5. Discontinuity spacing

▶ Introduction

-Definition: Distance between two joint intersections in a scanline

-Spacing and frequency: The frequency is the reciprocal of a mean spacing. The reciprocal of a single spacing is called repetition (or repetition value).

- Spacing
  - Total spacing: Distance between two adjacent joint intersections regardless of joint sets (X_t)
  - Set spacing: Distance between two adjacent joint intersections belonging to the same joint set (X_d)
  - Normal set spacing: Set spacing measured along the joint normal (X_n)

- Expression: $X_n = X_d \cos\theta$ cf. $\lambda_L = \dfrac{\lambda_l}{\cos\theta}$

(θ: acute angle between a scanline and joint normal)

$\lambda_S = \dfrac{1}{\overline{X_t}}$ (total linear frequency is the reciprocal of the mean total spacing)

$\lambda_l = \dfrac{1}{\overline{X_d}}$ ((set) linear frequency is the reciprocal of the mean set spacing)

$\lambda_L = \dfrac{1}{\overline{X_n}}$ (normal linear frequency is the reciprocal of the mean normal set spacing)

▶ Discontinuity spacing distributions

- Poisson distribution of joint intersections: Combined joint intersections of all joint sets are randomly located along scanlines on the whole. (refer to Fig. 5.4, p.127)

- Spacing follows negative exponential: (total) Spacing turned out by theoretical and observational approach to follow a negative exponential distribution when the joint intersections are randomly located.

Theoretical approach: When the number of joint intersections obeys Poisson distribution the probability that k of the intersections are located in a scanline whose length is x is as follow (λ is a linear frequency).

$$P(k,x) = \frac{e^{-\lambda x}(\lambda x)^k}{k!}$$

Then, $f(x)$, the probability density function of $P'(x)$ indicating a probability that total spacing X_t becomes x is as follow.

$$f(x) = \frac{d}{dx}F(x) = \frac{d}{dx}P'(X_t \leq x) = \frac{d}{dx}(1 - P(0,x)) = \frac{d}{dx}(1 - e^{-\lambda x}) = \lambda e^{-\lambda x}$$

: Negative exponential distribution (mean= $\int_0^\infty \lambda x e^{-\lambda x} dx = \frac{1}{\lambda}$)

▶ Rock Quality Designation

-Definition: Percentage of the summed length of core pieces which are longer than 10㎝.

- RQD and spacing/frequency: total spacing and total frequency are closely related with RQD.

-TRQD (Theoretical RQD)

Let the random variable of total spacing be x and PDF of x be $f(x)$. When the length of rock core is L,

① Total frequency: $\lambda = 1/\overline{x}$

No. of joint intersections: λL

No. of joint pairs whose spacing is x:$\lambda L f(x) dx$

② No. of joint pairs whose spacing $x > t$: $\int_t^L \lambda L f(x) dx$

Sum of spacing $x > t$ $\int_t^L \lambda L x f(x) dx$

③ Let threshold of RQD be t. Then TRQD$_t$ becomes:

$$TRQD_t = \frac{100}{L}\int_t^L \lambda L x f(x) dx = 100\lambda \int_t^L x f(x) dx$$

④ When the spacing obeys negative exponential distribution:

$$\begin{aligned} TRQD_t &= 100\lambda^2 \int_t^L x e^{-\lambda x} dx \qquad (L \text{ is large enough}) \\ &= 100\left[e^{-\lambda t}(1 + \lambda t) - e^{-\lambda L}(1 + \lambda L)\right] \\ &= 100 e^{-\lambda t}(1 + \lambda t) \\ &= 100 e^{-t/\overline{x}}(1 + t/\overline{x}) \end{aligned}$$

- TRQDt vs. Mean spacing: refer to Fig. 5.5 (p.130).

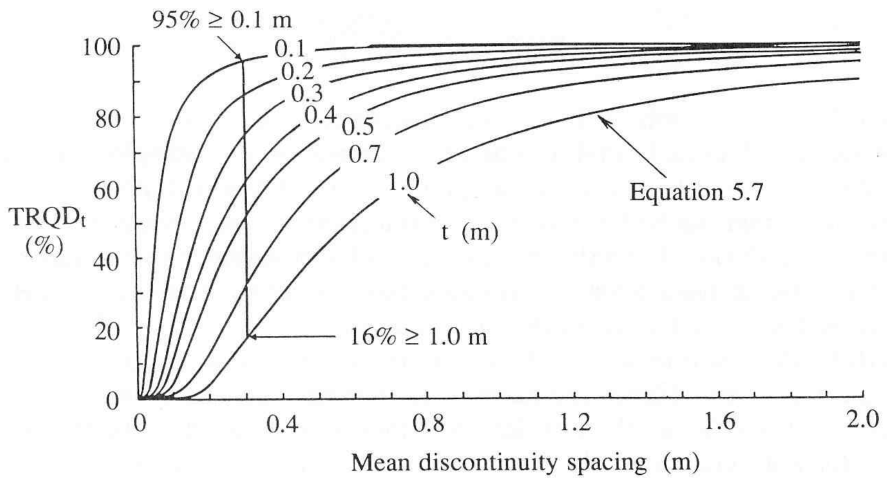

Figure 5.5 Variation of $TRQD_t$ with mean discontinuity spacing for a range of $TRQD_t$ threshold values t (after Priest and Hudson, 1976).

- TRQDt vs. Frequency: refer to Fig. 5.6 (p.131).

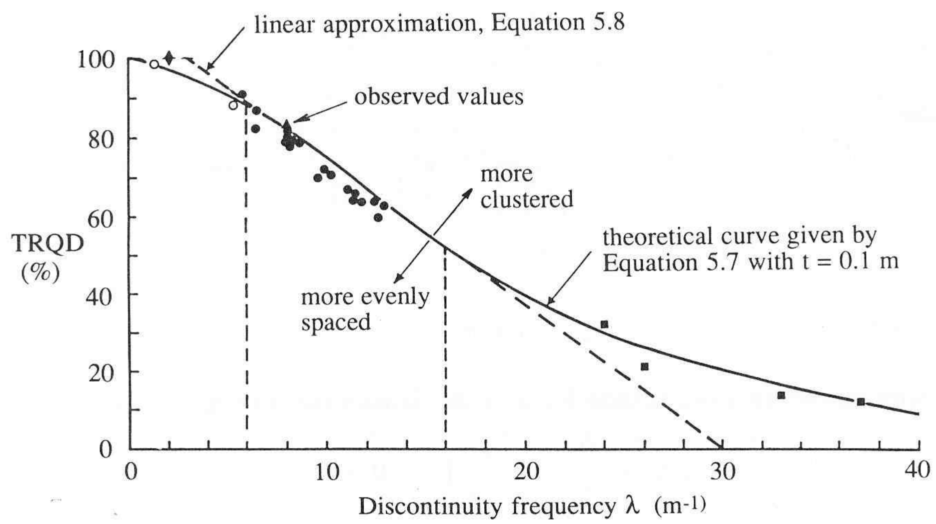

Figure 5.6 Relation between TRQD and mean discontinuity frequency (after Priest and Hudson, 1976).

$$TRQD_{0.1} \approx 110.4 - 3.68\lambda \tag{5.8}$$

The above equation provides a reasonable approximation for the range $6 < \lambda < 16\,m^{-1}$. It is worth noting that the ISRM (1978) has proposed the following approximate relation between RQD and the volumetric joint count J_v

$$\begin{aligned} RQD &= 115 - 3.3J_v && \text{for } J_v \geqslant 4.5\,m^{-1} \\ RQD &= 100 && \text{for } J_v < 4.5\,m^{-1} \end{aligned} \tag{5.9}$$

where J_v is defined as the sum of the number of joints per metre for each joint set present. The similarity between equation 5.8, which is based on funda-

▶ Accuracy and precision of discontinuity spacing estimates

- Error in spacing measurement:

Inaccuracy: Consistent error caused by persistent factor. ex) Spacing greater than L cannot be measured by a scanline of which length is L.

Imprecision: Inconsistent error caused by sampling size. As the sampling size increases the variance of mean spacing becomes smaller.

① Inaccuracy caused by short sampling lines

- Curtailment: Ignoring some samples whose values are greater than a certain level. It is classified into two groups according to whether the samples are counted/recorded or not: truncated (not counted) and censored (counted). Spacing greater than L is always truncated by a scanline of which length is L.

- Mathematical expression of curtailment

$f(x) \rightarrow f_L(x)$: $f_L(x) = \frac{f(x)}{F(L)}$

When $f(x) = \lambda e^{-\lambda x}$ $\quad f_L(x) = \frac{\lambda e^{-\lambda x}}{1 - e^{-\lambda L}}$

$$\bar{x} = \mu_{xL} = \frac{\lambda}{(1 - e^{-\lambda L})} \int_0^L x e^{-\lambda x}\, dx = \frac{1 - e^{-\lambda L} - \lambda L e^{-\lambda L}}{\lambda (1 - e^{-\lambda L})}$$

Eqn.(5.13) p.135

(As $L \rightarrow \infty$, $\bar{x}$ becomes $\frac{1}{\lambda}$)

$$\begin{aligned} Var(x) &= E(x^2) - (E(x))^2 \\ &= M_{x2} - \mu_{xL}^2 \\ &= \frac{\lambda}{(1 - e^{-\lambda L})} \int_0^L x^2 e^{-\lambda x}\, dx - \mu_{xL}^2 \\ &= \frac{2 - e^{-\lambda L}(2 + 2\lambda L + \lambda^2 L^2)}{\lambda^2 (1 - e^{-\lambda L})} - \mu_{xL}^2 \end{aligned}$$

(As $L \rightarrow \infty$, $Var(x)$ becomes $\frac{1}{\lambda^2}$)

Eqn.(5.14) p.135

$$\begin{aligned} TRQD_t &= 100\lambda' \int_t^L x f_L(x) dx \\ &= \frac{100}{\bar{x}} \int_t^L x f_L(x) dx \\ &= \frac{100}{1 - e^{-\lambda L} - \lambda L e^{-\lambda L}} \left(e^{-\lambda t}(1 + \lambda t) - e^{-\lambda L}(1 + \lambda L) \right) \end{aligned}$$

Eqn.(5.15) p.135

Fig. 5.7 p.136, Fig.5.8, Fig. 5.9

② Imprecision caused by small sample sizes

-Table 5.1 (p.141), Fig. 5.10 (p.142)
-As the scanline length increases the sample size of spacing also increases addressing the imprecision problem
-Application of the central limit theorem (a sample mean converges to $N(\mu,\sigma^2/n)$) to mean spacing enables us to estimate the confidence range of the mean spacing by using the standard normal distribution.