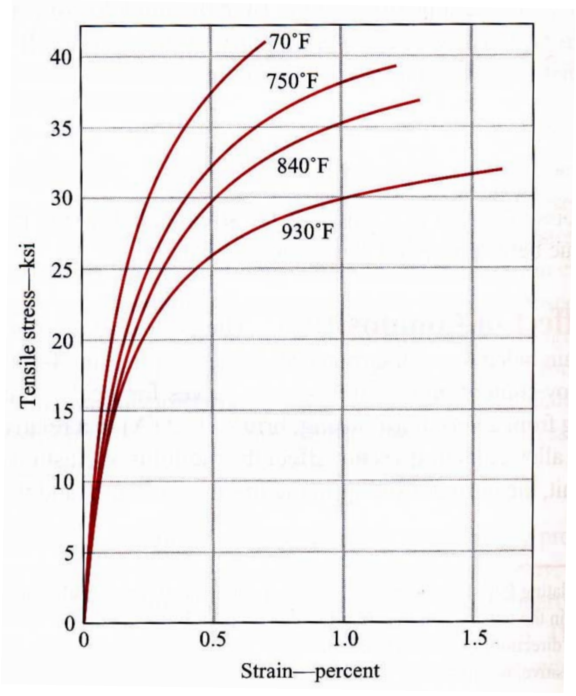
4.2 Stress-strain diagrams

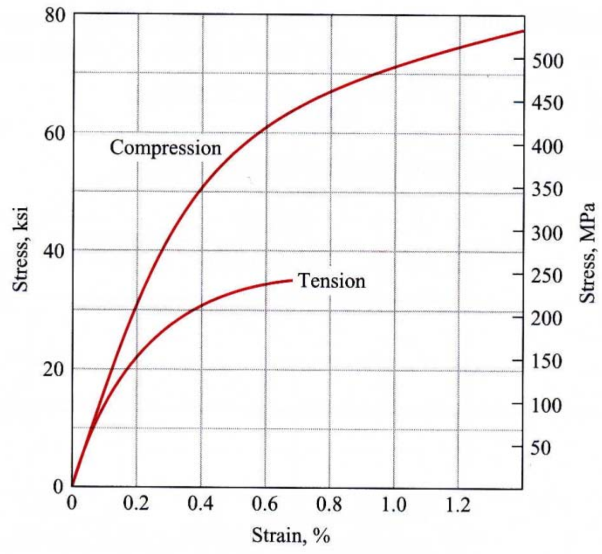
- Effect of composition: brittle to ductile

- Effect of temperature: ductility, ultimate strength…

- Effect of tension or compression: for brittle materials







4.2 Stress-strain diagrams

- Example Problem 4-1

A 100 kip axial load on a $1 \times 4 \times 90$-in., 4-in. side becomes 3.9986 in., length increases 0.09 in.

- Poisson's ratio

- Modulus of elasticity

- Modulus of rigidity

4.3 Generalized Hooke's law

- The deformations of an element for a combined loading can be determined by using the principle of superposition.

- The principle of superposition can be applied when
  1) Each effect (strain) is linearly related to the load.
  2) The effect of the first load does not significantly change the effect of the second load.

- Case of plane stress:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right) \longrightarrow \begin{aligned} \sigma_x &= \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) \\ \sigma_y &= \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) \end{aligned}$$

$$\varepsilon_z = -\frac{\nu}{E}\left(\sigma_x + \sigma_y\right)$$

4.3 Generalized Hooke's law

- Case of triaxial principal stresses:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right) \qquad \sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu\left(\varepsilon_y + \varepsilon_z\right)\right]$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right) \quad \longrightarrow \quad \sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_y + \nu\left(\varepsilon_z + \varepsilon_x\right)\right]$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right) \qquad \sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_z + \nu\left(\varepsilon_x + \varepsilon_y\right)\right]$$

- Out-of-plane principal strain:

$$\varepsilon_z = -\frac{\nu}{E}\left(\sigma_x + \sigma_y\right) = -\frac{\nu}{E}\left(\frac{E}{1-\nu^2}\right)\left(\varepsilon_x + \nu\varepsilon_y + \varepsilon_y + \nu\varepsilon_x\right)$$

$$= -\frac{\nu}{1-\nu}\left(\varepsilon_x + \varepsilon_y\right)$$

$$\left(\because \sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right), \quad \sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right)\right)$$

4.3 Generalized Hooke's law

- Expression of G in terms of E and ν: refer to p167~168.

$$G = \frac{E}{2\left(1+\nu\right)}$$

- Example Problem 4-2

An alloy steel (E=210 Gpa, ν=0.3) under a biaxial state of stress shows strains: ε_x=1394 μ, ε_y=-660 μ, γ_{xy}=2054 μ.

- σ_x , σ_y , τ_{xy}

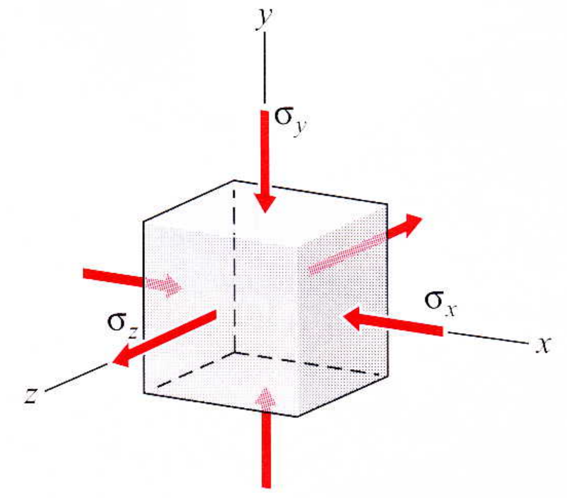
- Principal stresses and the max. shear stress with a sketch of a triangular element

4.3 Generalized Hooke's law

- Example Problem 4-4

There is a steel block (E=30,000 ksi, ν=0.3) of 10 × 10 × 10-in. under a uniformly distributed pressure of 30,000 psi in *x*- and *y*- directions. Deformation in the *z*-direction is 0.002 in.

- Determine σ_z.

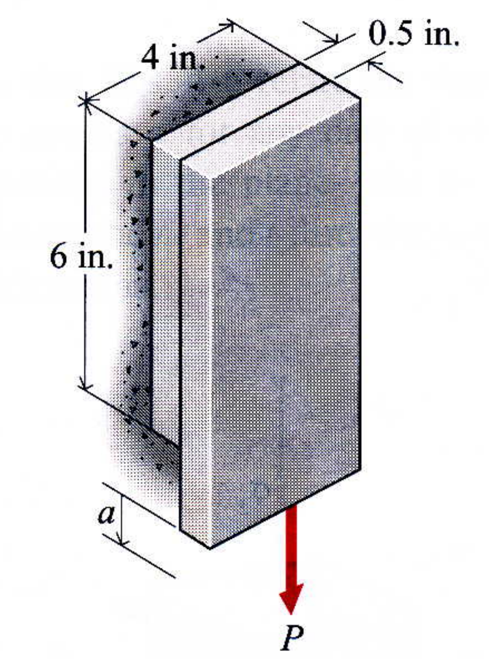


4.3 Generalized Hooke's law

- Example Problem 4-5

There is a rubber block of 0.5 × 6 × 4-in. attached to a wall and a steel plate. *P* is 30 *lb* and the rigid steel plate displaces downward a = 0.0003 in.

- Determine the shear modulus of the rubber.



4.4 Thermal strain

- The coefficient of thermal expansion, α, is approximately constant for a large range of temperatures.

$$\varepsilon_T = \alpha \Delta T$$

- The total strain is the sum of the strains by loads and temperature change.

$$\begin{aligned}\varepsilon_{total} &= \varepsilon_\sigma + \varepsilon_T \\ &= \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right) + \alpha \Delta T\end{aligned}$$

4.4 Thermal strain

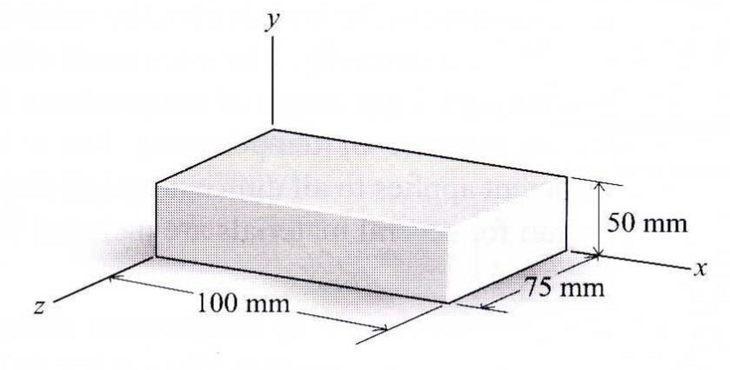
- Example Problem 4-6

An aluminum block (E=70 Gpa, α=22.5 μ/˚C) is subjected to a temperature change ΔT=20˚C

- Thermal strains: ε_{Tx}, ε_{Ty}, ε_{Tz}

- Deformations: δ_x, δ_y, δ_z

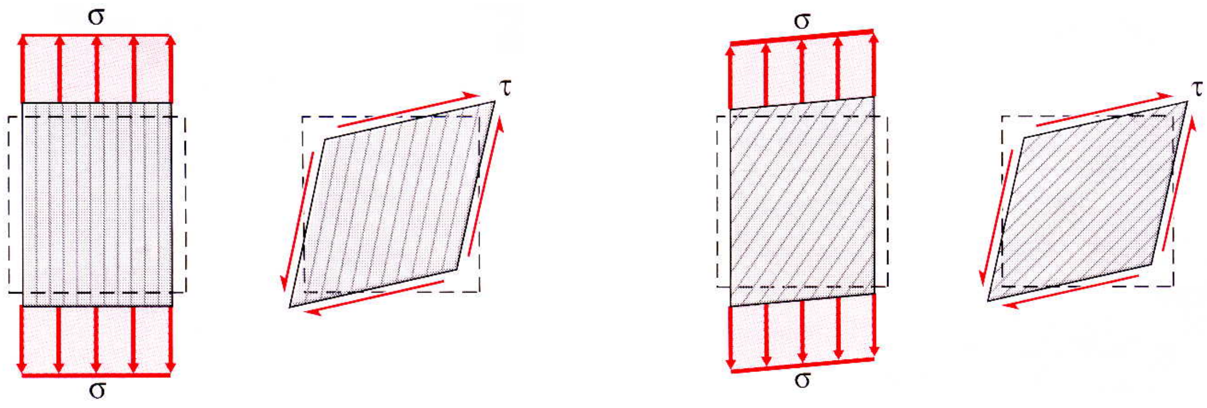
- Shearing strain: γ_{xy}



4.5 Stress-strain equations for orthotropic materials

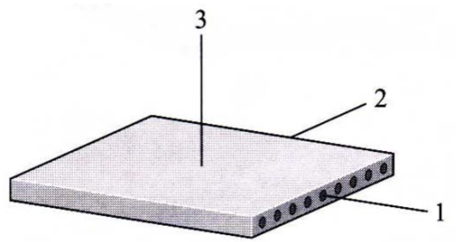
- Orthotropic materials have three mutually perpendicular planes of material symmetry.

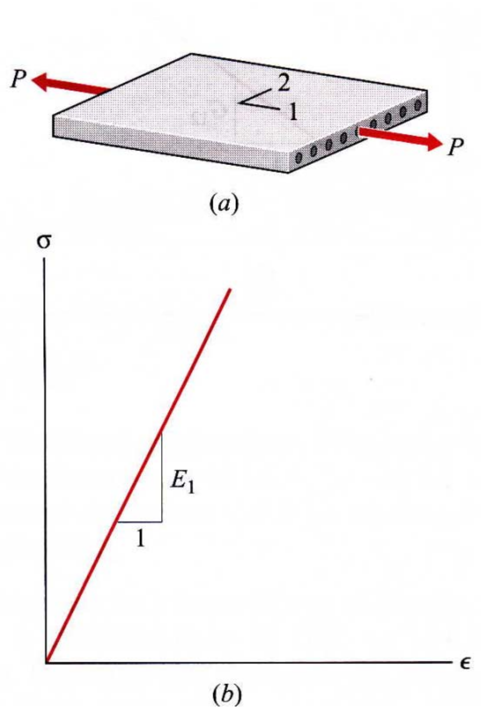
- Normal stress that is not in the natural axis direction of the orthotropic material produces not only normal strains but also shear strain

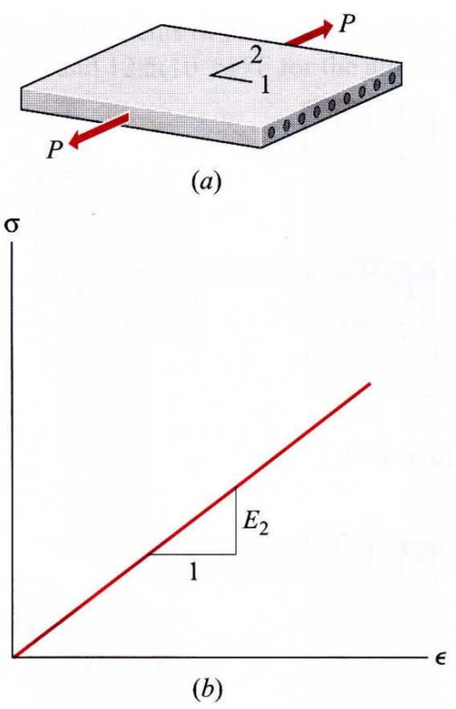


4.5 Stress-strain equations for orthotropic materials

- Orthotropic materials subjected to plane stress







$$\sigma_1 = \frac{P}{A} \quad \varepsilon_1 = \frac{\sigma_1}{E_1} \quad \nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}$$

$$\sigma_2 = \frac{P}{A} \quad \varepsilon_2^{'} = \frac{\sigma_2}{E_2} \quad \nu_{21} = -\frac{\varepsilon_1^{'}}{\varepsilon_2^{'}}$$

4.5 Stress-strain equations for orthotropic materials

$$\gamma_{12} = \frac{\tau_{12}}{G_{12}}$$

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \nu_{21}\frac{\sigma_2}{E_2}$$

$$\varepsilon_2 = \frac{\sigma_2}{E_2} - \nu_{12}\frac{\sigma_1}{E_1}$$

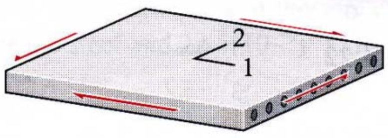
$$\longrightarrow$$

$$\tau_{12} = G_{12}\gamma_{12}$$

$$\sigma_1 = \frac{E_1}{1-\nu_{12}\nu_{21}}\left(\varepsilon_1 + \nu_{21}\varepsilon_2\right)$$

$$\sigma_2 = \frac{E_2}{1-\nu_{12}\nu_{21}}\left(\varepsilon_2 + \nu_{12}\varepsilon_1\right)$$

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}$$



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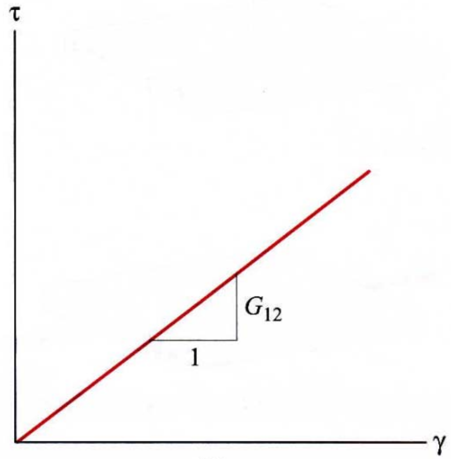

(b)

4.5 Stress-strain equations for orthotropic materials

- Example Problem 4-8

A unidirectional T300/5208 graphite/epoxy composite material is loaded in principal material directions: σ_1 = 50 ksi, σ_2 = 6 ksi, τ_{12} = 2 ksi.

- Normal and shear strains in the principal material directions

Table 4-1 Material Properties for Two Unidirectional Composites

Type	Material	E_1 GPa (ksi)	E_2 GPa (ksi)	ν_{12}	G_{12} GPa (ksi)
T300/5208	Graphite/ Epoxy	181 (26,300)	10.3 (1494)	0.28	7.17 (1040)
Scotchply 1002	Glass/ Epoxy	38.6 (5600)	8.27 (1199)	0.26	4.14 (600)