



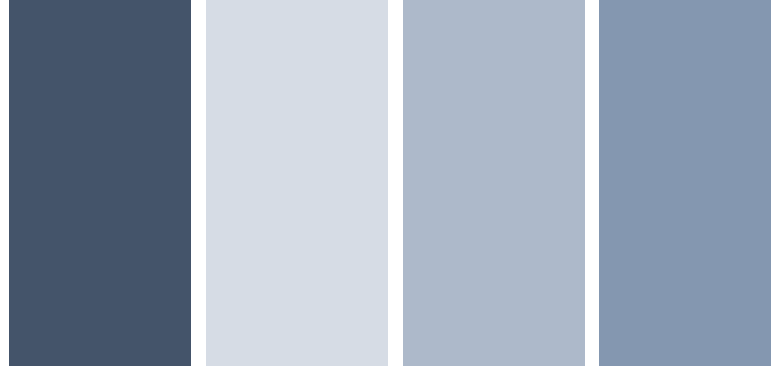
# Mechanics and Design

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## Chapter 4. Stability of Equilibrium

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# Elastic Stability

## Concept of stability

When slightly disturbed from an equilibrium configuration, does a system tend to return to its equilibrium position or does it tend to depart even further?

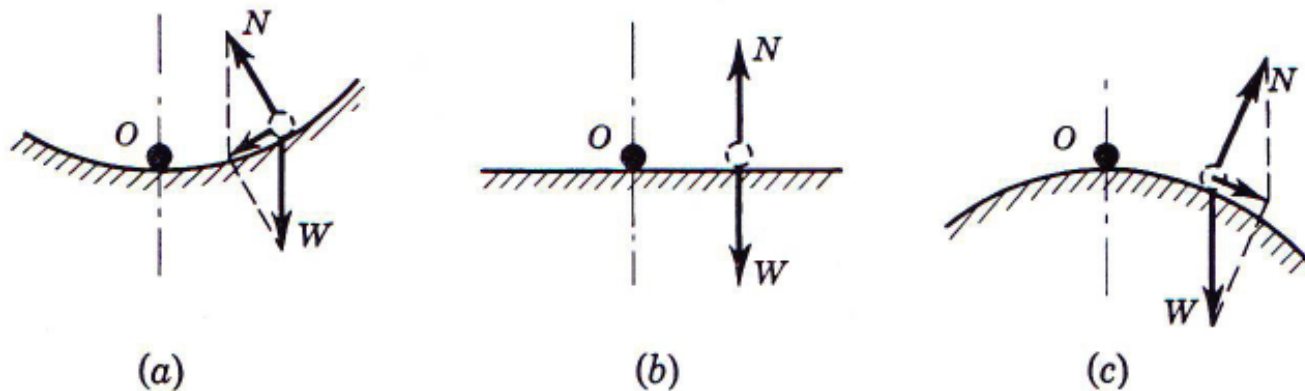
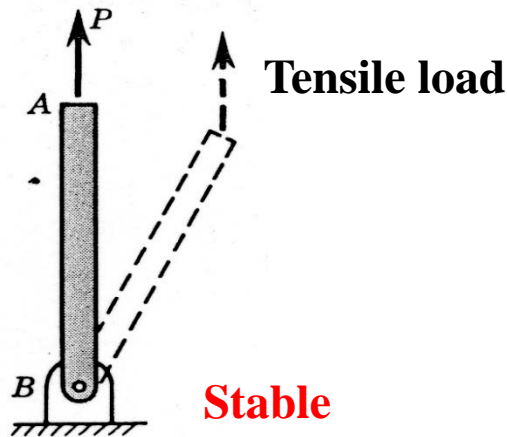


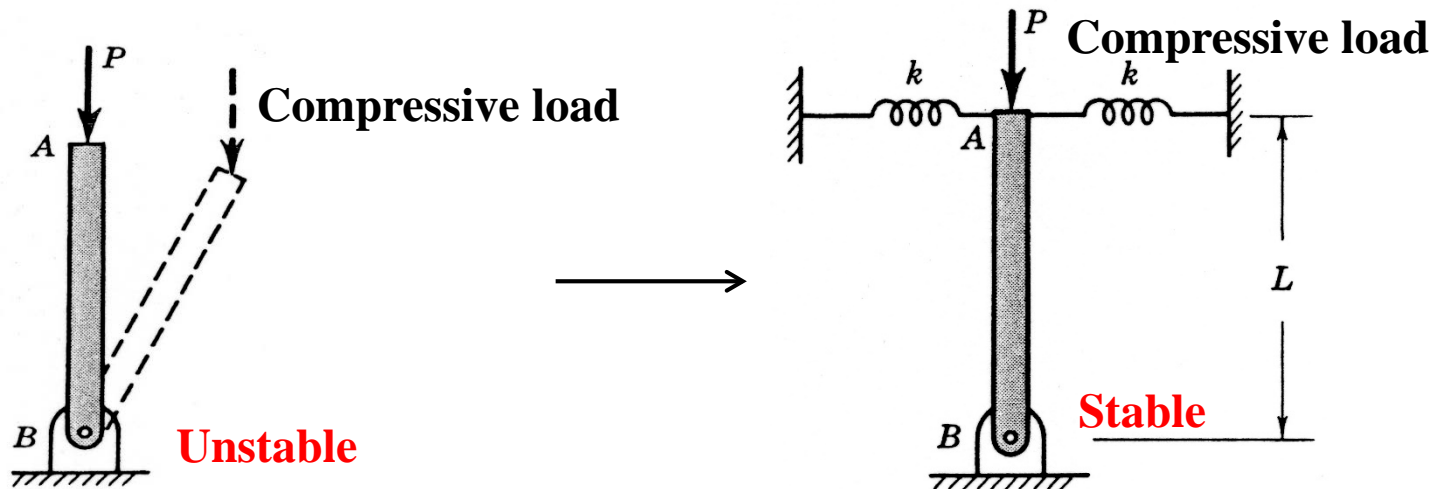
Fig. 4.1 Example of (a) stable, (b) neutral, and (c) unstable equilibrium.

# Elastic Stability

## Hinged bar with spring



The unstable structure can be stabilized by adding guy wires or transverse springs



# Elastic Stability

## Hinged bar with spring (Continued)

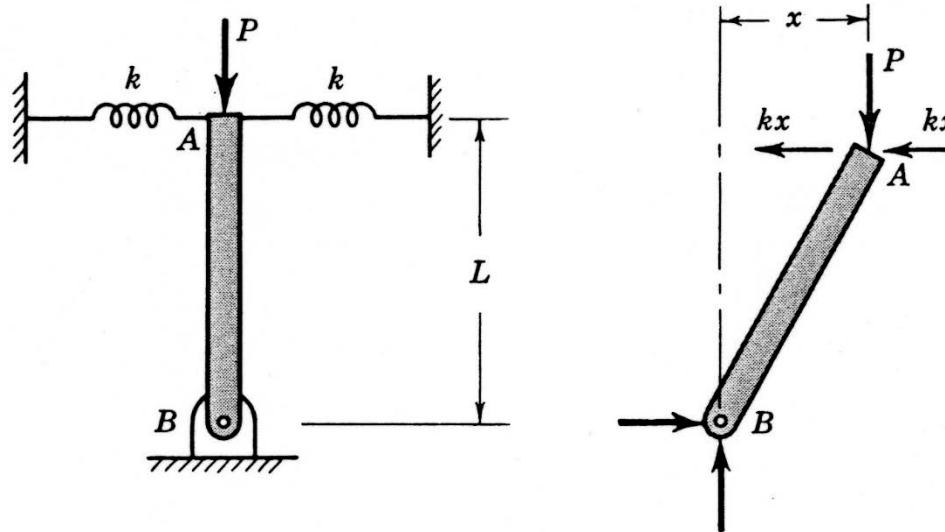


Fig. 4.4 Analysis of hinged bar in compression stabilized by springs

$$Px > 2kxL \quad (\text{unstable})$$

$$Px < 2kxL \quad (\text{stable}) \quad (4-1)$$

$$**Px = 2kL \quad (\text{critical load or buckling load})**$$

# Elastic Stability

## Hinged bar with spring (Continued)

Using force equilibrium, it can be drawn transverse displacement  $x$  due to load eccentricity

$$P(x + \epsilon) = 2kxL$$

$$x = \epsilon \frac{P}{2kL - P} \quad (4-2)$$

If  $P$  is not too close to the critical load the equilibrium displacement ( $x$ ) is small.

Else if  $P$  is larger than the critical load, the equilibrium displacement is infeasible. ( $<0$ )

It means the system is unstable.

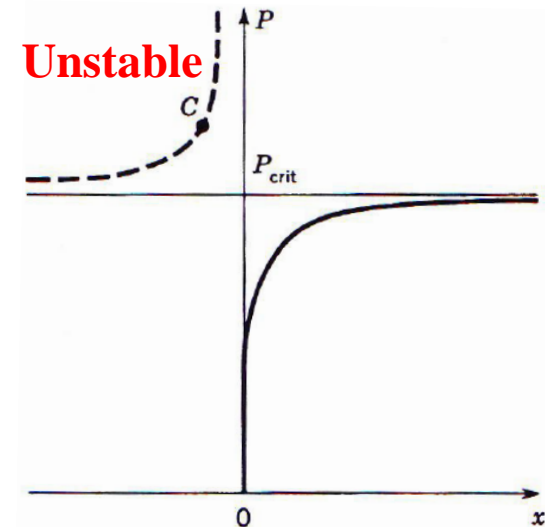
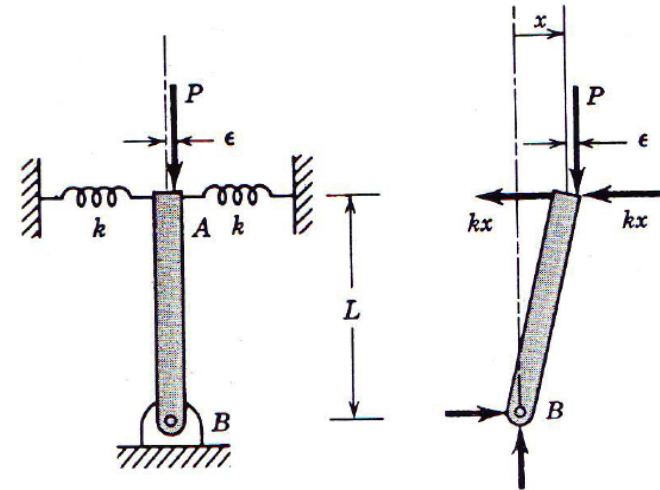


Fig 4.5. Transverse displacement  $x$  due to load eccentricity

# Elastic Stability

## Example of instability

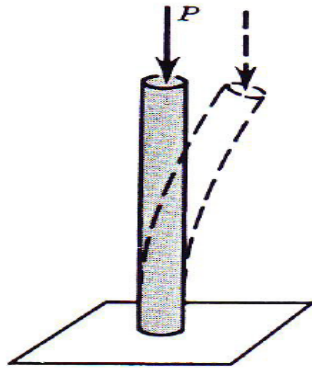


Fig. 4.6 compressive buckling of a shallow column

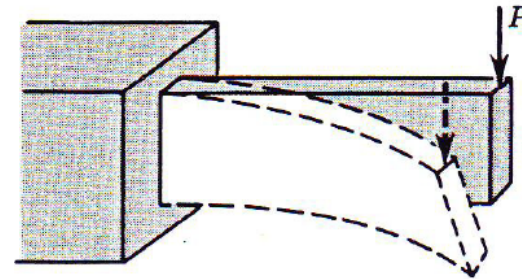


Fig. 4.7 Twist-bend buckling of a deep, narrow beam

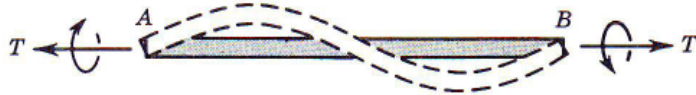


Fig. 4.8 Twist-bend buckling of a shaft in torsion.

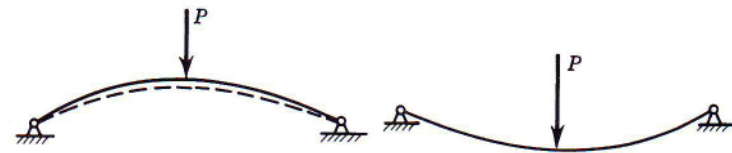


Fig. 4.9 “Snap-through” instability of a shallow curved member.

# Elastic Stability of Flexible Columns

## Governing differential equation

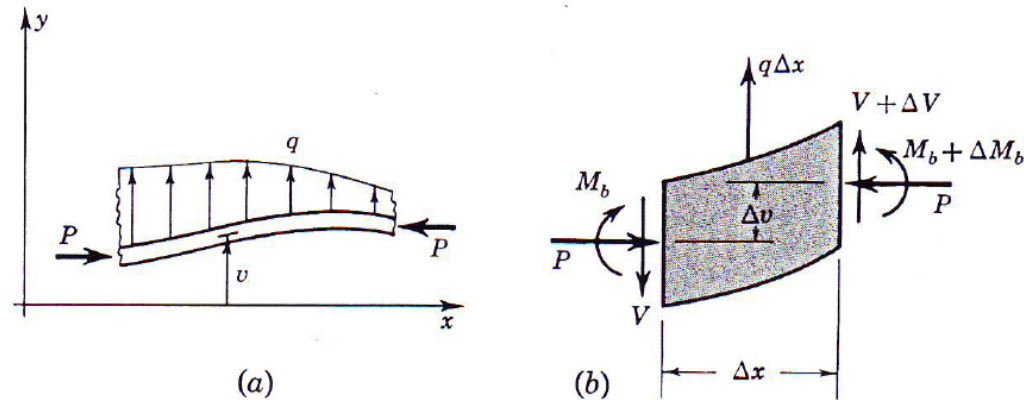


Fig. 4.10 (a) Beam subjected to longitudinal and transverse loads; (b) free-body sketch of element of beam.

Force equilibrium ;  $(V + \Delta V) - V + q\Delta x = 0$

$$\rightarrow \frac{dV}{dx} + q = 0$$

Moment equilibrium ;  $(M_b + \Delta M_b) - M_b + V \frac{\Delta x}{2} + (V + \Delta V) \frac{\Delta x}{2} + P\Delta v = 0$

$$\rightarrow \frac{dM_b}{dx} + V + P \frac{dv}{dx} = 0 \quad (4-4)$$

$$(4-3)$$

Using the fact that  $EI \frac{d^2 v}{dx^2} = M_b$  (4-5) and two equilibrium,

$$\boxed{\frac{d^2}{dx^2} \left( EI \frac{d^2 v}{dx^2} \right) + \frac{d}{dx} \left( P \frac{dv}{dx} \right) = q} \quad (4-6)$$



# Elastic Stability of Flexible Columns

## Example 1

Consider a column in a state of neutral equilibrium in the bent position.

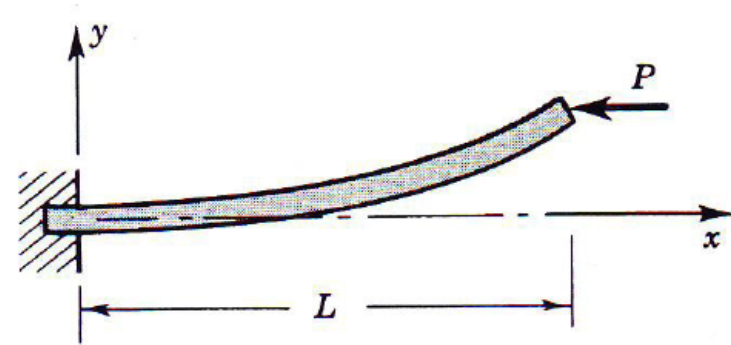


Fig. 4.11. example 1

$$\text{Boundary conditions ; } \left. \begin{array}{l} v = 0 \\ \frac{dv}{dx} = 0 \end{array} \right\} \text{ at } x = 0 \quad \left. \begin{array}{l} M_b = 0 \\ V = 0 \end{array} \right\} \text{ at } x = L \quad (4-7)$$

$$\left. \begin{array}{l} M_b = EI \frac{d^2v}{dx^2} = 0 \\ -V = \frac{d}{dx} \left( EI \frac{d^2v}{dx^2} \right) + P \frac{dv}{dx} = 0 \end{array} \right\} \text{ at } x = L$$

When  $EI$  and  $P$  are constants, the governing equation (4-6) is

$$EI \frac{d^4v}{dx^4} + P \frac{d^2v}{dx^2} = 0 \quad (4-9)$$

# Elastic Stability of Flexible Columns

## Example1 (Continued)

A solution to (4-9) for arbitrary values of the four constants is

$$v = c_1 + c_2 x + c_3 \sin \sqrt{\frac{P}{EI}} x + c_4 \cos \sqrt{\frac{P}{EI}} x \quad (4-10)$$

Substituting (4-10) into the four boundary conditions of (4-7) and (4-8)

$$\begin{aligned} c_1 + c_4 &= 0 \\ c_2 + c_3 \sqrt{\frac{P}{EI}} &= 0 \\ -c_3 \frac{P}{EI} \sin \sqrt{\frac{P}{EI}} L - c_4 \frac{P}{EI} \cos \sqrt{\frac{P}{EI}} L &= 0 \\ c_2 P &= 0 \end{aligned} \quad (4-11)$$

This is an eigenvalue problem.

$$c_2 = c_3 = 0 \quad \text{and} \quad c_4 = -c_1$$

Then the third equation becomes simply

## Elastic Stability of Flexible Columns

### Example1 (Continued)

$$c_1 \frac{P}{EI} \cos \sqrt{\frac{P}{EI}} L = 0 \quad (4-12)$$

This can be satisfied by having a value of  $P$  such that

$$\cos \sqrt{\frac{P}{EI}} L = 0 \quad (4-13)$$

The smallest value of  $P$  meeting this condition is

$$P = \frac{\pi^2}{4} \frac{EI}{L^2} \quad (\text{Critical load}) \quad (4-14)$$

Substituting back into (4-10), the corresponding deflection curve is

$$v = c_1 \left( 1 - \cos \frac{\pi x}{2 L} \right) \quad (4-15)$$

**For smaller value of  $P$  the straight column is stable.**

**For larger value of  $P$  the straight column is no longer stable. → Buckling**

# Elastic Stability of Flexible Columns

## Example2 with imperfection

Another insight into column buckling :  
**imperfection** in either the column or the loading

Consider flexible column held in equilibrium by a longitudinal compressive force  $P$  with eccentricity  $\epsilon$

It is equivalent to the state that flexible column held in equilibrium by the same compressive force  $P$  plus an end moment  $M_0$ .

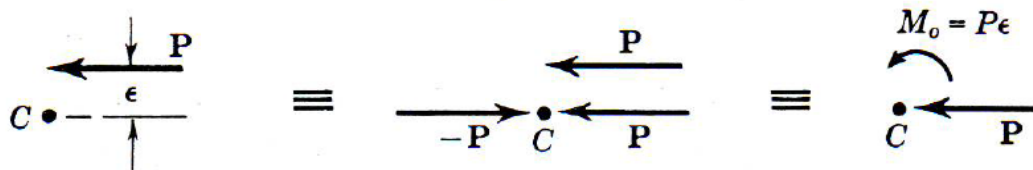
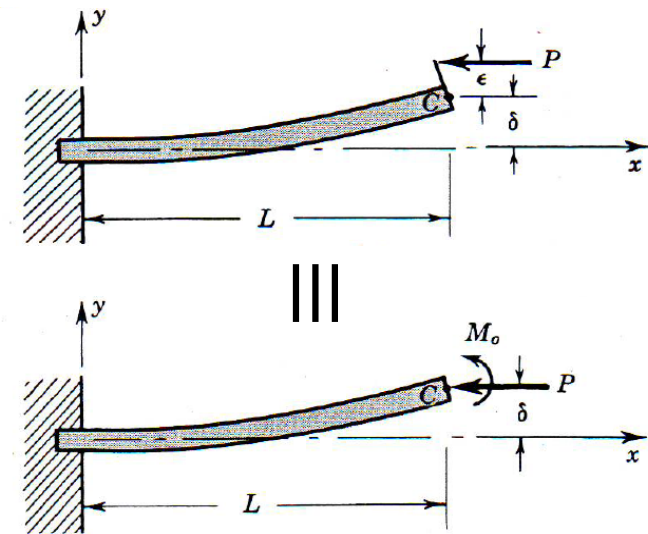


Fig 4.12. The equivalence of the two loadings

# Elastic Stability of Flexible Columns

## Example2 with imperfection (Continued)

$$\text{Boundary conditions ; } \left. \begin{array}{l} v = 0 \\ \frac{dv}{dx} = 0 \end{array} \right\} \text{ at } x = 0 \quad \left. \begin{array}{l} M_b = M_0 \\ V = 0 \end{array} \right\} \text{ at } x = L$$

Then, substituting (4-10) into given boundary condition

$$\begin{aligned} c_1 + c_4 &= 0 \\ c_2 + c_3 \sqrt{\frac{P}{EI}} &= 0 \\ -c_3 \frac{P}{EI} \sin \sqrt{\frac{P}{EI}} L - c_4 \frac{P}{EI} \cos \sqrt{\frac{P}{EI}} L &= \frac{M_0}{EI} \\ c_2 P &= 0 \end{aligned} \quad (4-16)$$

# Elastic Stability of Flexible Columns

## Example2 with imperfection (Continued)

In a similar way, we can derive corresponding deflection curve ;

$$v = \frac{M_0}{P} \frac{1 - \cos \sqrt{P/EI} x}{\cos \sqrt{P/EI} L}$$

At  $x = L$ ,

$$\begin{aligned} \delta = v(L) &= \frac{M_0}{P} \left( \sec \sqrt{\frac{P}{EI}} L - 1 \right) \\ &= \varepsilon \left( \sec \sqrt{\frac{P}{EI}} L - 1 \right) \quad (4-17) \end{aligned}$$

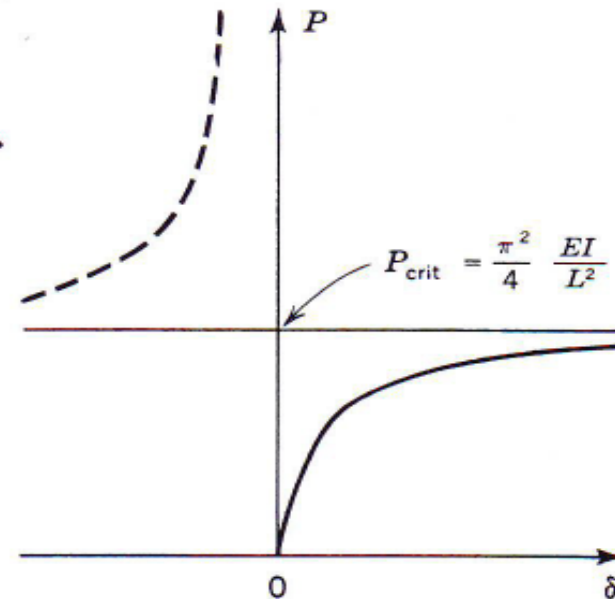
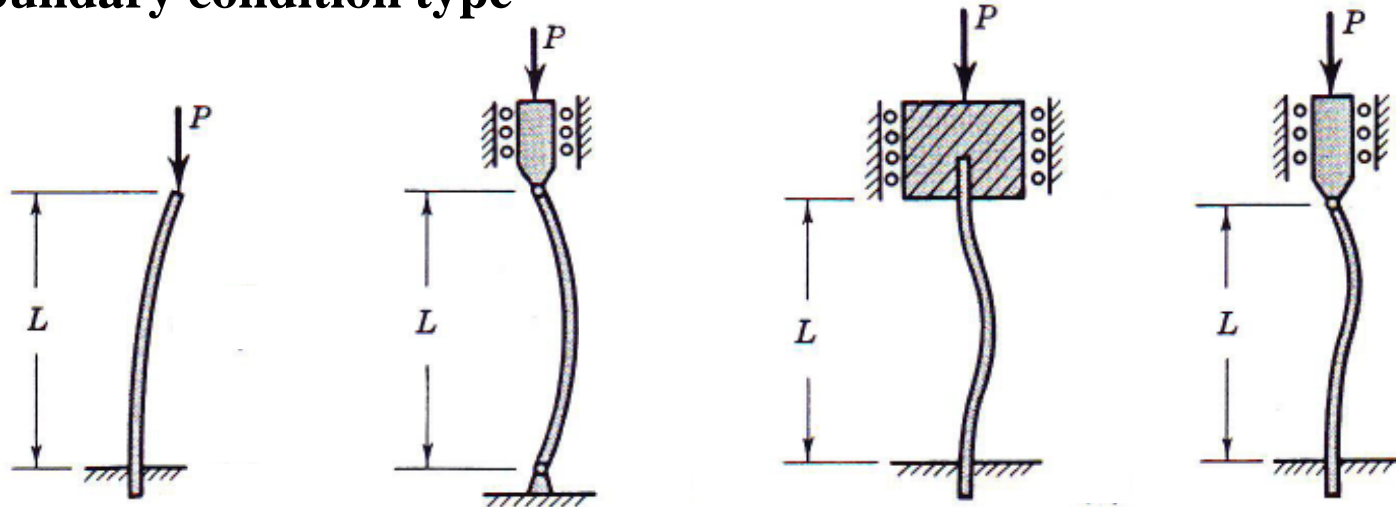


Fig. 4.13 Relation between compressive force  $P$  and transverse deflection  $\delta$  due to eccentricity  $\varepsilon$ .

# Elastic Stability of Flexible Columns

## Boundary condition type



(a) clamped-free (b) hinged-hinged (c) clamped-clamped (d) clamped-hinged

$$P_{\text{crt}} = \frac{cEI}{L^2}$$

	$c$
(a)	$\pi^2/4 = 2.47$
(b)	$\pi^2 = 9.87$
(c)	$20.2$
(d)	$4\pi^2 = 39.5$



## Elastic Stability of Flexible Columns

### Example of buckling in reality



**Sun kink** in rail tracks



**Lateral-torsional buckling**  
of an aluminium alloy plate girder



# Elastic Postbuckling Behavior

## Hinged body with nonlinear spring

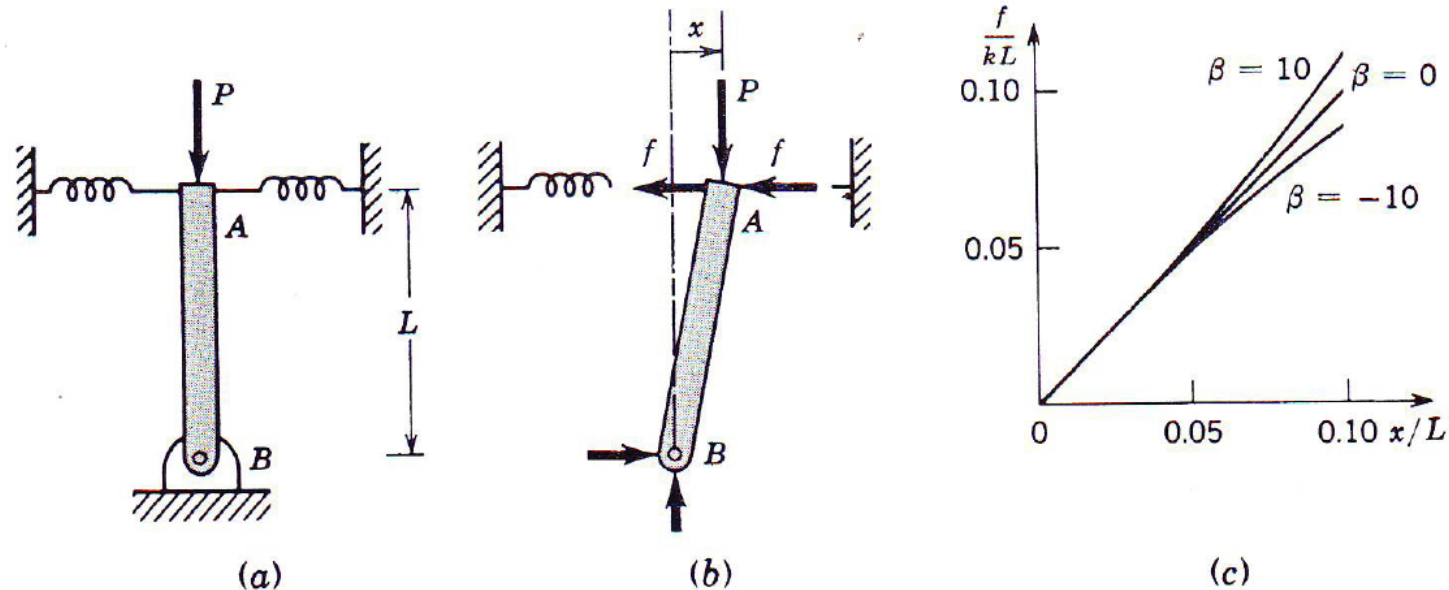


Fig. 4.14 Strut supported by nonlinear springs

$$f = kx \left( 1 + \beta \frac{x^2}{L^2} \right) \quad (4-19)$$

where  $\beta$  is a parameter which fixes the nature of the nonlinearity

$\beta > 0$ : stiffening spring

$\beta < 0$ : softening spring

# Elastic Postbuckling Behavior

## Hinged body with nonlinear spring (Continued)

From Fig. 4.16(b), moment equilibrium is

$$Px - 2kLx \left( 1 + \beta \frac{x^2}{L^2} \right) = 0 \quad \longrightarrow \quad x = 0 \text{ or } P = 2kL \left( 1 + \beta \frac{x^2}{L^2} \right)$$

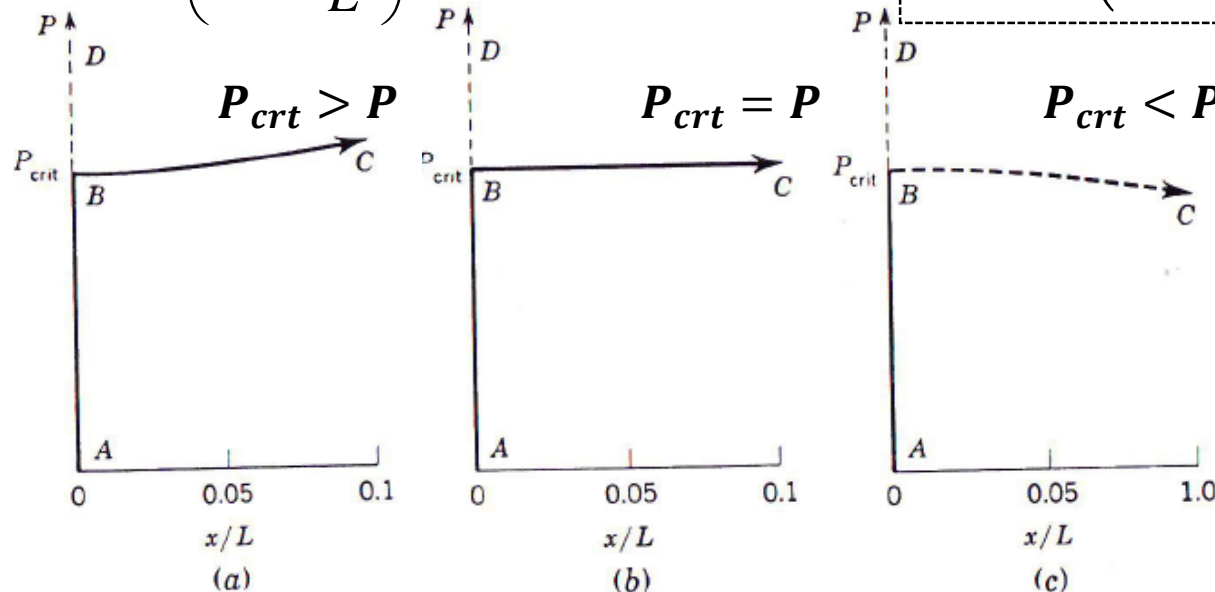


Fig. 4.15 Ideal postbuckling curves for (a)  $\beta = 10$ , (b)  $\beta = 0$ , (c)  $\beta = -10$

In every case the branch BD represents **unstable** equilibrium positions.

The branch BC represents

{	<b>stable</b> equilibrium positions.	for $\beta > 0$
	<b>neutral</b> equilibrium positions.	for $\beta = 0$
	<b>unstable</b> equilibrium positions.	for $\beta < 0$

# Elastic Postbuckling Behavior

## Hinged body with nonlinear spring (Continued)

When the load is positioned slightly off-center:

$$P(x + \varepsilon) = 2kLx \left( 1 + \beta \frac{x^2}{L^2} \right) \quad (4-20)$$

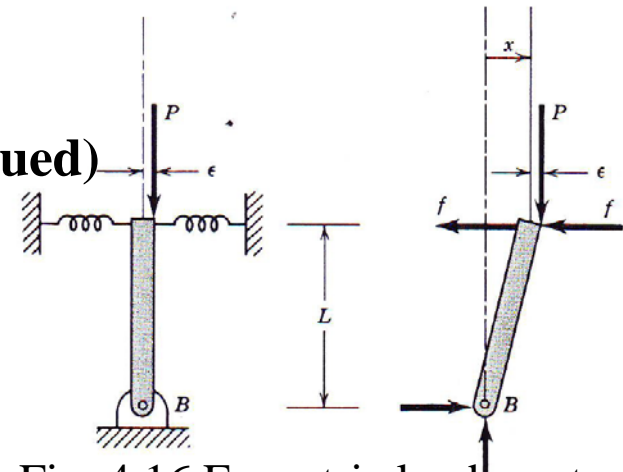


Fig. 4.16 Eccentric load on strut supported by nonlinear springs.

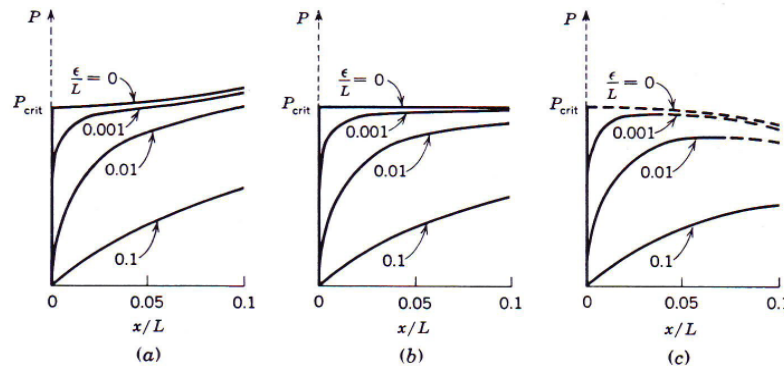


Fig. 4.17 Effect of imperfection parameter  $\varepsilon/L$  on postbuckling behavior for (a)  $\beta = 10$ , (b)  $\beta = 0$ , (c)  $\beta = -10$ .

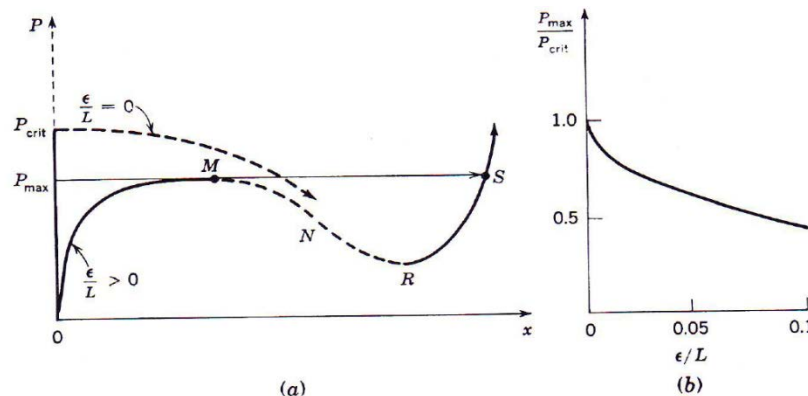


Fig. 4.18 Maximum load for softening nonlinearity ( $\beta = -10$ ) depends on magnitude of imperfection.

## Extension of Euler's Formula To Columns

### Free end A And Fixed end B

Behaves as the upper half of a pin-connected column.

- Effective length :  $L_e = 2L$
- Critical Load :

$$P_{crt} = \frac{\pi^2 EI}{4L} = \frac{\pi^2 EI}{L_e^2} \quad (4-21)$$

- Critical Stress :

$$\sigma_{crt} = \frac{\pi^2 E}{(L_e/r)^2} \quad (4-22)$$

$L_e/r$  : Effective slenderness ratio

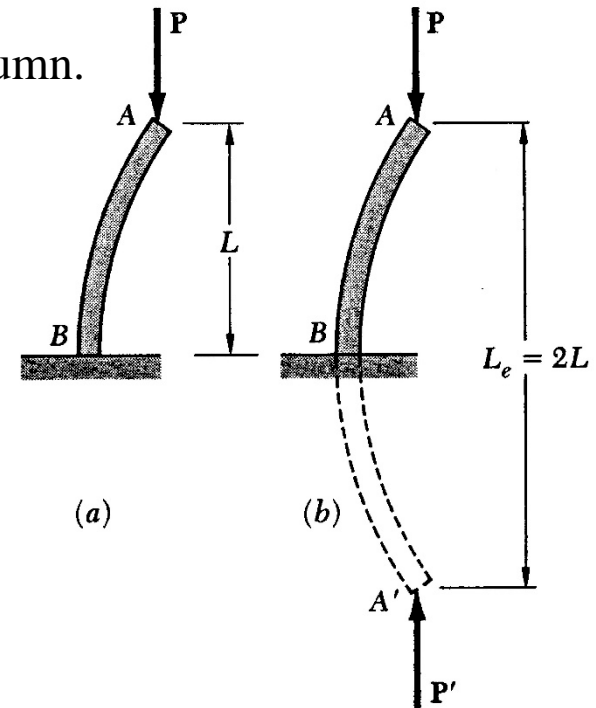


Fig. 4.19 Free end and fixed end

# Extension of Euler's Formula To Columns

## Two fixed ends A and B

The shear at C and the horizontal components of the reaction at A and B are 0.

Restraints upon AC and CB are identical.

Portion AC and BC: symmetric about its midpoint D and E.

- D and E are points of inflection ( $M = 0$ )

Portion DE must behave as a pin-ended column.

- The effective length is :  $L_e = L/2$

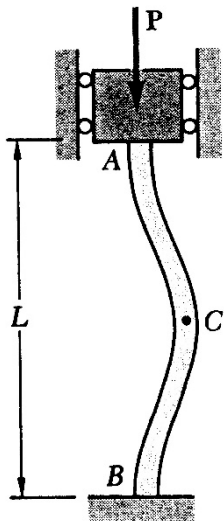


Fig. 4.20

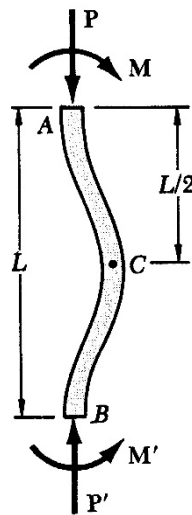


Fig. 4.21

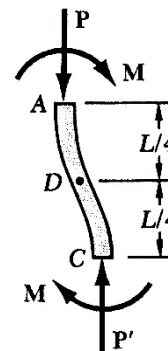


Fig. 4.22

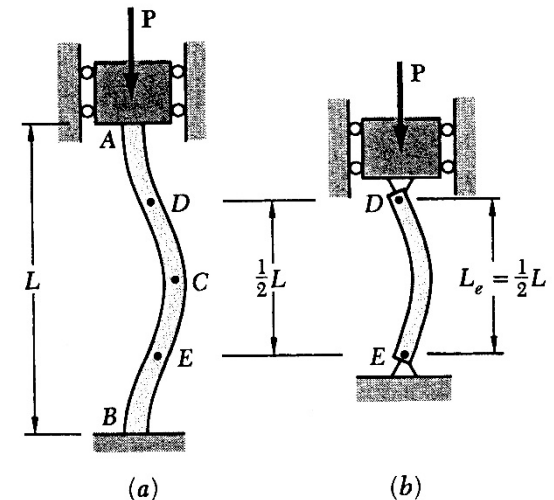


Fig. 4.23

## Extension of Euler's Formula To Columns

### One pin-connect end A and one fixed end B

Differential equation of the elastic curve:

$$M = -Py - Vx$$

$$\frac{d^2 y}{dx^2} = \frac{M}{EI} = -\frac{Py}{EI} - \frac{Vx}{EI} \quad \frac{d^2 y}{dx^2} + p^2 y = -\frac{Vx}{EI}$$

where  $p^2 = \frac{P}{EI}$  (4-23)

Particular solution is  $y = -\frac{V}{p^2 EI} x = -\frac{V}{P} x$

General solution is  $y = A \sin px + B \cos px - \frac{V}{P} x$

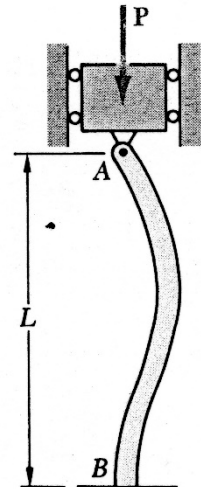


Fig. 4.24

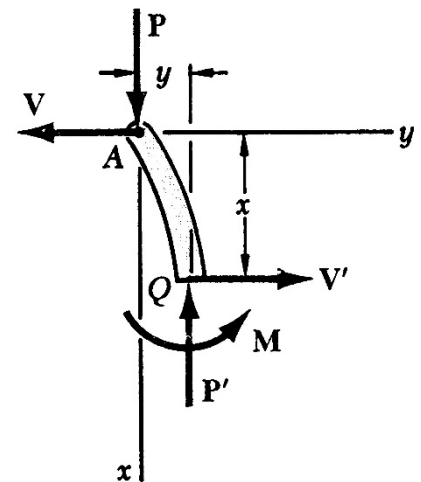


Fig. 4.25

## Extension of Euler's Formula To Columns

### One pin-connect end A and one fixed end B (Continued)

$$\text{Boundary conditions 1 ; } y(0) = 0 \rightarrow B = 0 \quad (4-24)$$

$$\text{Boundary conditions 2 ; } y(L) = 0 \rightarrow A \sin pL = \frac{VL}{P} \quad (4-25)$$

$$\left. \frac{dy}{dx} \right|_{x=L} = 0 \rightarrow A p \cos pL = \frac{V}{P} \quad (4-26)$$

From (4-25) and (4-26),

$$\tan pL = pL \rightarrow pL = 4.4934$$

From (4-23),

$$P_{crt} = \frac{20.19EI}{L^2}$$

From (4-21),

$$\frac{\pi^2 EI}{L_e^2} = \frac{20.19EI}{L^2} \rightarrow L_e = 0.699L \approx 0.7L$$

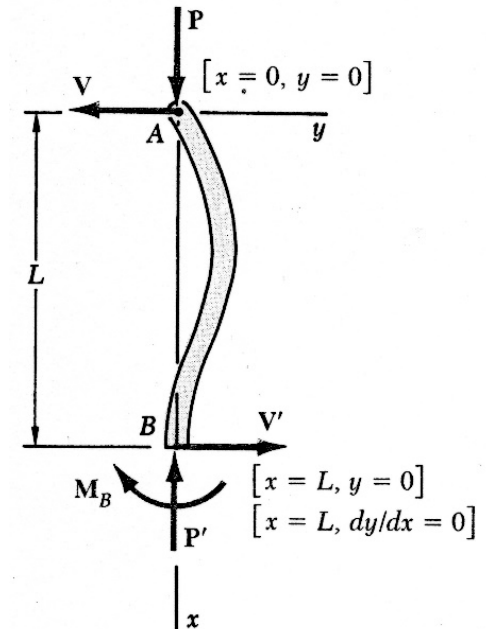


Fig. 4.26

# Extension of Euler's Formula To Columns

## Effective length of column for various end conditions

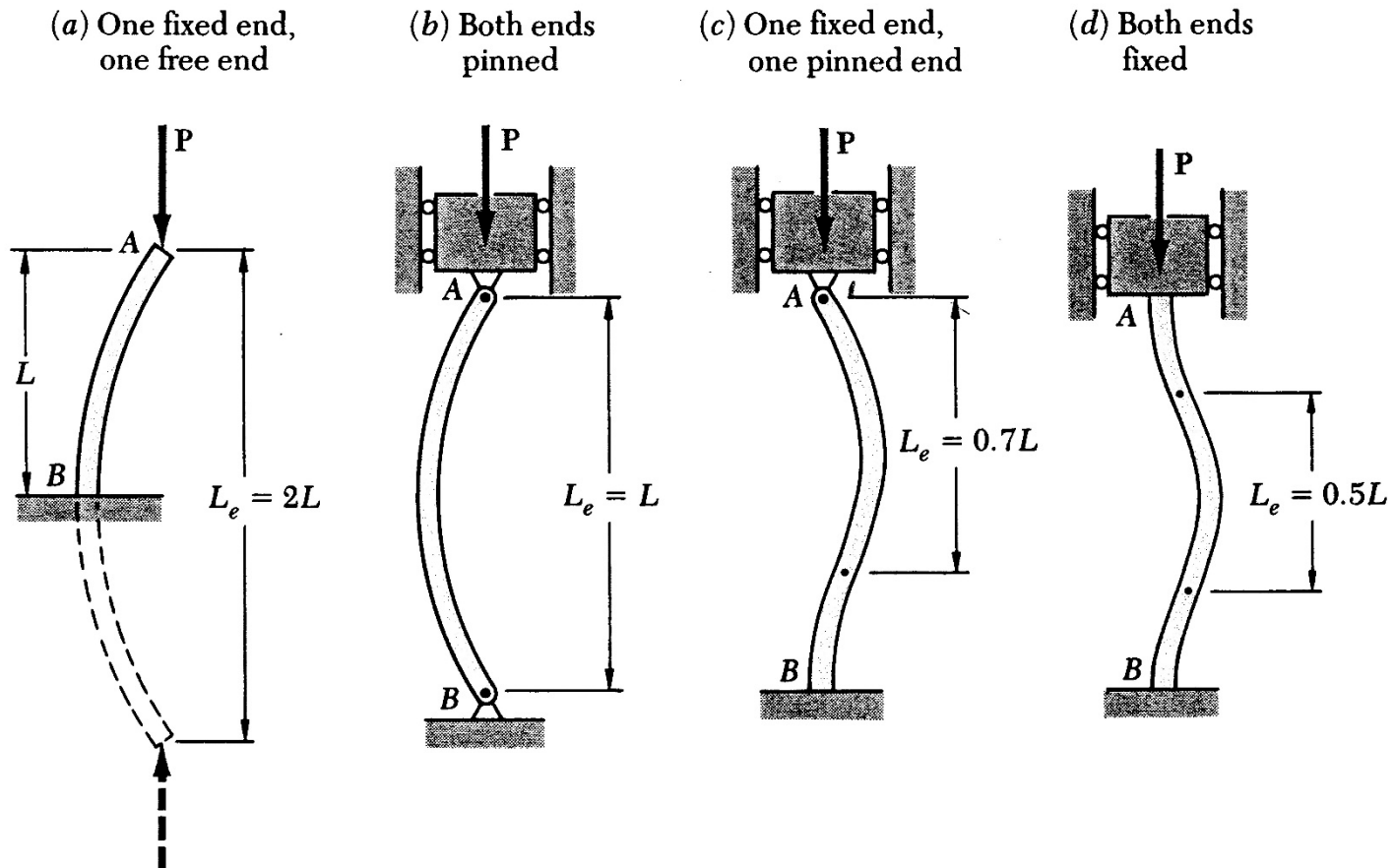


Fig. 4.27 Effective length of column for various end conditions



## Extension of Euler's Formula To Columns

### Example 1

An aluminum column of length  $L$  and rectangular cross section has a fixed end B and supports a centric load at A. Two smooth and rounded fixed plates restrain end A from moving in one of the vertical planes of symmetry of the column, but allow it to move in the other plane.

- (a) Determine the ratio  $a/b$  of the two sides of the cross section corresponding to the most efficient design against buckling.
- (b) Design the most efficient cross section for the column, knowing that  $L=500$  mm,  $E=70$  GPa,  $P=20$  kN, and that a factor safety of 2.5 is required.

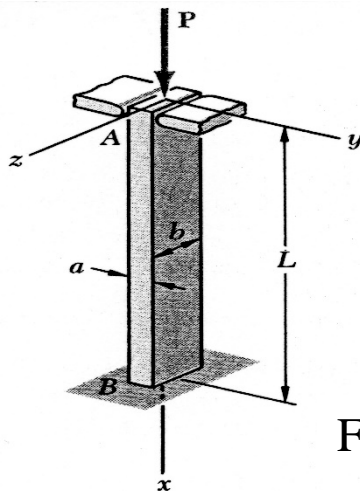


Fig. 4.28 example 1 illustration

\* Gere, James, James M. Gere, and Barry J. Goodno. *Mechanics of materials*. Nelson Education, 2012.

## Extension of Euler's Formula To Columns

### Example 1 (Continued)

#### Buckling in x, y plane

- Effective length with respect to buckling in this plane:  $L_e = 0.7L$  .
- Radius of gyration:  $r_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{ba^3}{12ab}} = \frac{a}{\sqrt{12}}$
- Effective slenderness ratio:  $\frac{L_e}{r_z} = (0.7L)/(\frac{a}{\sqrt{12}})$

#### Buckling in x, z plane

- Effective length with respect to buckling in this plane:  $L_e = 2L$
- Radius of gyration:  $r_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{ab^3}{12ab}} = \frac{b}{\sqrt{12}}$
- Effective slenderness ratio:  $\frac{L_e}{r_z} = (2L)/(\frac{b}{\sqrt{12}})$

#### (a) Most effective design.

The critical stresses corresponding to the possible modes of buckling are equal.

$$\sigma_{crt} = \frac{\pi^2 E}{(L_e/r)^2} \quad ; \quad \frac{0.7L}{a/\sqrt{12}} = \frac{2L}{b/\sqrt{12}} \rightarrow \frac{a}{b} = 0.35$$

\* Gere, James, James M. Gere, and Barry J. Goodno. *Mechanics of materials*. Nelson Education, 2012.

## Extension of Euler's Formula To Columns

### Example 1 (Continued)

#### (b) Design for given data.

$$P_{cr} = (F.S.)P = (2.5)(20\text{kN}) = 50\text{kN}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{50 \times 10^3 \text{ N}}{0.35b^2} \quad (A = ab = (0.35b)b)$$

$$L = 0.5\text{m} \rightarrow L_e/r_y = 3.464/b$$

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e / r)^2} = \frac{50 \times 10^3 \text{ N}}{0.35b^2} = \frac{\pi^2 (70 \times 10^9 \text{ Pa})}{(3.464 / b)^2}$$

$$b = 39.7\text{mm} \quad a = 0.35b = 13.9\text{mm}$$

\* Gere, James, James M. Gere, and Barry J. Goodno. *Mechanics of materials*. Nelson Education, 2012.

# Eccentric Loading : The Secant Formula

## Governing differential equation

- Portion AQ:

- Bending moment at Q is

$$M = -Py - M_A = -Py - Pe \quad (4-28)$$

$$\frac{d^2 y}{dx^2} = \frac{M}{EI} = -\frac{P}{EI} y - \frac{Pe}{EI}$$

$$\frac{d^2 y}{dx^2} + p^2 y = -p^2 e \quad (4-29)$$

$$\text{where, } p^2 = \frac{P}{EI}$$

- General solution of (4-29):

$$y = A \sin px + B \cos px - e \quad (4-30)$$

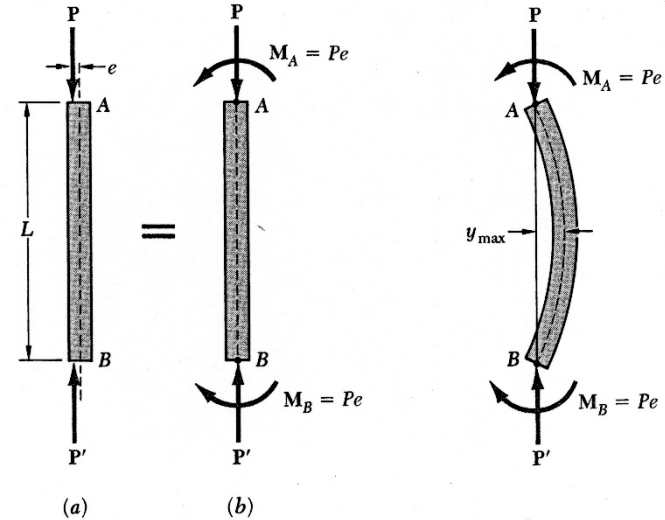


Fig. 4.29

Fig. 4.30

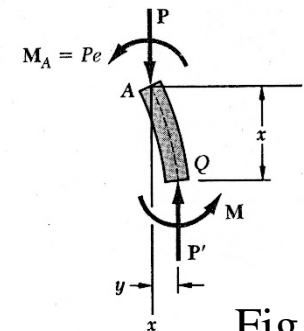


Fig. 4.31

## Eccentric Loading : The Secant Formula

### Governing differential equation (Continued)

– Using boundary condition ;  $y(0) = 0$   $y(L) = 0$

$$\rightarrow A = e \tan \frac{pL}{2} \quad B = e$$

$$\therefore y = e \left( \tan \frac{pL}{2} \sin px + \cos px - 1 \right) \quad (4-31)$$

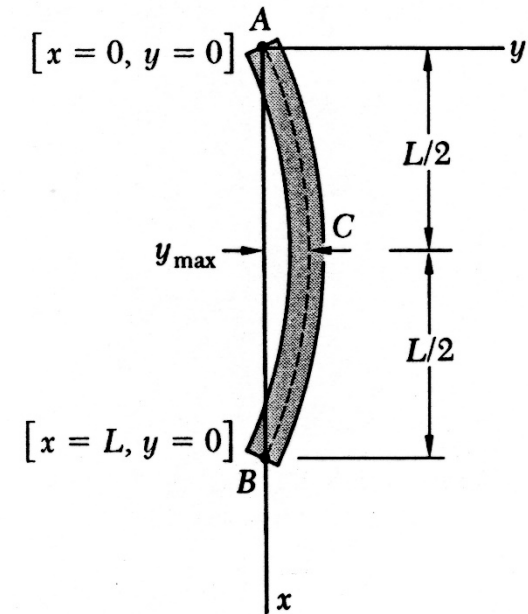


Fig. 4.32

## Eccentric Loading : The Secant Formula

### Governing differential equation (Continued)

The value of the maximum deflection is obtained by setting.  $x = L / 2$

$$\begin{aligned} y_{\max} &= e \left( \tan \frac{pL}{2} \sin \frac{pL}{2} + \cos \frac{pL}{2} - 1 \right) \\ &= e \left( \frac{\tan \frac{pL}{2} \cos^2 \frac{pL}{2}}{\cos \frac{pL}{2}} - 1 \right) \\ y_{\max} &= e \left( \sec \frac{pL}{2} - 1 \right) \end{aligned} \quad (4-32)$$

$$y_{\max} = e \left[ \sec \left( \sqrt{\frac{P}{EI}} \frac{L}{2} \right) - 1 \right] \quad \left( p^2 = \frac{P}{EI} \right) \quad (4-33)$$

## Eccentric Loading : The Secant Formula

### Governing differential equation (Continued)

$y_{\max}$  becomes infinite when

$$\sqrt{\frac{P}{EI}} \frac{L}{2} = \frac{\pi}{2} \quad (4-34)$$

While the deflection does not actually become infinite, and  $P$  should not be allowed to reach the critical value which satisfies (4-34).

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (4-35)$$

Solving (11-30) for  $EI$  and substituting into (4-33),

$$y_{\max} = e \left[ \sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] \quad (4-36)$$

## Eccentric Loading : The Secant Formula

### Governing differential equation (Continued)

The maximum stress:  $\sigma_{\max} = \frac{P}{A} + \frac{M_{\max} c}{I}$  (4-37)

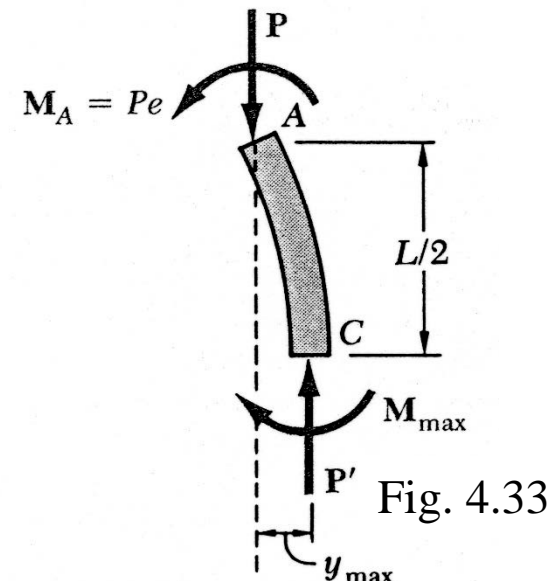
• Portion AC:  $M_{\max} = Py_{\max} + M_A = P(y_{\max} + e)$

$$\sigma_{\max} = \frac{P}{A} + \frac{(y_{\max} + e)c}{I} = \frac{P}{A} \left[ 1 + \frac{(y_{\max} + e)c}{r^2} \right] \quad (4-38)$$

– Substituting  $y_{\max}$

$$\sigma_{\max} = \frac{P}{A} \left[ 1 + \frac{ec}{r^2} \sec \left( \sqrt{\frac{P}{EI}} \frac{L}{2} \right) \right] \quad (4-39)$$

$$= \frac{P}{A} \left[ 1 + \frac{ec}{r^2} \sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right] \quad (4-40)$$





## Eccentric Loading : The Secant Formula

### The Secant Formula

Since the maximum stress does not vary linearly with the load  $P$ , the principle of superposition does not apply to the determination of the stress due to the simultaneous application of several loads; the resultant load must first be computed, and (4-39) or (4-40) may be used to determine the corresponding stress. For the same reason, any given factor of safety should be applied to the load, and not to the stress.

(4-39): Making  $I = Ar^2$

$$\frac{P}{A} = \frac{\sigma_{\max}}{1 + \frac{ec}{r^2} \sec\left(\frac{1}{2} \sqrt{\frac{P}{EA}} \frac{L_e}{r}\right)} \quad (4-41)$$

# Eccentric Loading : The Secant Formula

## The Secant Formula

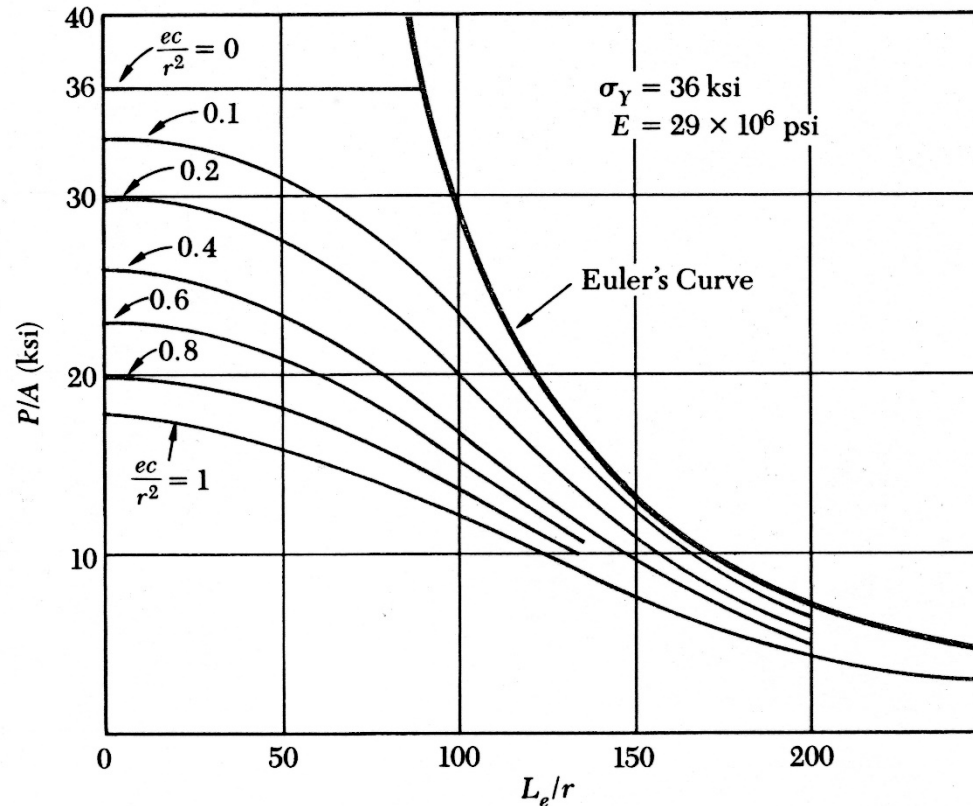


Fig. 4.34 Load per unit area,  $P/A$ , causing yield in column

For a steel column  $E = 29 \times 10^6 \text{ psi}$   $\sigma_Y = 36 \text{ ksi}$

## Eccentric Loading : The Secant Formula

### The Secant Formula

For all small value of  $L_e / r^2$ , the secant is almost equal to 1:

$$\frac{P}{A} = \frac{\sigma_{\max}}{1 + \frac{ec}{r^2}} \quad (4-42)$$

For large values of  $L_e / r^2$ , the curves corresponding to the various values of the ratio  $ec / r^2$  get very close to Euler's curve defined by (4-22), and thus that the effect of the eccentricity of the loading on the value of  $P/A$  becomes negligible.

# Eccentric Loading : The Secant Formula

## Example 1

The uniform column AB consists of an 8-ft section of structural tubing having the cross section shown.

- Using Euler's formula and a factor of safety of two, determine the allowable centric load for the column and the corresponding normal stress.
- Assuming that the allowable load, found in part a, is applied as shown at a point 0.75 in. from the geometric axis of the column, determine the horizontal deflection of the top of the column and the maximum normal stress in the column. Use

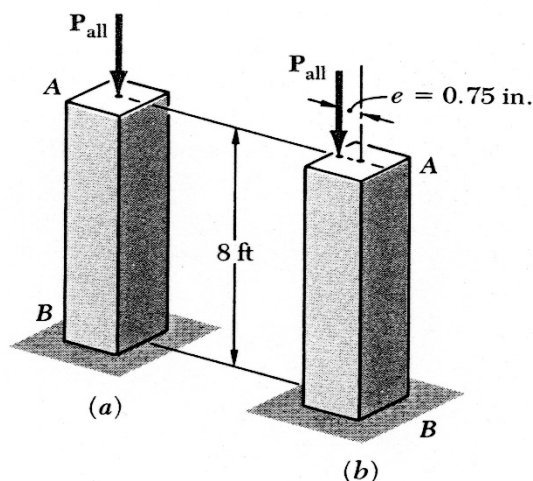


Fig. 4.35 Example

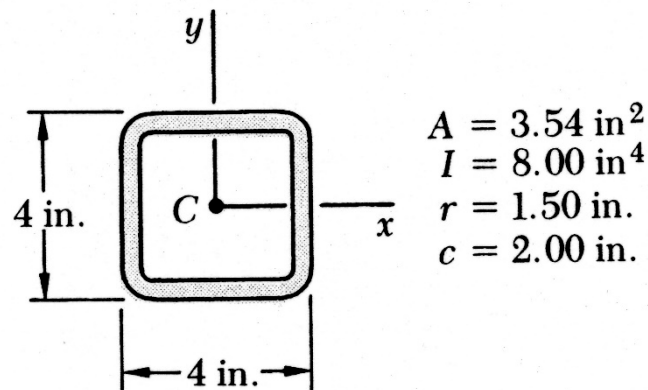


Fig. 4.36 Structural property

\* Gere, James, James M. Gere, and Barry J. Goodno. *Mechanics of materials*. Nelson Education, 2012.

## Eccentric Loading : The Secant Formula

### Example 1

#### Effective Length

One end fixed and one end free:  $L_e = 2(8ft) = 16ft = 192in$

#### Critical Load

Using Euler's formula,

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29 \times 10^6 \text{ psi})(8.00 \text{ in}^4)}{(192 \text{ in})^2} = 62.1 \text{ ksi} \cdot \text{in}^2$$

#### (a) Allowable Load and Stress

For a factor of safety of 2:  $P_a = \frac{P_{cr}}{F.S} = \frac{62.1 \text{ ksi} \cdot \text{in}^2}{2} = 31.1 \text{ ksi} \cdot \text{in}^2$

$$\sigma_a = \frac{P_a}{A} = \frac{31.1 \text{ ksi} \cdot \text{in}^2}{3.54 \text{ in}^2} = 8.79 \text{ ksi}$$

# Eccentric Loading : The Secant Formula

## Example 1

### (b) Eccentric Load.

Column AB (Fig. 4.39) and its loading are identical to the upper half of the upper half of the Fig. 4.39.

- Horizontal deflection of point A:

$$y_{\max} = e \left[ \sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) - 1 \right] = (0.75 \text{ in}) \left[ \sec \left( \frac{\pi}{2\sqrt{2}} \right) - 1 \right]$$

$$= 0.939 \text{ in}$$

- Maximum normal stress:

$$\sigma_{\max} = \frac{P}{A} \left[ 1 + \frac{ec}{r^2} \sec \left( \frac{\pi}{2} \sqrt{\frac{P}{P_{cr}}} \right) \right]$$

$$= \frac{31.1 \text{ kips}}{3.54 \text{ in}^2} \left( 1 + \frac{(0.75 \text{ in})(2 \text{ in})}{(1.50 \text{ in})^2} \sec \frac{\pi}{2\sqrt{2}} \right)$$

$$= 22.0 \text{ ksi}$$

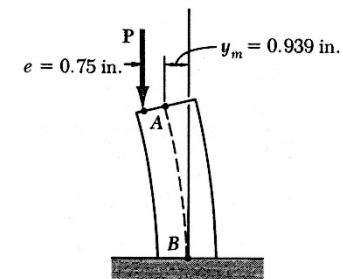
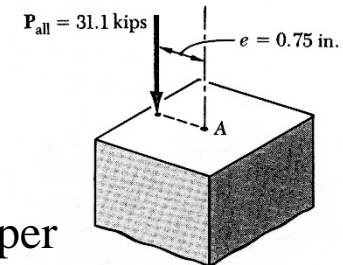


Fig. 4.37

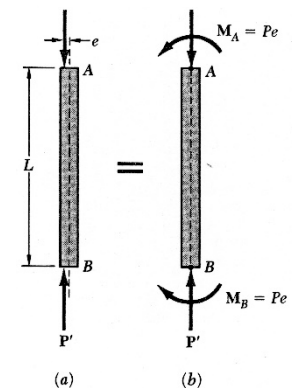


Fig. 4.38

\* Gere, James, James M. Gere, and Barry J. Goodno. *Mechanics of materials*. Nelson Education, 2012.

## Trivial Things Support Our Life!



당인의 ‘추풍환선사녀도’



기홍도의 ‘씨름’



## Engineers' Roles

### 국산무기 사고 · 군납장비 결함 관련 일지

2009년 12월9일

K-21 육군 전투장갑차, 도하훈련 도중 엔진 정지로 침수



2010년 7월29일

K-21, 전남 장성군 육군기계화학교 수상조종훈련장에서 침수, 부사관 1명 사망

8월6일

K-1 전차, 경기 파주시 무건리 사격장에서 훈련 중 포신 피열



31일

국방부, "신형 전투화 4000여 켤레에서 뒷굽이 떨어져 나가는 현상 발생해 감사 착수" 발표

9월8일

육군, "2005년부터 K-9 자주포 엔진 38개에서 '캐비테이션' 현상 발생했으며, 원인은 값싼 부동액 때문" 이라고 밝힘



15일

국방부, 육군 K계열 무기 결함 관련 보도자료에서 "K-9 자주포 엔진 설계에 이상이 있다" 며 입장 번복

29일

- 국방부 감사관실, 신형 전투화 감사 결과 "규격 제정과 품질 검사에 소홀했다" 고 밝힘. 국방부는 관련자 5명을 징계 처리하고 이 중 2명을 군 검찰에 수사 의뢰



- 해군 유도탄고속함 '한상국함', 최종테스트 결과 고속주행 시 '갈지자' 운항 드러남

10월7일

서종표 의원, "한상국함이 화재 위험까지 안고 있다" 고 주장



8일

김장수 의원, "K-11 복합소총 화기 검사에서 불량률 47.5% 달한다" 고 공개

14일

황의돈 육군참모총장, "K-11 안전성 확보될 때까지 전력화 연기하겠다" 고 밝힘



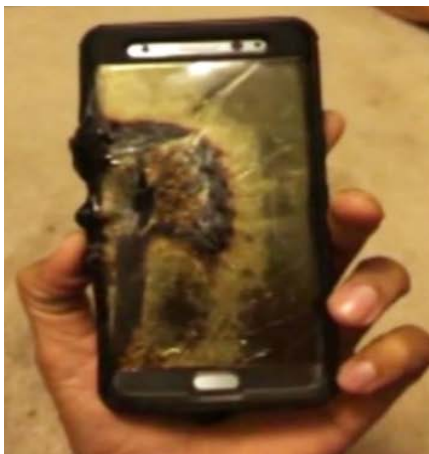
## Engineers' Roles



Minnesota I-35W Bridge Collapse  
(2007.08.)



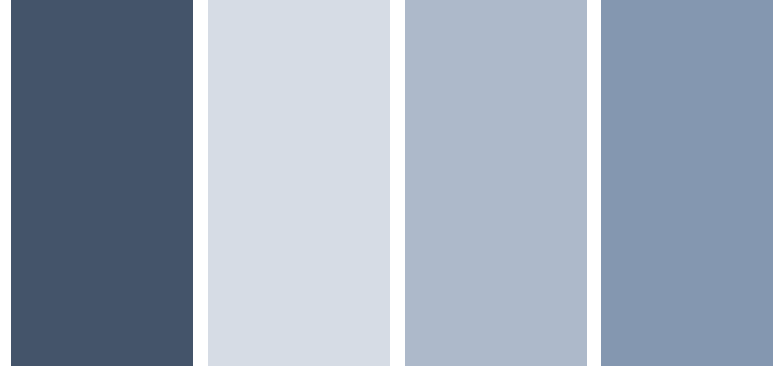
CNG버스 폭발사고  
(2010.08.)



갤럭시 노트7 폭발  
(2016.09.)



효성울산공장 가스폭발  
(2016.09.)



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FOR LISTENING**