CH. 5

STRESS-STRAIN-TEMPERATURE RELATIONS

5.1 Introduction

▶ The presence of only three equations of equilibrium for the six components of stress and the addition of three components (u, v, w in Ch. 4-10) of displacement in the six equations relating strain to displacement indicates;

→ Further relations are needed before the equations can be solved to determine the distributions of stress and strain in a body; i.e., the distribution of stress and strain will depend on the material behavior of the body.

▶ Two avenues of approach are suggested to investigate the relations between stress and strain;

i) Atomic level

→ Relations based on experimental evidence at the atomic level with theoretical extension to the macroscopic level

ii) Macroscopic level

→ Relations based on experimental evidence at the macroscopic level

▶ <u>Discussions in this chapter</u>

i) The stress-strain behavior of a wide variety of structural materials, including metals, wood, polymers, and composite materials

ii) Elastic, plastic, and viscoelastic behavior

iii) Various mathematical models are established to describe elasticity and plasticity.

iv) The theory of linear isotropic elasticity

v) Design criteria for yielding of ductile materials, fracture of brittle materials, and fatigue under repeated loading

▶ Definitions

1 ▷ Elastic deformation

→ The deformation that returns to its origin shape on release of load

2 ▷ Plastic deformation

→ The deformation which depends on the applied load, is independent of time, and remains on release of load

3 ▷ Strain hardening

→ Increase in the load required for further plastic deformation

4 ▷ Ductile structure

→ Structure for which the plastic deformation before fracture is much larger than the elastic deformation

5 ▷ Brittle structure

→ Structure which exhibits little deformation before fracture

6 ▷ Fatigue

→ Progressive fracture under repeated load

7 ▷ Notch brittle

→ A larger part might have a large elastic region surrounding the plastic zone when the crack started to grow so that the overall deformation of the part would only be little more than the elastic deformation. Such a part or structure would be called notch-brittle.

8 ▷ Creep

→ Time-dependent part of the deformation

9 ▷ Elastic after effect or recovery

→ Elastic spring back followed by a relatively slow unfolding

10 ▷ Visco-elasticity

→ A mixture of creep and elastic aftereffects at room temperature (Ex: long chin polymers)

5.2 Tensile Test

▶ Tensile test

→ Test in which a relatively slender member is pulled in the direction of its axis

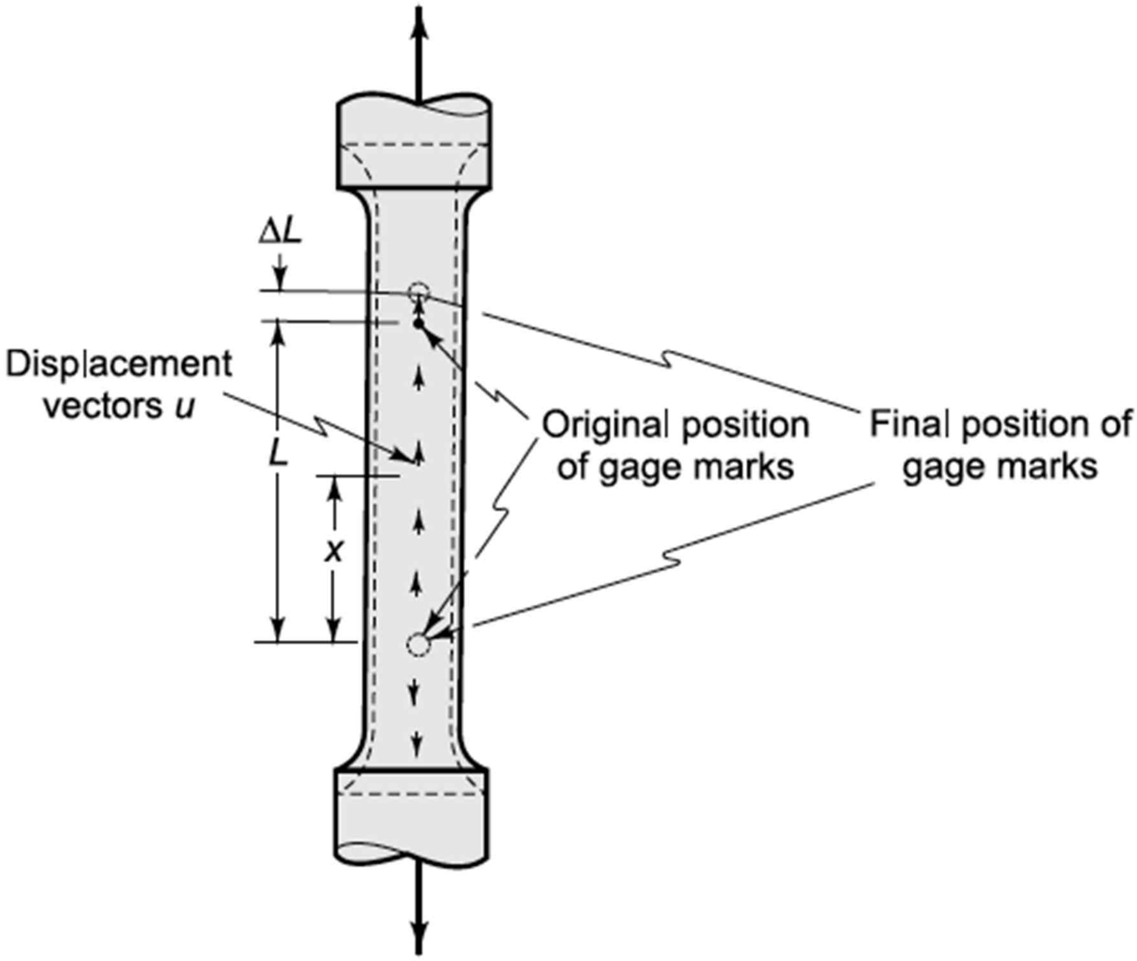


Fig. 5.4 *Displacements in a tensile test*

→ Our aim is to use tensile test data to formulate quantitative stress-strain relations which, when incorporated with equilibrium and compatibility requirements, will produce theoretical predictions in agreement with the experimental results in complicated situations.

→ The elongation and lateral contraction are also noted as the test proceeds.

▷ From Fig. 5.4, if the displacements vary uniformly over the gage length L,

$$u = (x/L)\Delta L$$

$$\therefore \varepsilon_x = \partial u/\partial x = \Delta L/L \tag{5.1}$$

▶ Stress-strain diagram & definitions

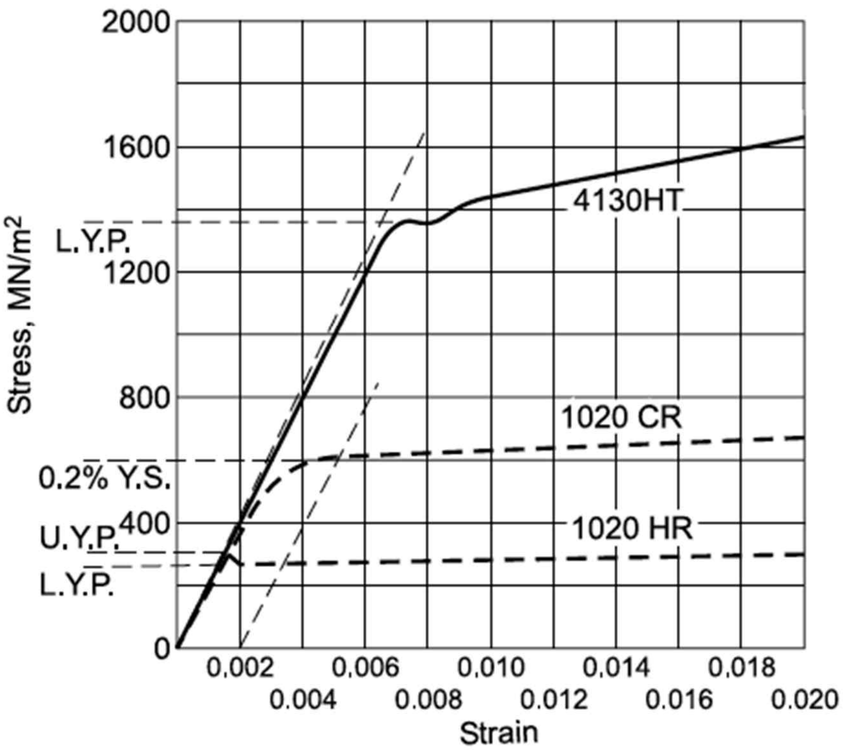


Fig. 5.5(a) *Stress-strain curves for three steels.*
- - - - Mild steel, hot-rolled (1020 HR)
– – Mild steel, cold-rolled (1020 CR)
—— 0.3% C, 0.5% Mn, 0.25% Si, 0.9% Cr, balance Fe (4130 HT)
Heat treatment: Oil quenched from 870°C, tempered at 315°C

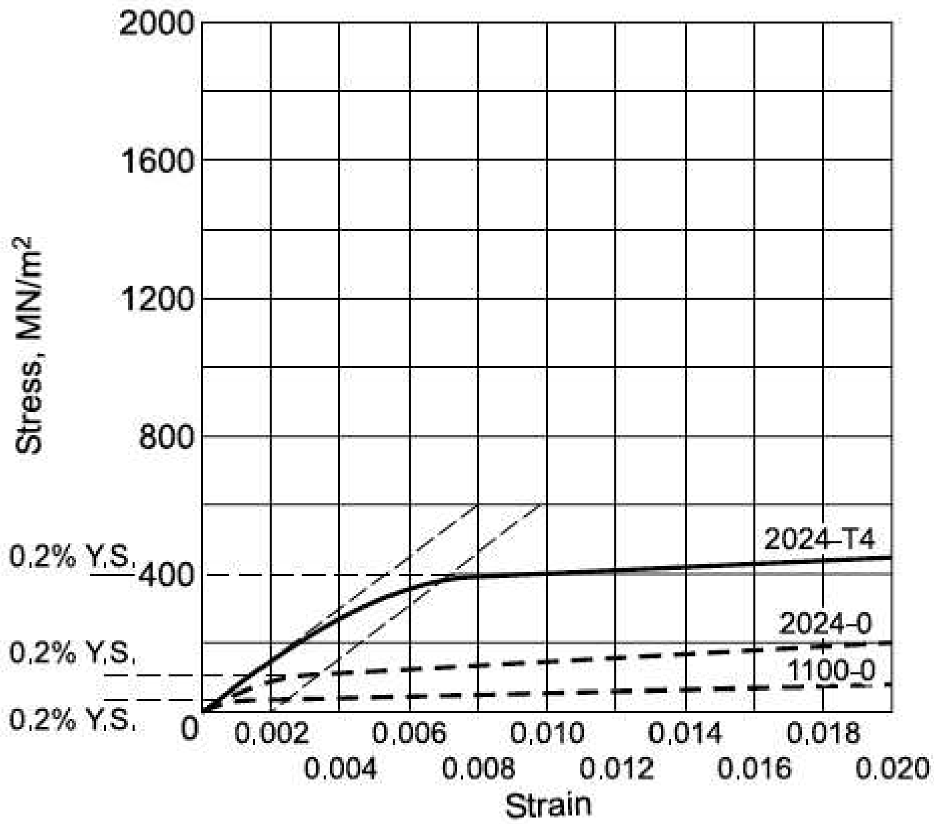
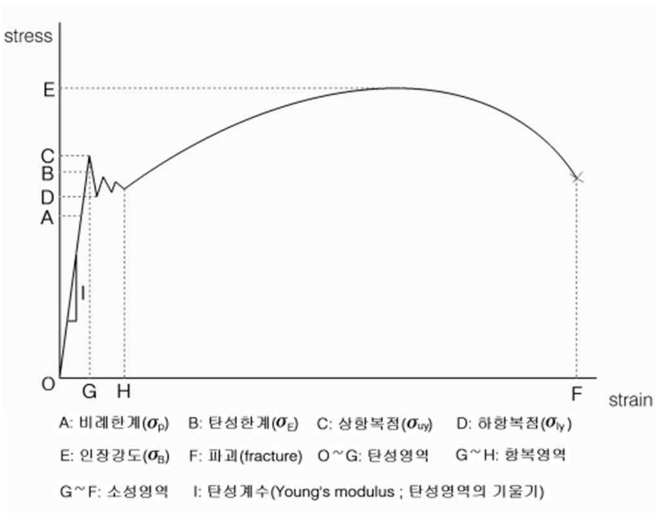
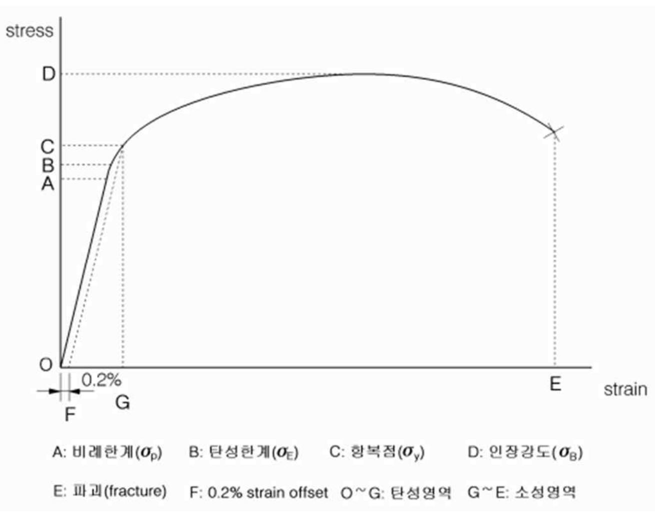


Fig. 5.5(b) *Stress-strain curves for aluminum and two aluminum alloys.*

- - - - *Commercially pure aluminum, annealed (1100-0)*
– – *4.6% Cu, 1.5% Mg, 0.7% Mn, balance Al, annealed (2024-0)*
—— *4.6% Cu, 1.5% Mg, 0.7% Mn, balance Al (2024-T4)*
Water quenched from 490°C, aged 24 hr at 120°C





1 ▷ Proportional limit

The greatest stress for which the stress is still proportional to the strain

2 ▷ Elastic limit

The greatest stress which can be applied without resulting in any permanent strain on release of stress

cf. For the materials shown in Fig. 5.5, the proportional and elastic limits coincide.

cf. Neither the proportional nor the elastic limits can be determined precisely, for they deal with the limiting cases of zero deviation from linearity and of no permanent set.

3 ▷ Yield strength

Standard practice to report a quantity called the yield strength, which is the stress required to produce a certain arbitrary plastic deformation

cf. Yield strength ≡ Offset yield stress (ductile material)

4 ▷ Upper yield point

Plastic deformation first begins

→ The upper yield point is very sensitive to rate of loading and accidental bending stresses or irregularities in the specimen. (from the Cottrell effect in the mechanics of materials)

5 ▷ Lower yield point

Subsequent plastic deformation which occurs at a lower stress

→ Because L.Y.P is the material property, so the lower yield point should be used for design purposes.

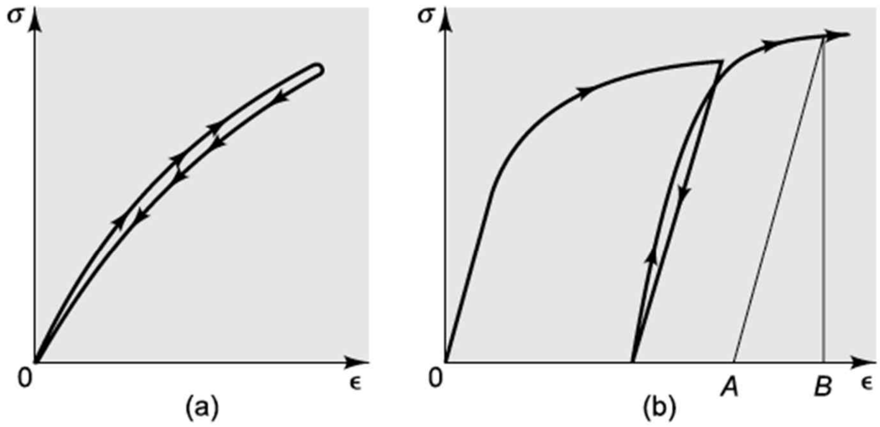


Fig. 5.6 *Effect of unloading and reloading. (a) Elastic; (b) plastic*

6 ▷ Flow stress(Strength)

As plastic deformation is continued, the stress required for further plastic flow, termed the flow strength, rises.

7 ▷ Strain hardening

The characteristic of the material in which further deformation requires an increase in the stress usually is referred to as strain-hardening of the material.

8 ▷ Brittle material

For glass, its behavior is entirely elastic and the stress at which fracture occurs is much greater in compression than in tension.

→ This is a usual characteristic of brittle materials.

9 ▷ From Fig. 5.6 (b)

The result of loading and unloading;

cf. In materials at room temperature, the strain rate change can be observed but is much less.

cf. For most ductile materials the stress-strain curves for tension and compression are nearly the same for strains small compared to unity, and in the following theoretical developments we shall assume that they are identical

5.3 Idealization of Stress-Strain Curve

→ Because we wish the mathematical part of our analysis to be as simple as possible, consistent with physical reality, we shall idealize the stress-strain curves of Fig. 5.5 into forms which can be described by simple equations.

→ The appropriateness of any such idealization will depend on the magnitude of the strains being considered, and this in turn will depend upon whatever practical problem is being studied at the moment.

→ In many cases we must design structures so as to accommodate or produce certain desired deformations. Examples of such applications are the design of springs, safety valves, bumpers, crash panels, shear pins, and so on.

→ Springs must accommodate the desired deformations repeatedly and reproducibly. In such cases the material must operate below the elastic limit.

→ Crash panels and automobile bumpers should not deform permanently under normal usage, but should deform plastically and so limit deceleration in case of an accident. Here an approximation is needed for both the elastic and plastic regions.

→ Shear pins and rupture discs are sacrificial parts, which are intended to fracture completely at certain loads, and for such structures the elastic deformations may be of no importance at all.

→ Another use of the mechanics of deformable bodies is in the design of metal forming and metal cutting processes. In metal forming, the strains usually are relatively small compared to unity, and elastic springback may be a problem. In metal cutting, the strains may be even greater than unity, and elastic effects often are negligible.

→ Perhaps the most important use of the mechanics of deformable bodies occurs in designing elements so that failure will not occur. Failures include excessive deformation, fracture, fatigue, mechanical wear, etc.

▶ Fracture

→ Fracture is the most dangerous mode of failure.

i) Brittle structures are those that fracture with little plastic deformation compared with the elastic deformation.

→ We may base all our calculations for these materials on a linear relation between stress and strain.

ii) For ductile structures there is as yet no quantitative theory which will predict fracture.

① Our lack of knowledge of the distributions of stress and strain in the plastic region in front of a crack

② Our lack of knowledge of strain around the holes that grow from inclusions and coalesce to cause fracture

→ It will be necessary to have available stress-strain relations which are reasonable approximations for large plastic strains.

iii) Fatigue: the repetitions of stress eventually produce fine cracks which grow very slowly at first and then extend rapidly across the entire part.

→ Since fatigue can occur even if the stresses are below the yield strength, it is sufficient for most practical design purposes to know the relation between the stress and the strain within the elastic region.

cf. Safety factor "n"

$$n = \frac{\text{Actual strength } (= \text{Strength of material})}{\text{Required strength } (= \text{Maximum allowable stress})} > 1$$

▶ Other failures

i) A small corrosion pit will cause a local stress concentration which will in turn create an electromotive force between the highly stressed and the less stressed regions. This electromotive force in turn accelerates the corrosion, and the process can lead to the development of cracks and final fracture of the part.


→ As in fatigue, the phenomenon may occur when stresses are below the yield strength so that the elastic stress-strain assumptions are of practical use.

ii) The most common form of mechanical failure is by wear. The laws governing the overall friction and wear between two surfaces seem to depend primarily on the total force transmitted across the two surfaces rather than on the local distribution of the force.

→ The local distributions of stress and strain are unimportant, and one may assume that the two bodies in contact are perfectly rigid.

The preceding discussion of problems arising in the mechanics of solids shows that there is a need for a variety of stress-strain relations, depending on the problem at hand.

▶ Six ideal model

1 ▷ Rigid material

i) A rigid material is one which has no strain regardless of the applied stress.

ii) This idealization is useful in studying the gross motions and forces on machine parts to provide for adequate power and for resistance to wear.

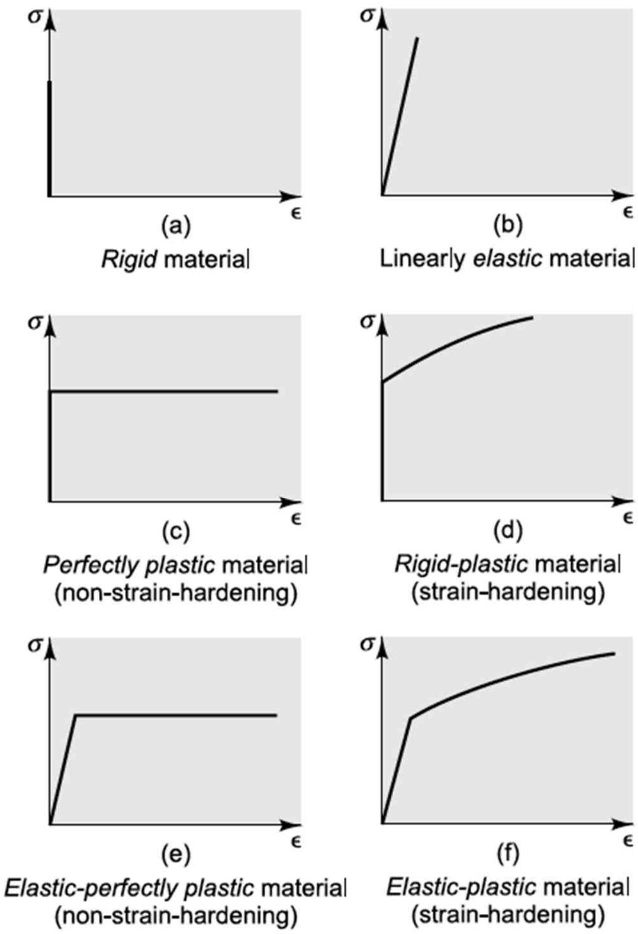


Fig. 5.7 *Idealized models of material behavior*

2 ▷ Linearly elastic material

i) A linearly elastic material is one in which the strain is proportional to the stress.

ii) This idealization is useful when we are designing for small deformations, for stiffness, or to prevent fatigue or fracture in brittle structures.

3 ▷ Rigid plastic material

i) A rigid-plastic material is one in which elastic and time-dependent deformations are neglected.

ii) Such idealizations are useful in designing structures for their maximum loads and in studying many machining and metal-forming problems, and in some detailed studies of fracture.

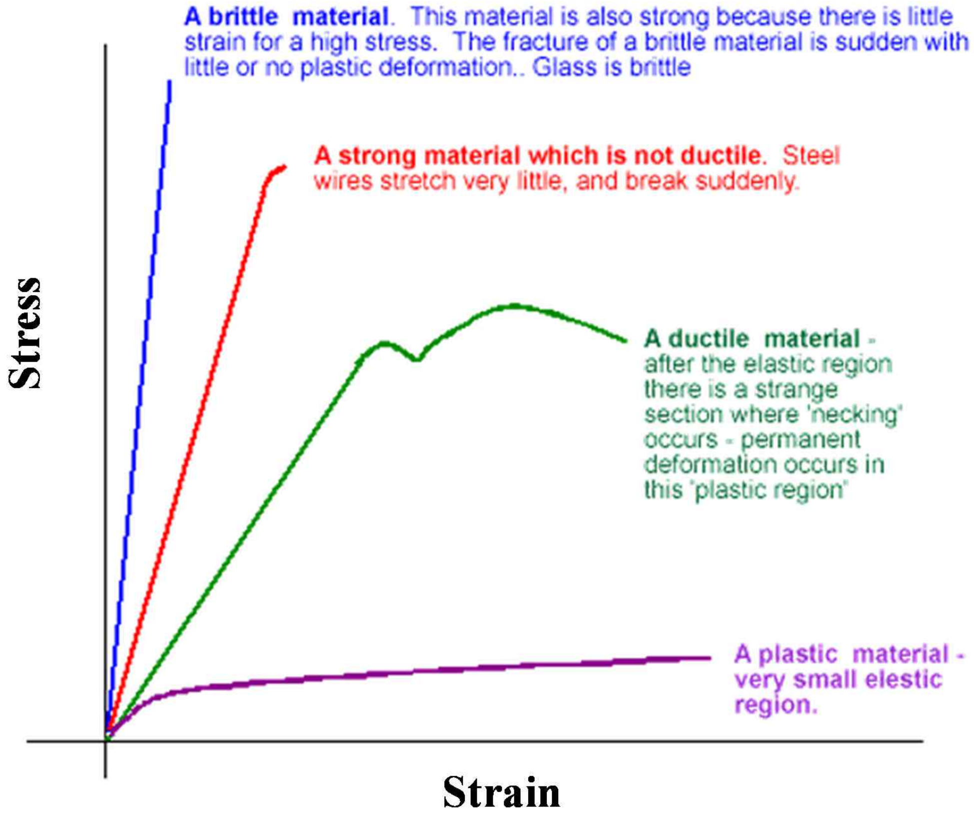
cf. The material that strain-hardening may be neglected (Fig. 5.7 (c)) is termed perfectly plastic material.

4 ▷ Elastic-plastic material

i) An elastic-plastic material is one in which both elastic and plastic strains are present; strain-hardening may or may not be assumed to be negligible (Figs. 5.7 (f) and (e)).

ii) These idealizations are useful in designing against moderate deformations when carrying out detailed studies of the mechanisms of fracture, wear, and friction.

▶ Examples of the behaviors of other materials



Other idealizations could be made, but these models are those of most practical use from the standpoint of mathematical simplicity. To illustrate the construction of an idealized stress-strain curve of a material and also to illustrate the use of such a curve, we consider the following example.

▶ <u>Example 5.1</u> Two coaxial tubes, the inner one of 1020 CR steel and cross-sectional area A_s, and the outer one of 2024-T4 aluminum alloy and of area A_a, are compressed between heavy, flat end plates, as shown in Fig 5.8. We wish to determine the load-deflection curve of the assembly as it is compressed into the plastic region by an axial force P.

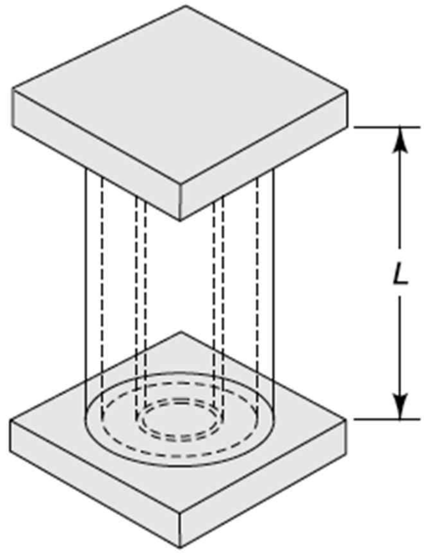


Fig. 5.8 *Example 5.1*

▷ Geometry

$$\epsilon_s = \epsilon_a = \epsilon = \delta / L \qquad \text{(a)}$$

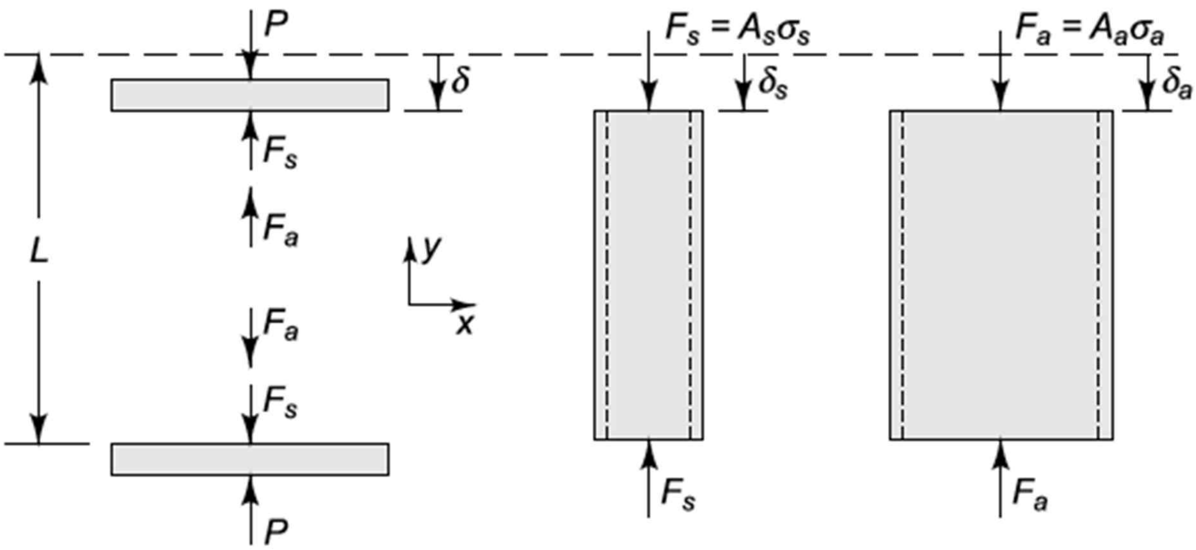


Fig. 5.9 *Idealized model for Example 5.1*

▷ Stress-strain relation

→ We can, with reasonable accuracy, idealize both these curves as being of the elastic-perfectly plastic type of Fig. 5.7 (e).

→ From Fig 5.10, we see that there are three regions of strain which are of interest as we compress the assembly.

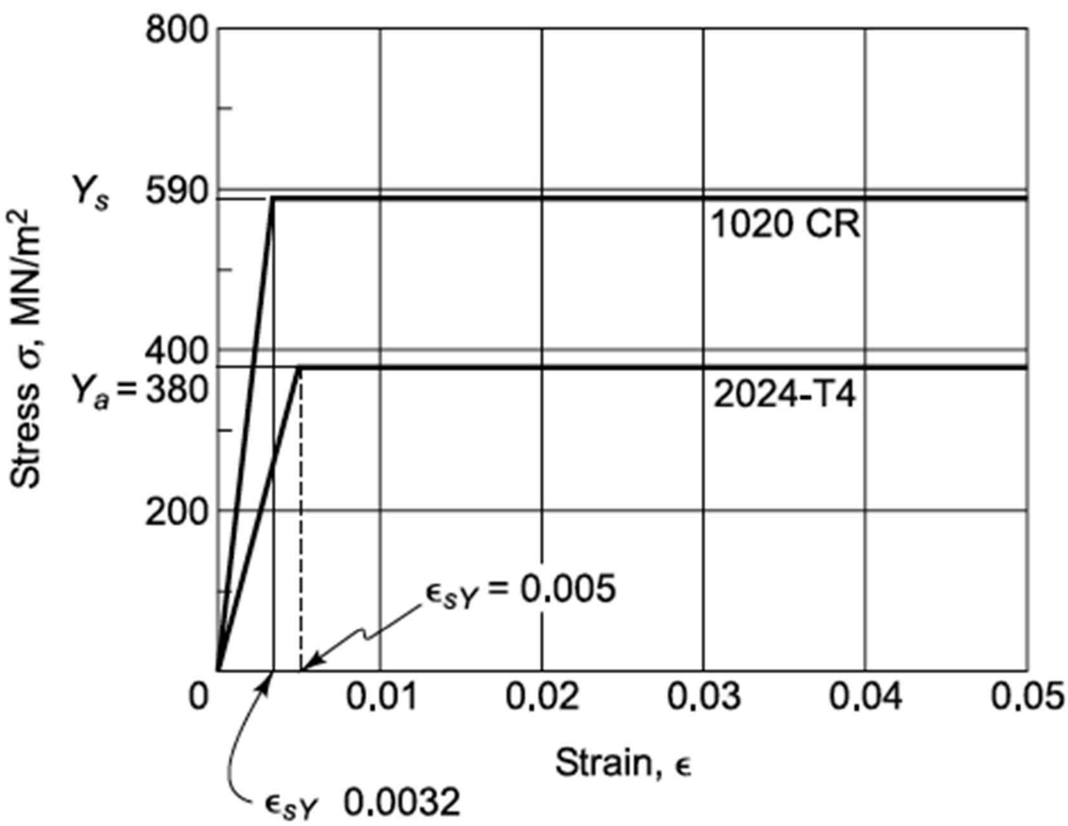


Fig. 5.10 *Idealized stress-strain curves for Example 5.1*

i) For $0.0000 \le \epsilon \le 0.0032$

$$\begin{cases} \sigma_s = E_s \epsilon_s = E_s \epsilon \\ \sigma_a = E_a \epsilon_a = E_a \epsilon \end{cases} \tag{b}$$

where, $\begin{cases} E_s = \dfrac{590}{0.0032} = 184\ \mathrm{GN/m^2} \\ E_a = \dfrac{380}{0.005} = 76\ \mathrm{GN/m^2} \end{cases}$

ii) For $0.0032 \le \epsilon \le 0.0050$

$$\begin{cases} \sigma_s = Y_s = 590\ \mathrm{MN/m^2} \\ \sigma_a = E_a \epsilon_a = E_a \epsilon \end{cases} \tag{c}$$

iii) For $0.0050 \le \epsilon$

$$\begin{cases} \sigma_s = Y_s = 590\ \mathrm{MN/m^2} \\ \sigma_a = Y_a = 380\ \mathrm{MN/m^2} \end{cases} \tag{d}$$

▷ Equilibrium

i) In Fig. 5.9 the top plate is in equilibrium when

$$\sum F_y = \sigma_s A_s + \sigma_a A_a - P = 0 \tag{e}$$

ii) Combining (e) with (b), (c), and (d) in succession, we obtain the load deformation curve of Fig. 5.11.

cf. We now turn to the generalization of these idealized uniaxial stress-strain relations for application to more general situations, where any or all components of stress and strain may be present.

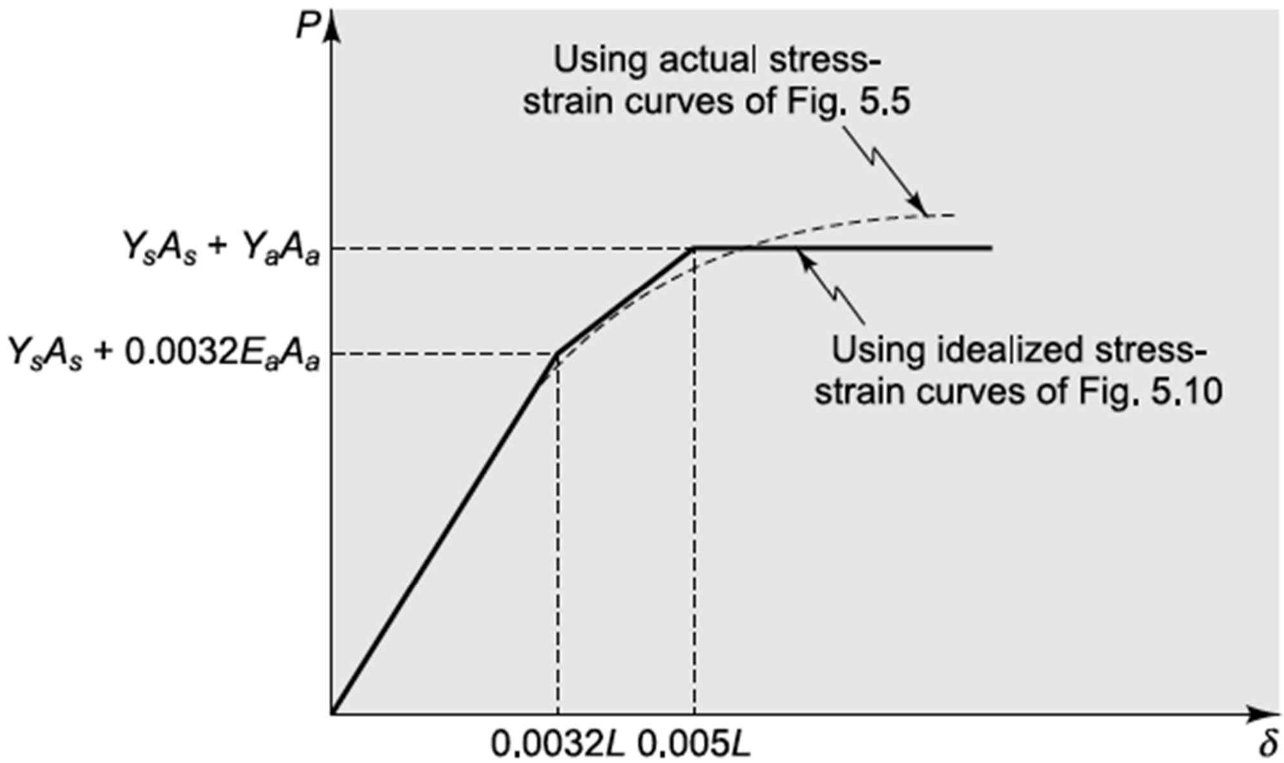


Fig. 5.11 *Load-deformation curve for Example 5.1*