Chapter 6 Estimation of deflection

6.1 Introduction

-Adequate strength
-Deflection: secondary concern
-Series of elastic analysis: Hinge by hinge method
-Deflection at collapse in one-step analysis:
slope deflection and virtual work method
-Deflection theorem

6.2 Deflection at collapse and working load

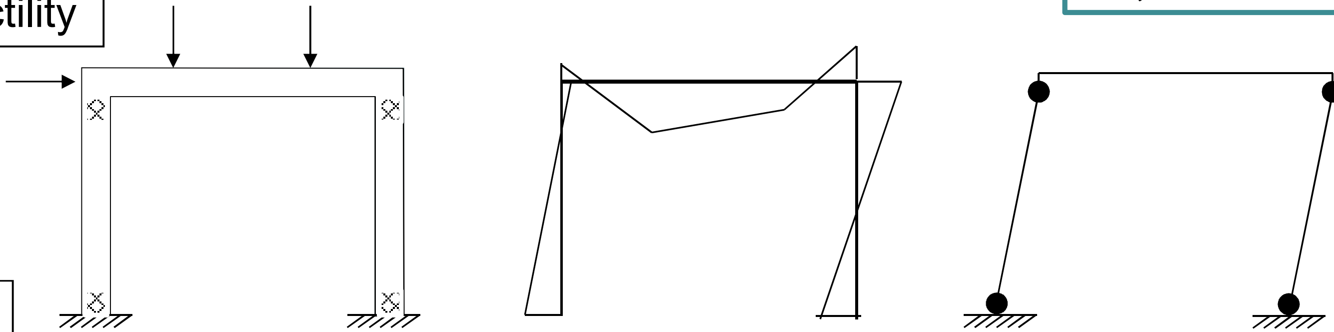
$$\delta_w = \frac{\delta_c}{\lambda}$$

Ductility requirement

$$\frac{\Delta_p}{\Delta_y} \approx (\mu_\Delta - 1)\frac{l}{l'} = \frac{(\mu_{c\Delta} - 1)\Delta_{cm}}{\Delta_b + \Delta_j + (\Delta_{cm} + \Delta_{cv}) + \Delta_f}$$

Target System ductility

$$\mu_\Delta \cong R$$



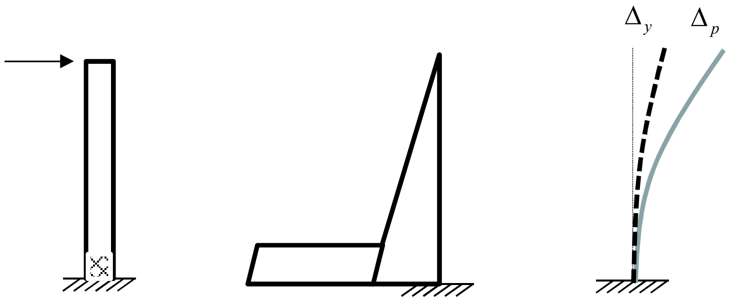
Member ductility

$$\mu_{column}$$

$$\mu_{c\Delta} = (\mu_\Delta - 1)\frac{l}{l'}\left(\frac{\Delta_b + \Delta_j + \Delta_{cv} + \Delta_f}{\Delta_{cm}} + 1\right) + 1$$

Section ductility

$$\mu_{cur}$$



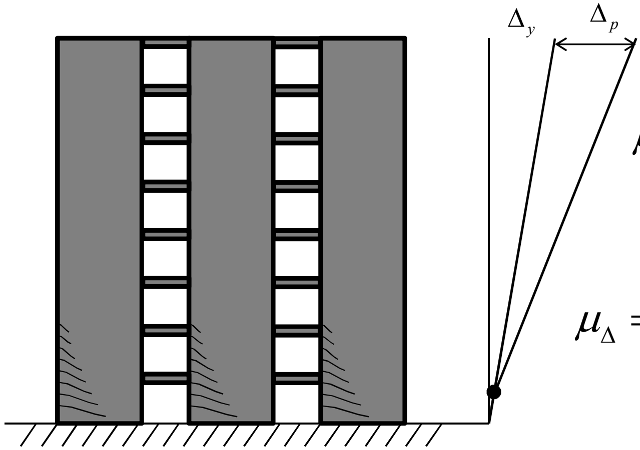
$$\Delta_y = \frac{\phi_y l^2}{3}$$

$$\Delta_p = \theta_p (l - 0.5 l_p) = (\phi_m - \phi_y) l_p (l - 0.5 l_p)$$

$$\mu_\Delta = 1 + 3(\mu_\phi - 1)\frac{l_p}{l}\left(1 - 0.5\frac{l_p}{l}\right)$$

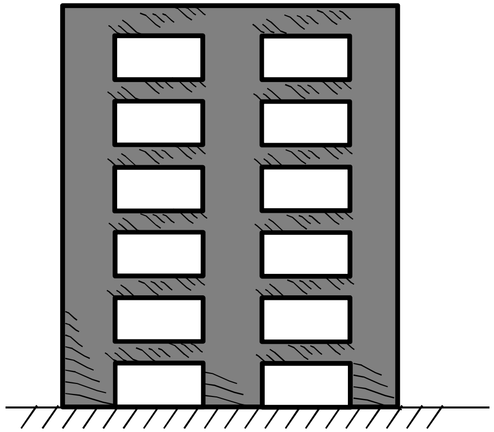
$$\mu_\phi = 1 + \frac{(\mu_\Delta - 1)}{3\frac{l_p}{l}\left(1 - 0.5\frac{l_p}{l}\right)}$$

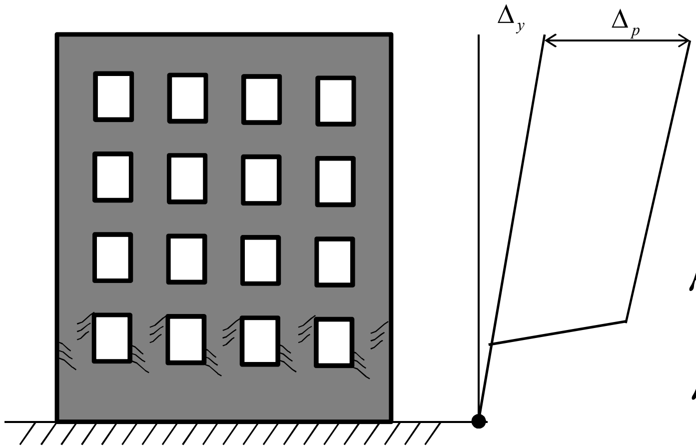
Masonry structure systems



$$\mu_\Delta = 1 + \frac{\theta_p}{\Delta_y}\left(h_w - \frac{l_p}{2}\right)$$

$$\mu_\Delta = 1 + \frac{3}{2A_r}\left(\mu_\phi - 1\right)\left(1 - \frac{1}{4A_r}\right)$$



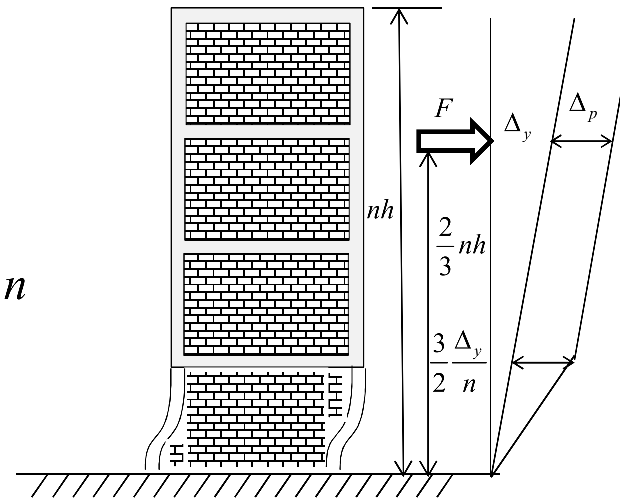


$$\Delta_p = \left(\mu - 1\right)\Delta_y$$

$$\Delta_{y1} = \frac{\Delta_y}{2n}$$

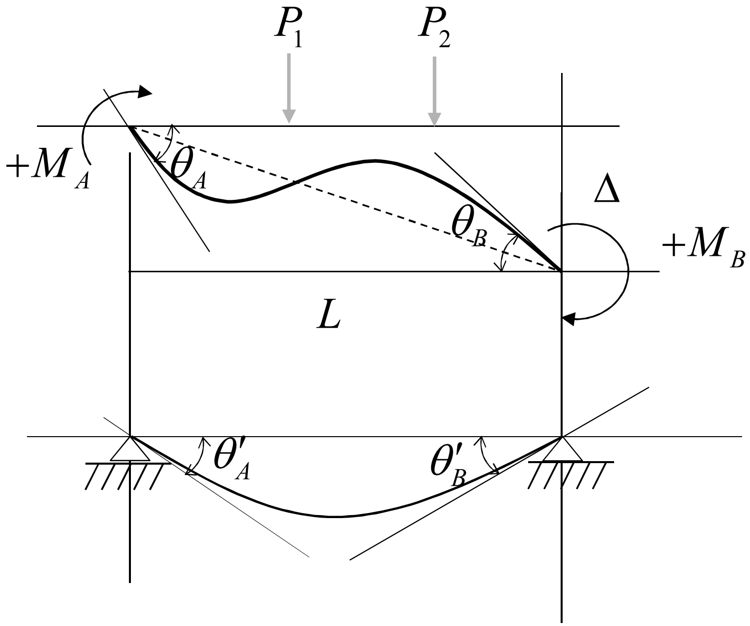
$$\mu_\Delta = 1 + 0.5\left(\mu_{\Delta p} - 1\right)/n$$

$$\mu_{\Delta p} = 2n\left(\mu_\Delta - 1\right) + 1$$

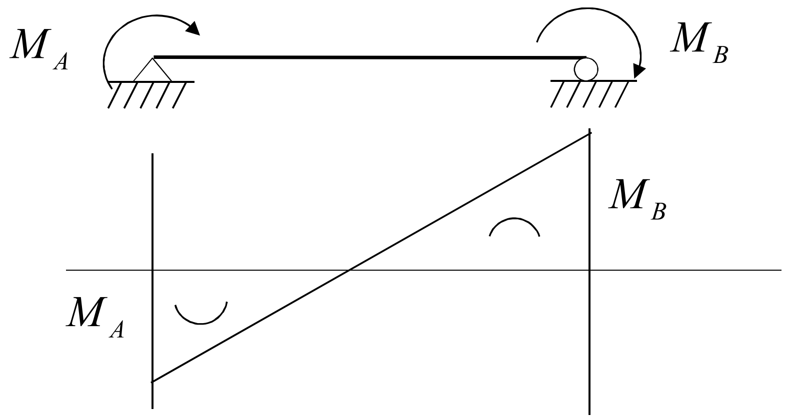


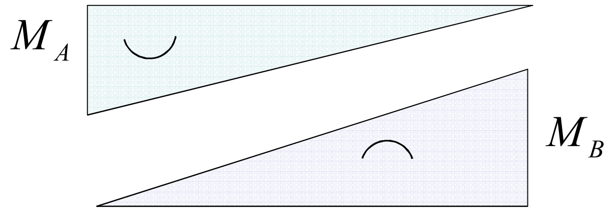
$$\mu_\Delta = 1 + \frac{\Delta_p}{\Delta_y}$$

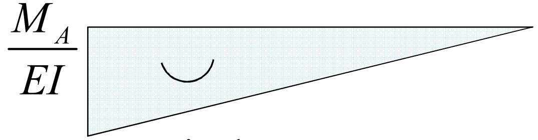
$$\mu_1 = 1 + \frac{2}{3}n\left(\mu_\Delta - 1\right)$$



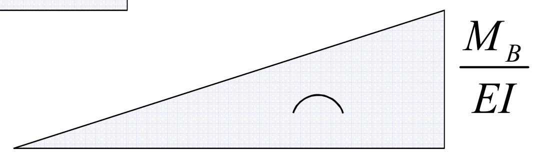
$$\theta_A = \theta'_A + \frac{\Delta}{L} + \text{due to end moments}$$



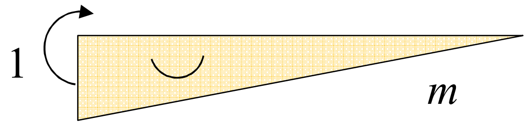




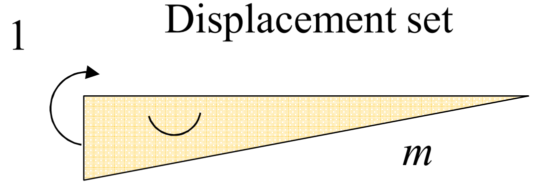
Displacement set



Displacement set



Equilibrium set



Equilibrium set

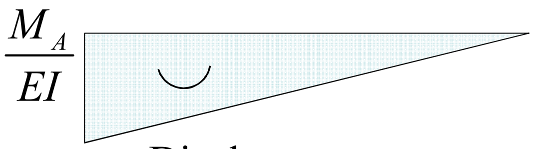
$$\theta''_A = \frac{1}{3}\frac{M_A}{EI}L$$

$$\theta''_B = -\frac{1}{2}\frac{M_B}{EI}L\frac{1}{3} = -\frac{1}{6}\frac{M_B L}{EI}$$

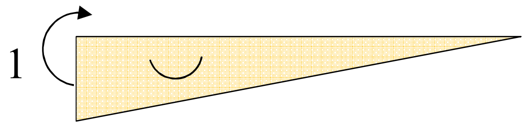
$$\theta_A = \theta'_A + \frac{\Delta}{L} + \text{due to end moments}$$

$$\theta_A = \theta'_A + \frac{\Delta}{L} + \theta''_A$$

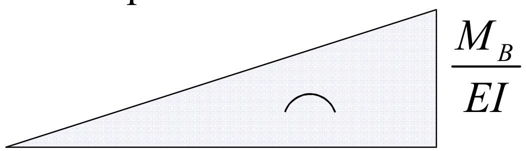
$$= \theta'_A + \frac{\Delta}{L} + \frac{1}{3EI}\left(M_{AB} - \frac{1}{2}M_{BA}\right)$$



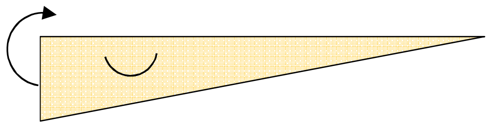
Displacement set



m Equilibrium set



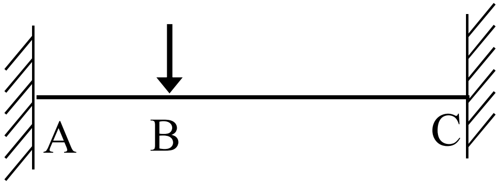
Displacement set

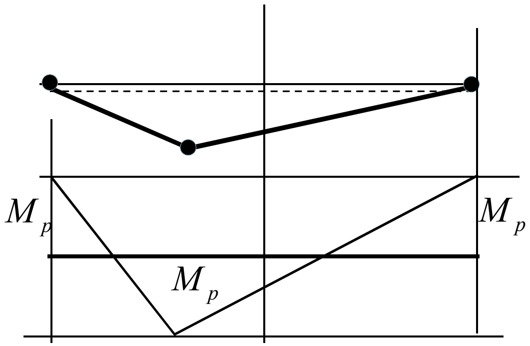


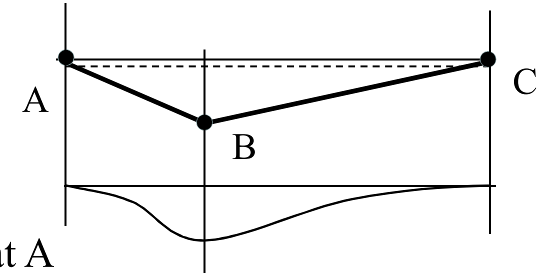
m Equilibrium set

Ex. 6.3.1

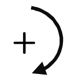
Find the deflection at B at collapse.
Assume the last PH to form, in turn at
A, B, and C





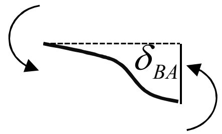


Last PH at A



Sign convention

$M_{AB} = -M_p$



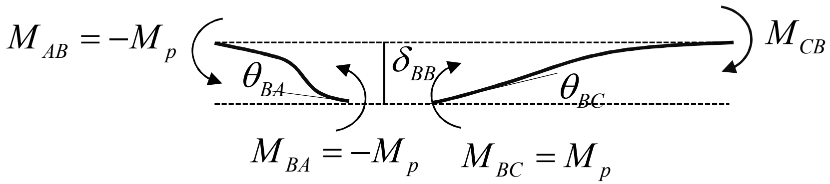
$M_{BA} = -M_p$

$$\theta_A = \theta'_A + \frac{\delta_{BA}}{L/3} + \frac{L/3}{3EI}\left(M_{AB} - \frac{M_{BA}}{2} \right) = 0$$

$$\frac{\delta_{BA}}{L/3} + \frac{L}{9EI}\left(-M_p + \frac{M_p}{2} \right) = 0$$

$$\delta_{BA} = \frac{M_p L^2}{54EI}$$

Last PH at B



$$\theta_{BA} = \frac{\delta_{BB}}{L/3} + \frac{L/3}{3EI}\left(-M_p + \frac{M_p}{2}\right)$$

$$= \frac{3\delta_{BB}}{L} - \frac{M_p L}{18EI}$$

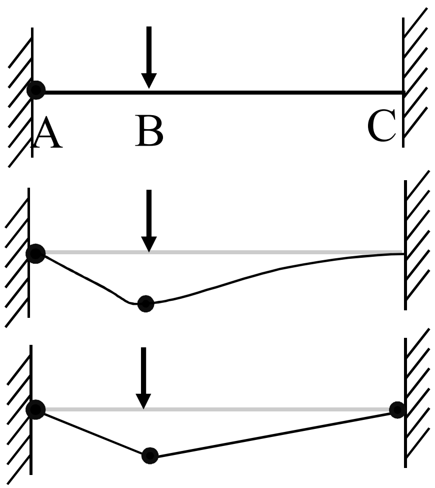
$$\theta_{BC} = -\frac{\delta_{BB}}{2L/3} + \frac{2L/3}{3EI}\left(M_p - \frac{M_p}{2}\right)$$

$$= -\frac{3\delta_{BB}}{2L} + \frac{M_p L}{9EI}$$

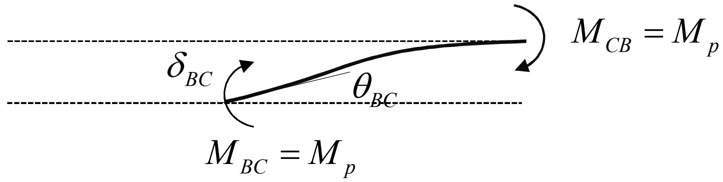
Since $\theta_{BA} = -\theta_{BC}$

$$\frac{9\delta_{BB}}{2L} = \frac{3M_p L}{18EI}$$

$$\delta_{BB} = \frac{M_p L^2}{27EI}$$



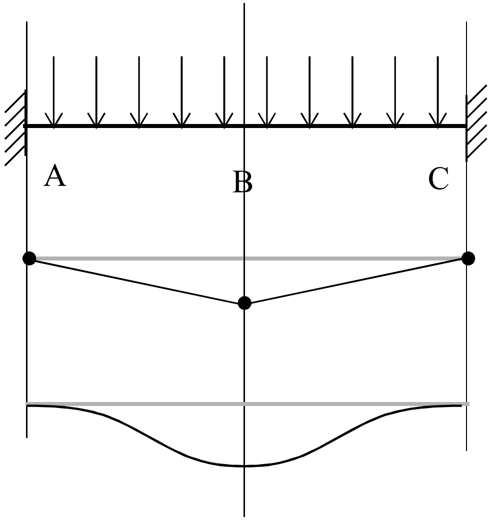
Last PH at C



$$\theta_C = -\frac{\delta_{BC}}{2L/3} + \frac{2L/3}{3EI}\left(M_p - \frac{M_p}{2}\right) = 0$$

$$\delta_{BC} = \frac{2M_p L^2}{27EI}$$

Last PH @	δ_B
A	1/54
B	1/27
C	2/27



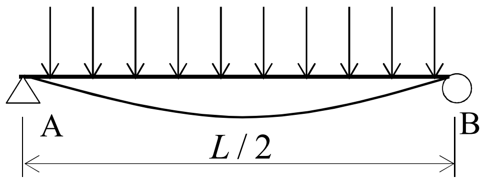
Last PH at A

$$M_{AB} = -M_p$$

$$M_{BA} = -M_p$$

$$\theta_A = \theta'_A + \frac{\delta_{BA}}{L/2} + \frac{L/2}{3EI}\left(-M_p + \frac{M_p}{2}\right) = 0$$

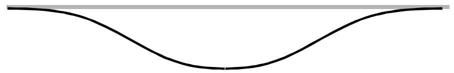
$$\theta'_A =$$



$$\theta'_A = \frac{M_p L}{12EI}, w = \frac{16M_p}{L^2}$$

$$\frac{M_p L}{12EI} + \frac{\delta_{BA}}{L/2} - \frac{L/2}{3EI}\frac{M_p}{2} = 0$$

$$\delta_{BA} = 0$$



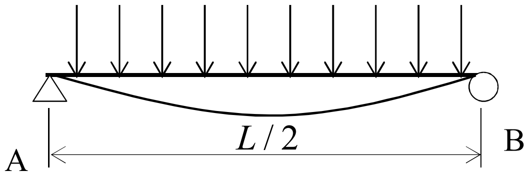
Last PH at B

$$M_{AB} = -M_p$$

$$M_{BA} = -M_p$$

$$\theta_{BA} = \theta'_B + \frac{\delta_{BB}}{L/2} + \frac{L/2}{3EI}\left(-M_p + \frac{M_p}{2}\right) = 0$$

$$\theta'_B =$$



$$\theta'_B = -\frac{M_p L}{12EI}, w = \frac{16M_p}{L^2}$$

$$-\frac{M_p L}{12EI} + \frac{\delta_{BB}}{L/2} - \frac{M_p L}{12EI} = 0$$

$$\delta_{BA} = \frac{M_p L^2}{12EI}$$

6.4 Dummy load method

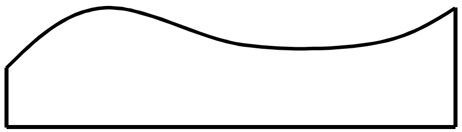
Unit load method

Virtual work method

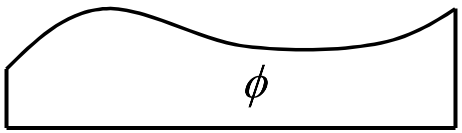
For given moment distribution → $\dfrac{M}{EI} = \phi$

Curvature distribution

Find δ at a specific point

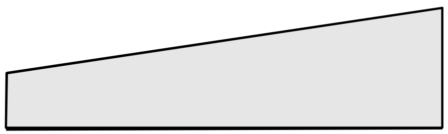


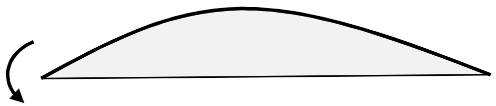
M



$\dfrac{M}{EI} = \phi$

1





$$1 \times \delta = \int m \times \phi dx$$

$$1 \times \delta = \int m \times \frac{M}{EI} dx$$

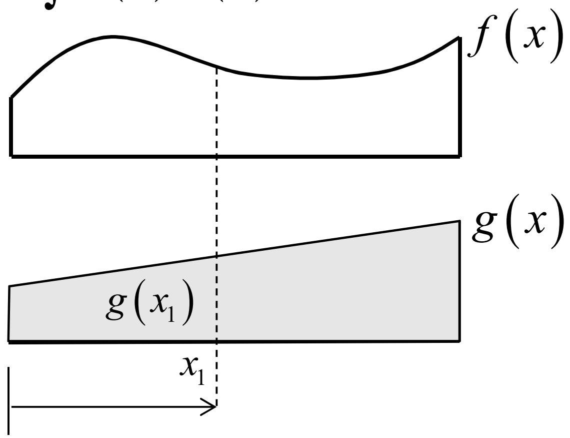
equilibrium set

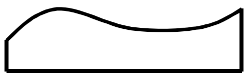
Displacement set

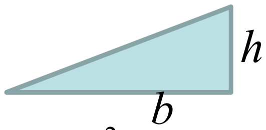
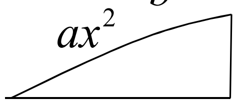
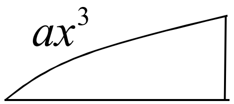
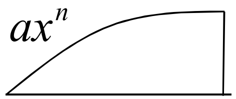
$$1 \times \theta = \int m \times \phi dx$$

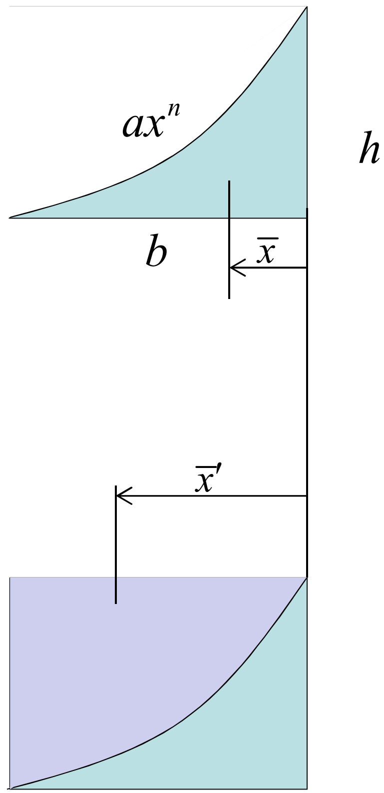
$$1 \times \theta = \int m \times \frac{M}{EI} dx$$

$$\int f(x)g(x)dx = ?$$



$\int fg dx =$  $\times g(x_1)$

	Area	Centroid
 h, b	$\frac{1}{2}bh$	$\frac{2}{3}b$
 ax^2	$\frac{2}{3}bh$	$\frac{5}{8}b$
 ax^3	$\frac{3}{4}bh$	$\frac{3}{5}b$
 ax^n	$\frac{n}{n+1}bh$	$\frac{n+3}{2(n+2)}b$



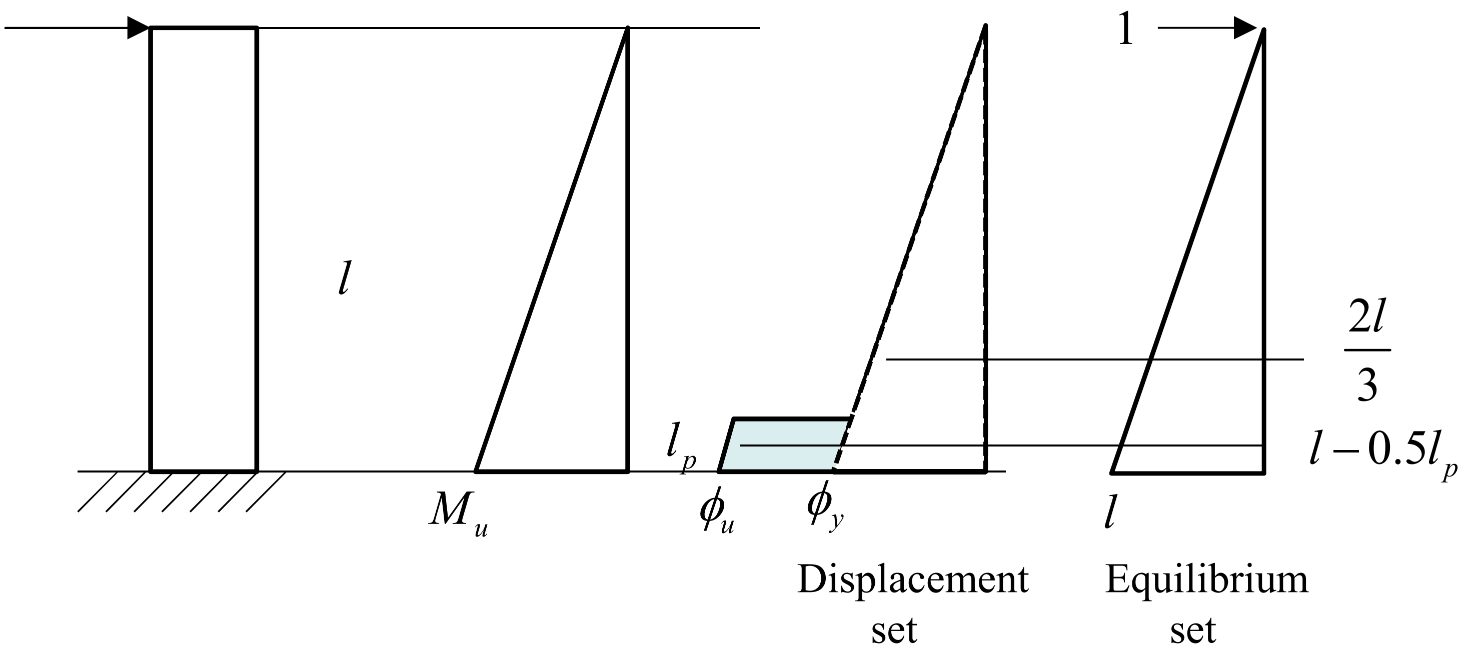
$$A = \frac{1}{n+1}bh$$

$$\overline{x} = \frac{b}{n+2}$$

$$bh\frac{b}{2} = bh\frac{1}{n+1}\frac{b}{n+2} + bh\left(1 - \frac{1}{n+1}\right)\overline{x}'$$

$$\overline{x}' = \left[\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right]\frac{n+1}{n}b$$

$$= \frac{n^2+3n}{2n(n+2)}b = \frac{n+3}{2(n+2)}b$$

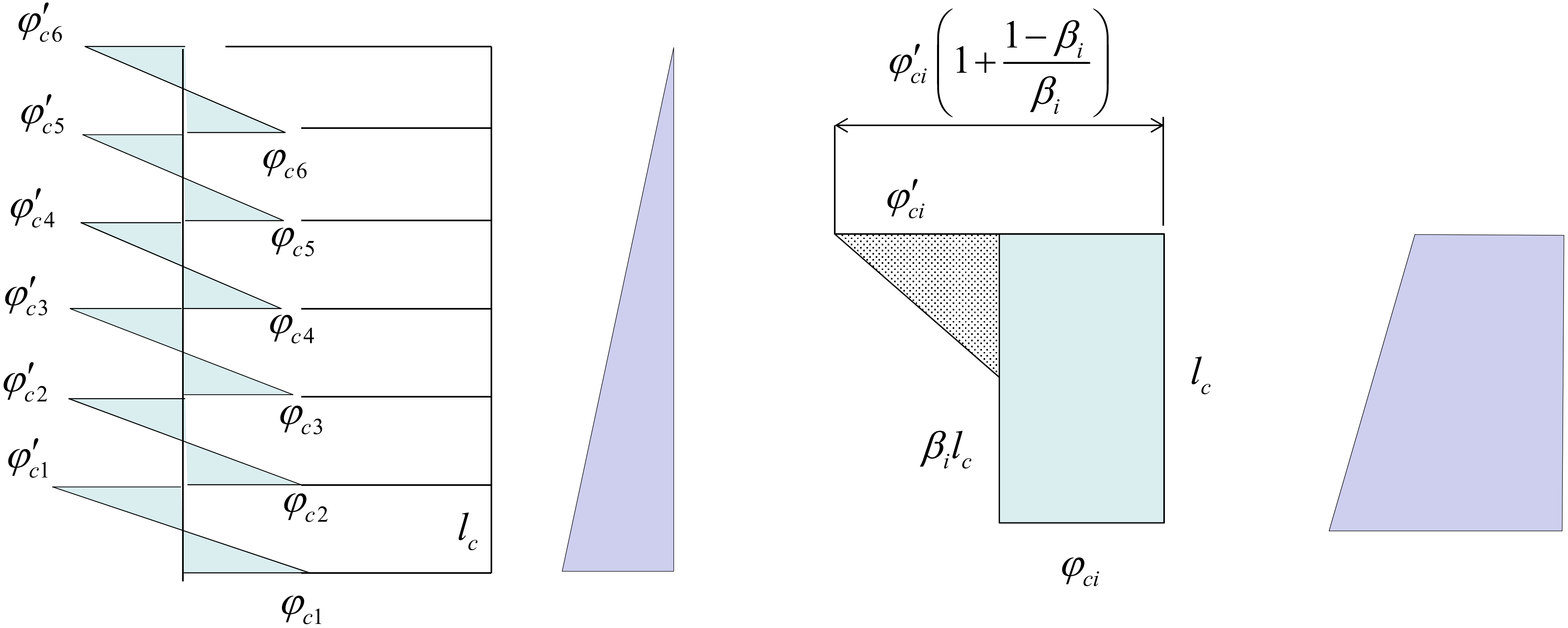


$$1\times\delta=\int m\times\phi dx$$

$$1\times\delta=\int m\times\frac{M}{EI}dx$$

$$\Delta_u=\left(\frac{\phi_y l}{2}\frac{2l}{3}\right)+\left(\phi_u-\phi_y\right)l_p\left(l-0.5l_p\right)$$

area area



$$\Delta_y=\varphi_{c1}l_c\left(rl_c-\frac{l_c}{2}\right)-\varphi_{c1}\left(1+\frac{1-\beta_1}{\beta_1}\right)\frac{l_c}{2}\left(rl_c-\frac{2l_c}{3}\right)+\varphi_{c2}l_c\left(rl_c-\frac{3l_c}{2}\right)-\varphi_{c2}\left(1+\frac{1-\beta_2}{\beta_2}\right)\frac{l_c}{2}\left(rl_c-\frac{5l_c}{3}\right)+\cdots$$

$$+\varphi_{ci}l_c\left[rl_c-\left(i-\frac{1}{2}\right)l_c\right]-\varphi_{ci}\left(1+\frac{1-\beta_i}{\beta_i}\right)\frac{l_c}{2}\left[rl_c-\left(i-\frac{1}{3}\right)l_c\right]+\cdots+\varphi_{cr}\frac{l_c^2}{2}-\varphi_{c2}\left(1+\frac{1-\beta_r}{\beta_r}\right)\frac{l_c^2}{6}$$

$$=\frac{l_c^2}{6}\sum_{i=r}\frac{\varphi_{ci}}{\beta_i}\left[6\beta_i\left(r-i+0.5\right)-3\left(r-i\right)-1\right]$$

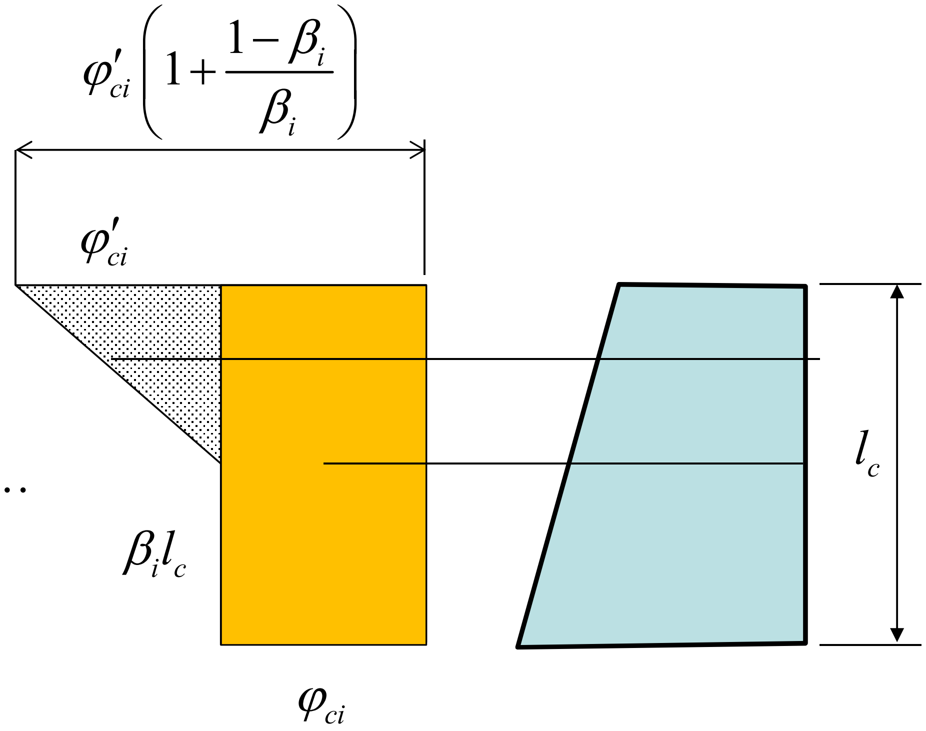
$$\Delta_y = \varphi_{c1} l_c \left(r l_c - \frac{l_c}{2} \right) - \varphi_{c1} \left(1 + \frac{1-\beta_1}{\beta_1} \right) \frac{l_c}{2} \left(r l_c - \frac{2 l_c}{3} \right)$$

$$+ \varphi_{c2} l_c \left(r l_c - \frac{3 l_c}{2} \right) - \varphi_{c2} \left(1 + \frac{1-\beta_2}{\beta_2} \right) \frac{l_c}{2} \left(r l_c - \frac{5 l_c}{3} \right) + \cdots$$

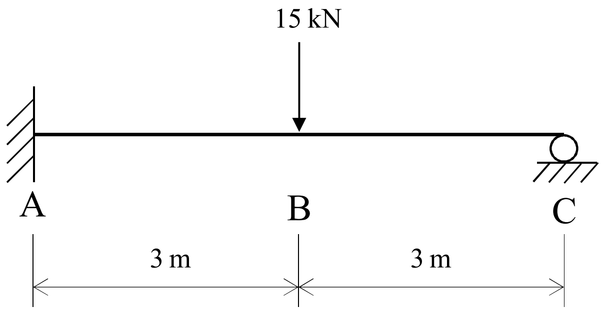
$$+ \varphi_{ci} l_c \left[r l_c - \left(i - \frac{1}{2} \right) l_c \right] - \varphi_{ci} \left(1 + \frac{1-\beta_i}{\beta_i} \right) \frac{l_c}{2} \left[r l_c - \left(i - \frac{1}{3} \right) l_c \right] + \cdots$$

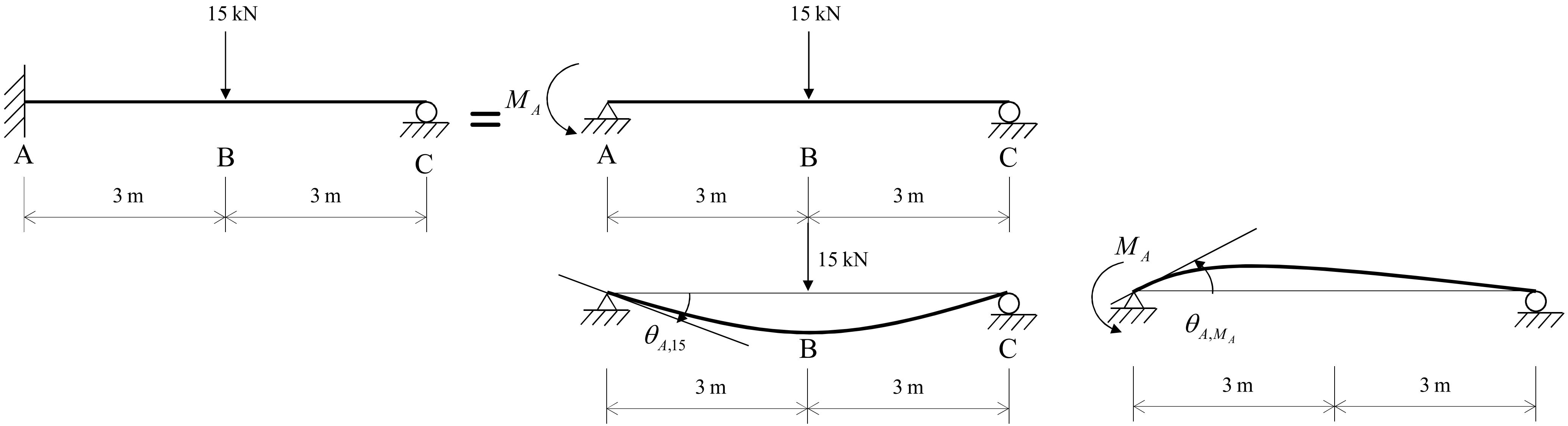
$$+ \varphi_{cr} \frac{l_c^2}{2} - \varphi_{c2} \left(1 + \frac{1-\beta_r}{\beta_r} \right) \frac{l_c^2}{6}$$

$$= \frac{l_c^2}{6} \sum_{i=r} \frac{\varphi_{ci}}{\beta_i} \left[6\beta_i (r - i + 0.5) - 3(r-i) - 1 \right]$$

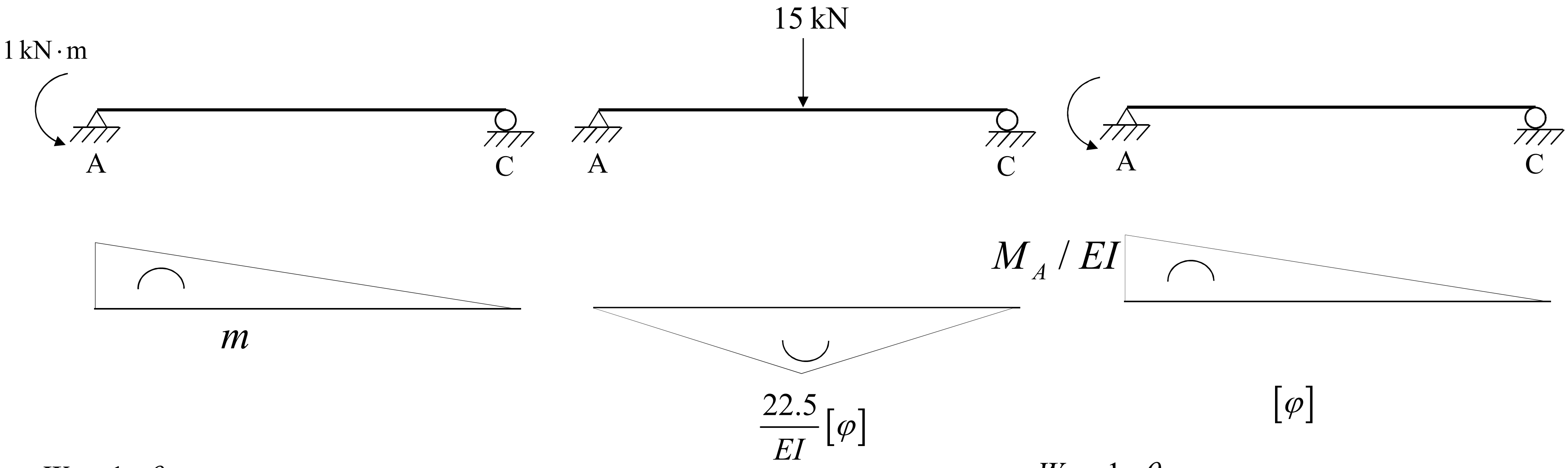


Find the deflection at B





$$\theta_{A,15} + \theta_{A,M_A} = 0$$



$$W_E = 1 \times \theta_{A,15}$$

$$W_I = \int_0^6 m \times \varphi dx = -\frac{1}{2} \times 3 \times \frac{22.5}{EI} \times \left[\frac{2}{3} + \frac{1}{3}\right] = -\frac{33.75}{EI}$$

$$\theta_{A,15} = -\frac{33.75}{EI}$$

$$W_E = 1 \times \theta_{A,M_A}$$

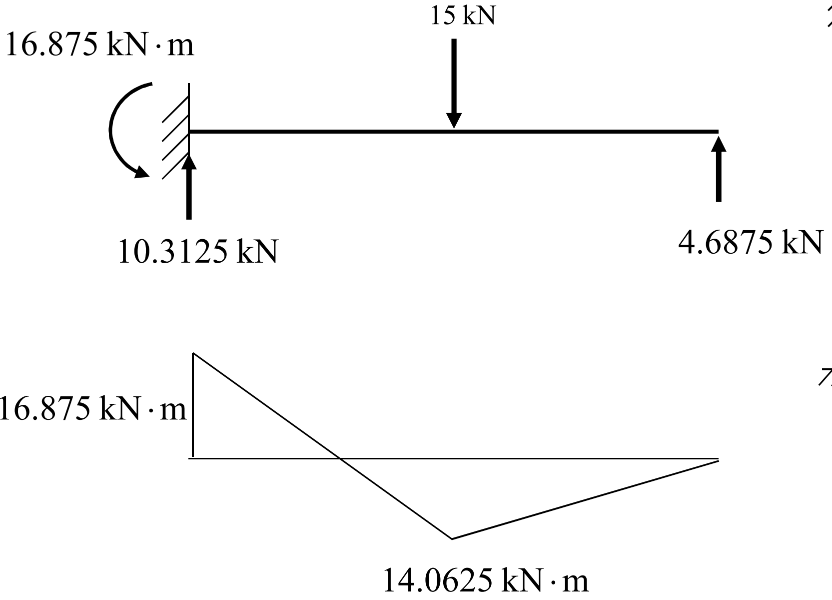
$$W_I = \int_0^6 m \times \varphi dx = \frac{1}{2} \times 6 \times \frac{M_A}{EI} \times \left[\frac{2}{3}\right] = \frac{2M_A}{EI}$$

$$\theta_{A,M_A} = \frac{2M_A}{EI}$$

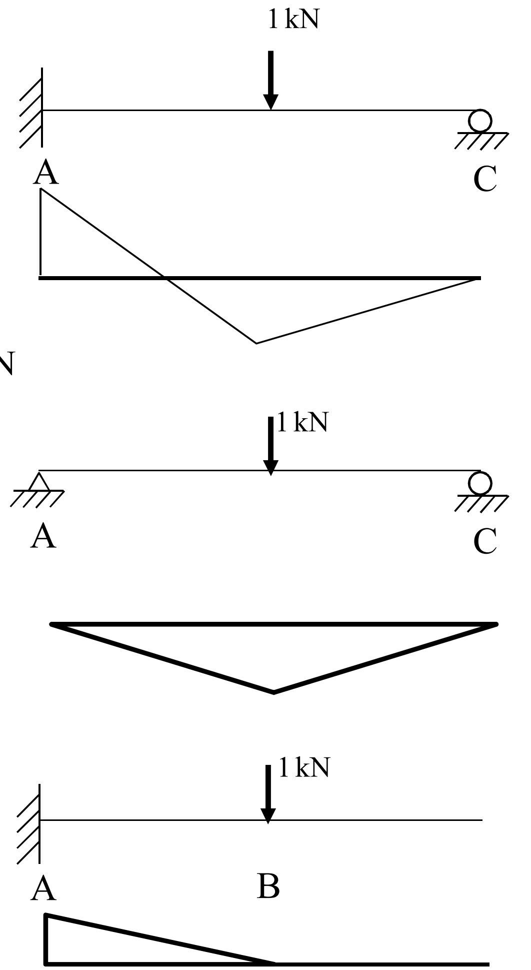
$$\theta_{A,15} + \theta_{A,M_A} = 0$$

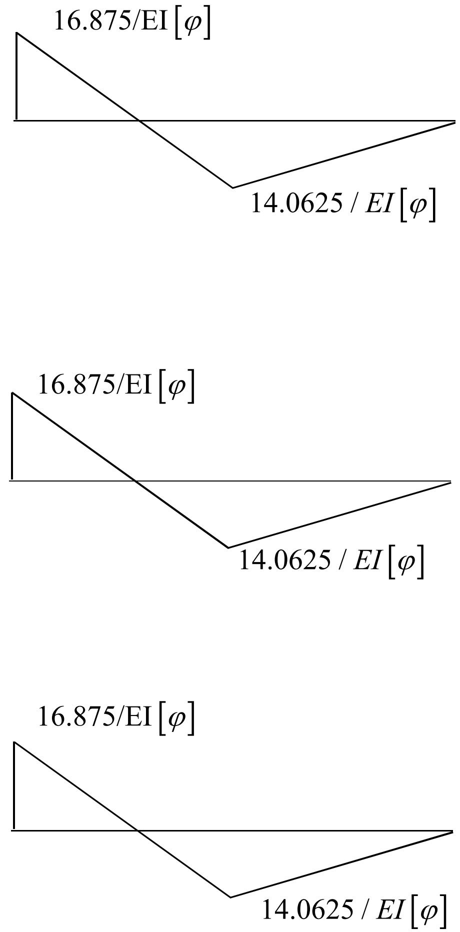
$$-\frac{33.75}{EI} + \frac{2M_A}{EI} = 0$$

$$M_A = 16.875 \text{ kN-m}$$

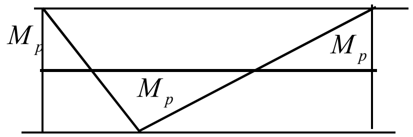


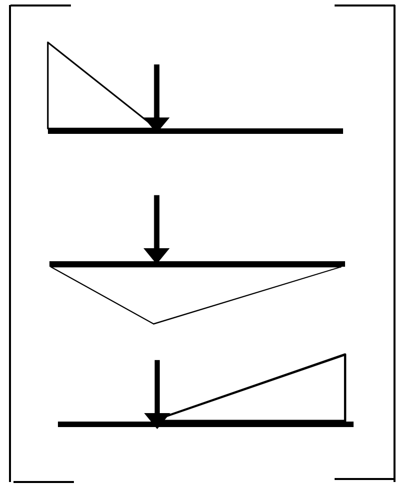
Equilibrium system	Displacement system





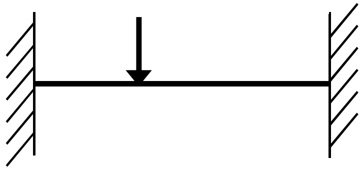
6.5 Deflection theorem

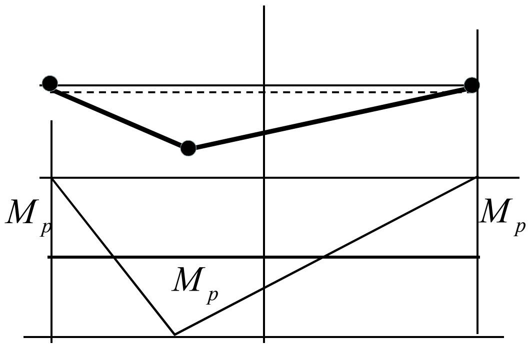


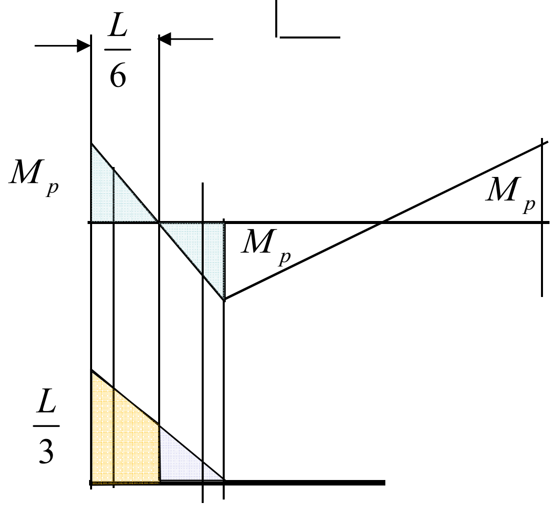


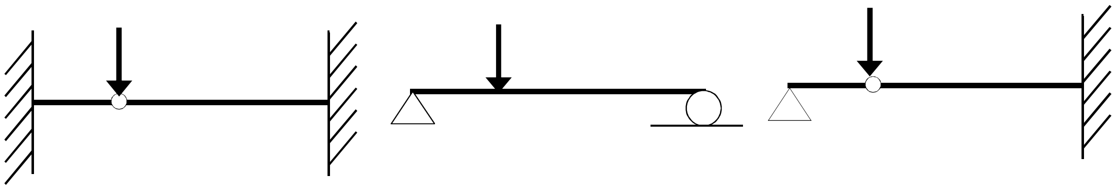
The correct deflection at the collapse load is the maximum value obtained from various trials.

6.6 Simple beam







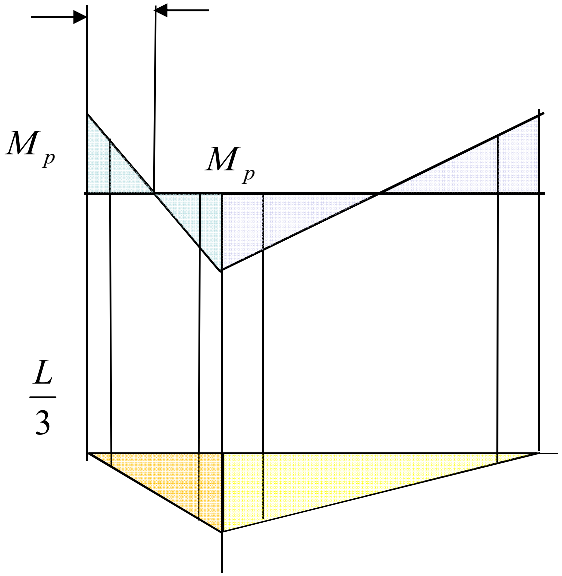


Last plastic hinge

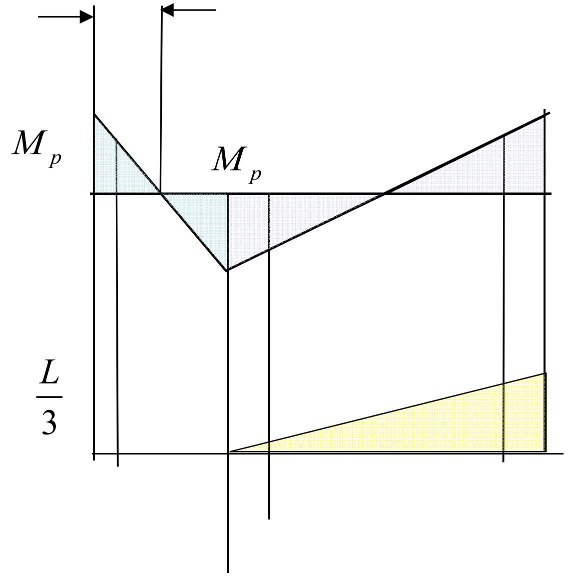
$$A_1\bar{y}_1 + A_2\bar{y}_2$$

$$= \frac{1}{2}\frac{L}{6}\frac{M_P}{EI}\times\frac{L}{3}\frac{5L}{6} - \frac{1}{2}\frac{L}{6}\frac{M_P}{EI}\times\frac{L}{3}\frac{L}{6}$$

$$= \frac{1}{2}\frac{1}{6}\frac{M_P}{EI}\times\frac{L}{3}\frac{2L}{3} = \frac{M_pL^2}{54EI}$$



$$A_1\overline{y}_1 + A_2\overline{y}_2 + A_3\overline{y}_3 + A_4\overline{y}_4$$
$$= \frac{1}{2}\times\frac{L}{6}\times\frac{M_p}{EI}\times\frac{1}{6}\times\frac{2L}{9} - \frac{1}{2}\times\frac{L}{6}\times\frac{M_p}{EI}\times\frac{5}{6}\times\frac{2L}{9}$$
$$+ \frac{1}{2}\times\frac{L}{3}\times\frac{M_p}{EI}\times\frac{5}{6}\times\frac{2L}{9} - \frac{1}{2}\times\frac{L}{3}\times\frac{M_p}{EI}\times\frac{1}{6}\times\frac{2L}{9}$$
$$= \frac{1}{2}\times\frac{L}{6}\times\frac{M_p}{EI}\times\frac{2L}{9}\left[\frac{1}{6} - \frac{5}{6} + 2\times\frac{5}{6} - 2\times\frac{1}{6}\right]$$
$$= \frac{M_p L^2}{27EI}$$



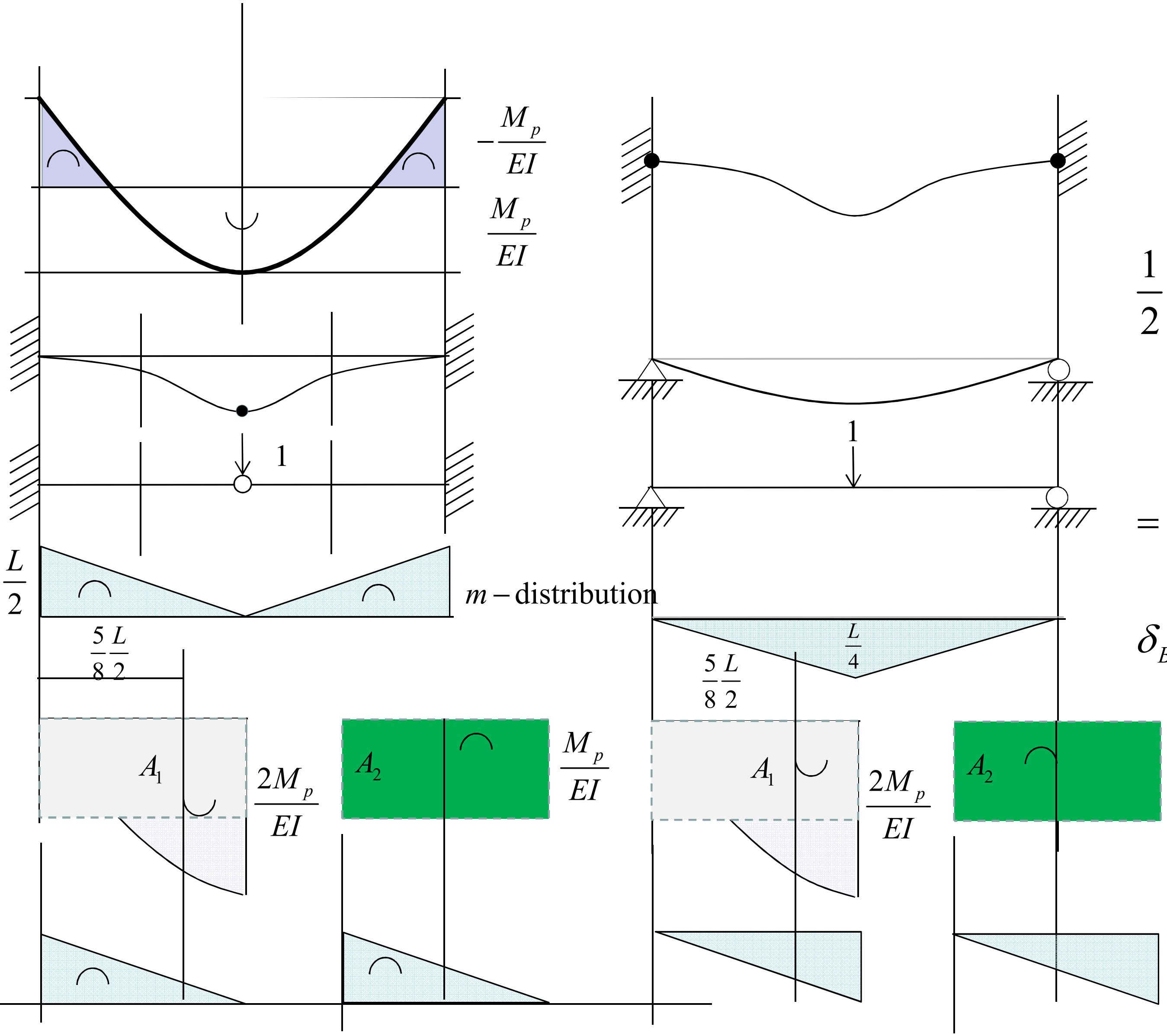
$$A_3\overline{y}_3 + A_4\overline{y}_4$$
$$= \frac{1}{2}\times\frac{L}{3}\times\frac{M_p}{EI}\times\frac{1}{6}\times\frac{2L}{3} - \frac{1}{2}\times\frac{L}{3}\times\frac{M_p}{EI}\times\frac{5}{6}\times\frac{2L}{3}$$
$$= \frac{1}{2}\times\frac{L}{3}\times\frac{M_p}{EI}\times\frac{2L}{3}\left[-\frac{1}{6} + \frac{5}{6}\right]$$
$$= \frac{2M_p L^2}{27EI}$$

Displacement system: curvature distribution

The first PH occurs at the center

The structure system after the first PH

Equilibrium system after the first PH



$\frac{M_p}{EI}$

$$1\times\delta=\int m\times\phi dx$$

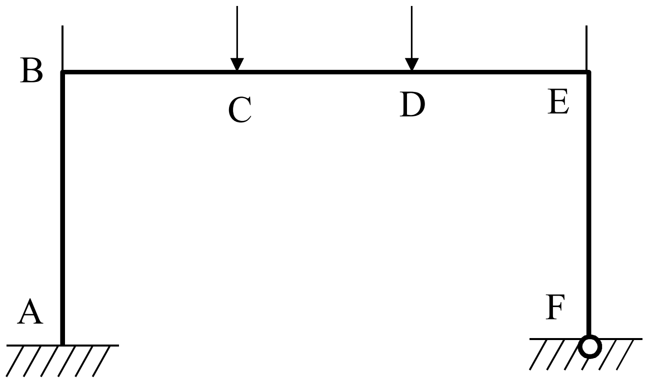
$$1\times\delta=\int m\times\frac{M}{EI}dx$$

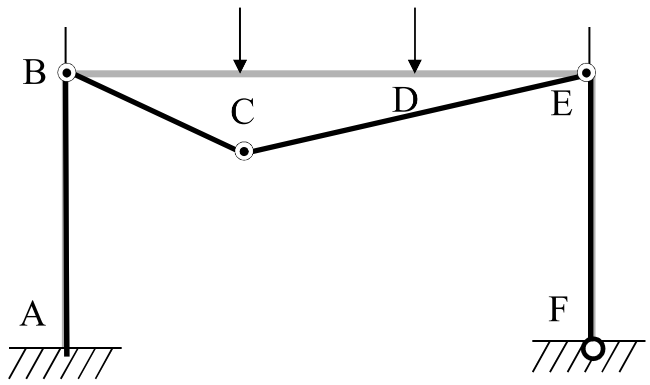
$$\delta_{BA}=-\frac{2}{3}\frac{L}{2}\frac{2M_p}{EI}\frac{3}{8}\frac{L}{4}+\frac{L}{2}\frac{M_p}{EI}\frac{1}{2}\frac{L}{4}=0$$

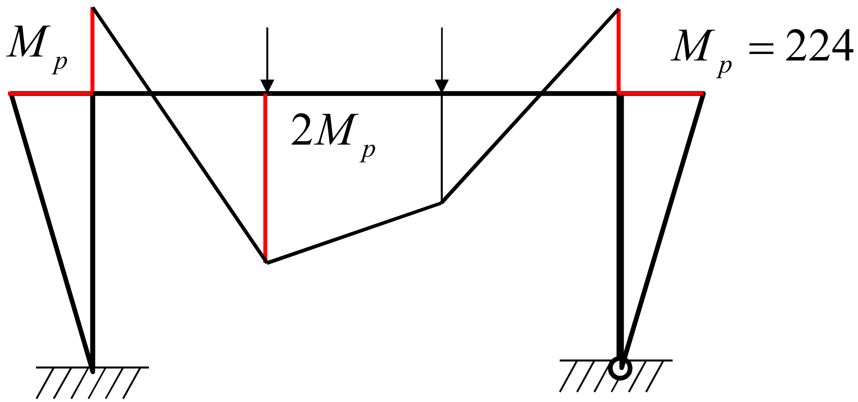
$$\frac{1}{2}\delta_{BB}=\frac{2}{3}\frac{L}{2}\frac{2M_p}{EI}\frac{5}{8}\frac{L}{4}-\frac{L}{2}\frac{M_p}{EI}\frac{1}{2}\frac{L}{4}=\frac{M_pL^2}{EI}\left(\frac{5}{48}-\frac{1}{16}\right)$$

$$\delta_{BB}=\frac{M_pL^2}{12EI}$$

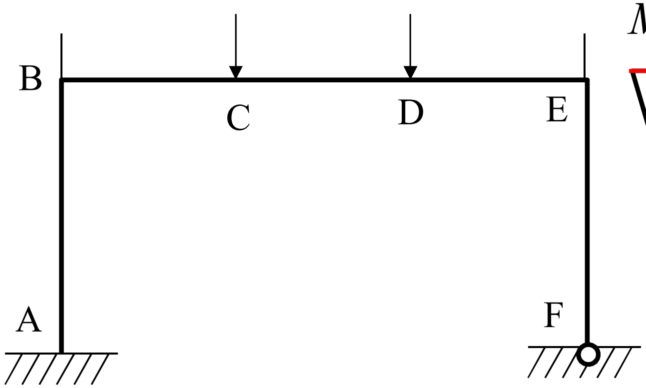
6.7 Simple frames

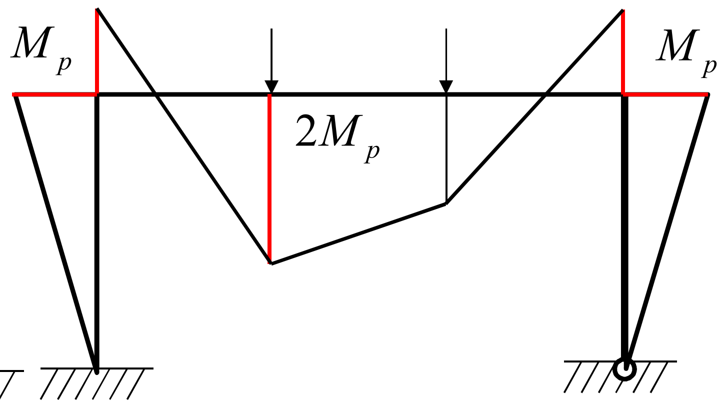


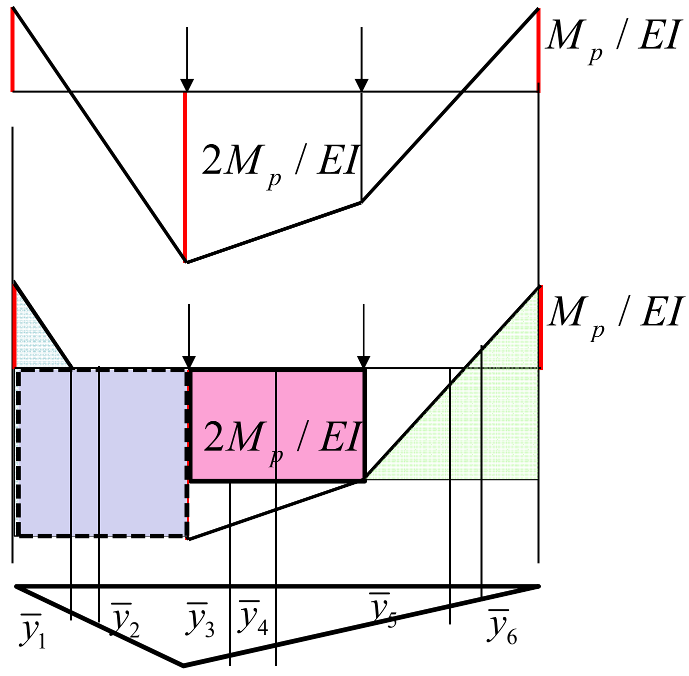




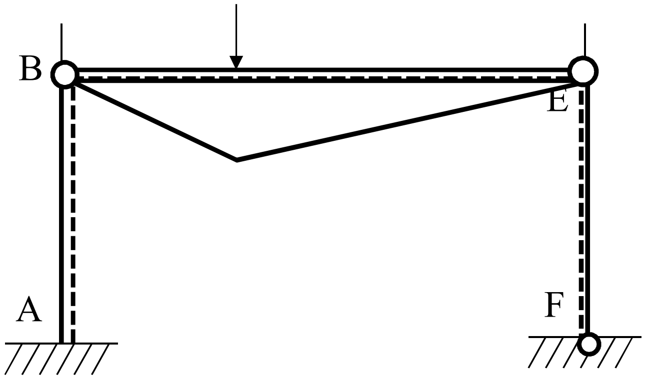
6.7 Simple frames

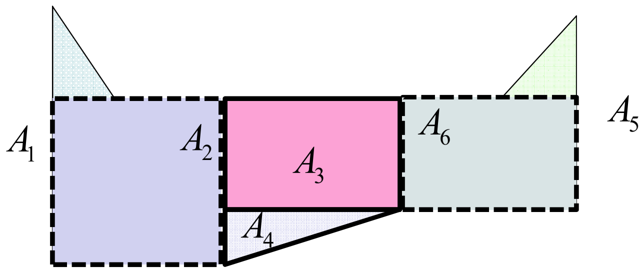






Last PH @ C





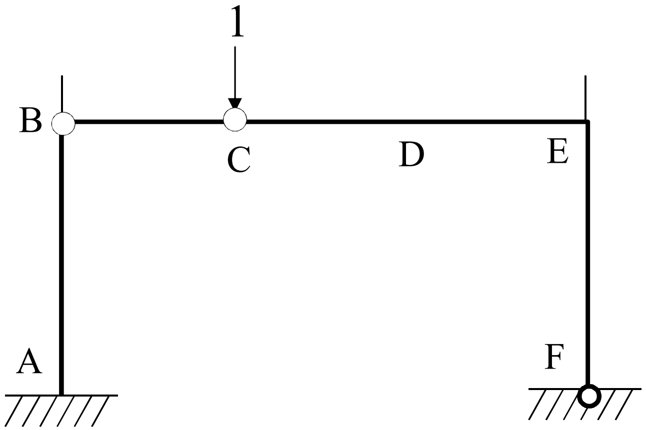
$$EI\delta_{CC} = A_1\overline{y}_1 - A_2\overline{y}_2 + A_3\overline{y}_3 + A_4\overline{y}_4 + A_5\overline{y}_5 - A_6\overline{y}_6$$

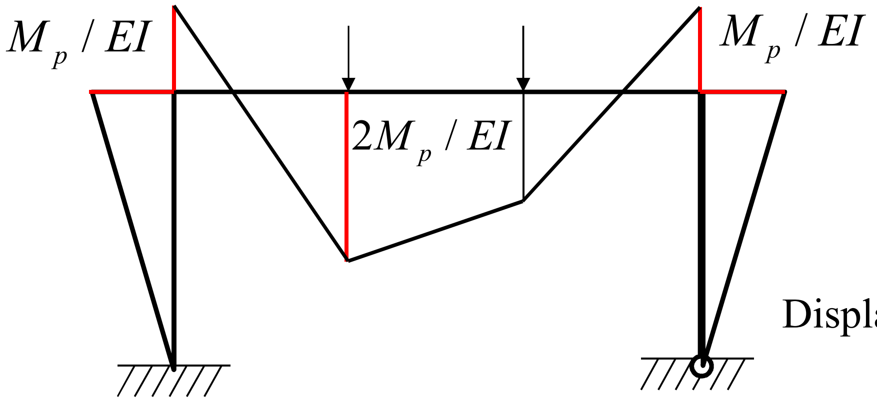
$$= \frac{1}{2}(244+488)\times 10\times\frac{2}{3}\times\frac{20}{3} - 244\times 10\times\frac{1}{2}\times\frac{20}{3}$$

$$422\times 10\times\left(\frac{10}{3}+\frac{1}{2}\times\frac{10}{3}\right)+\frac{1}{2}\times 66\times 10\times\left(\frac{10}{3}+\frac{2}{3}\times\frac{10}{3}\right)$$

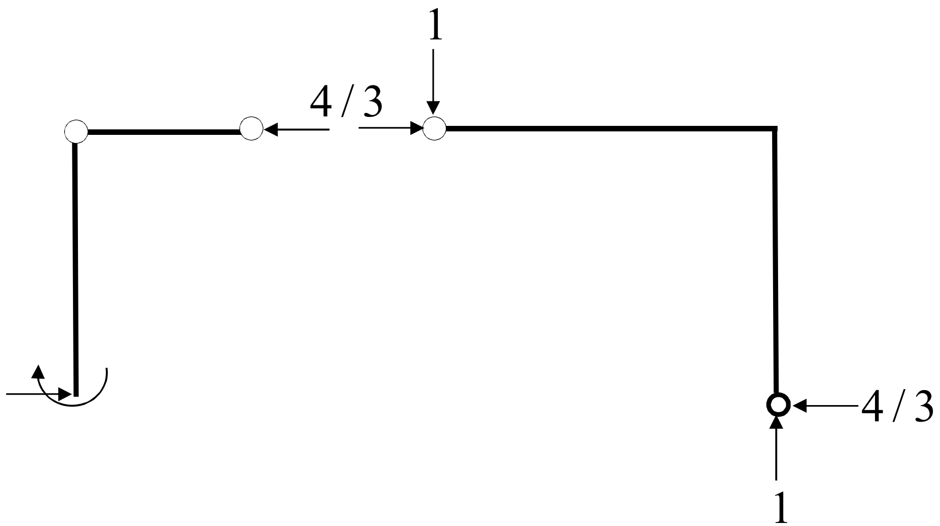
$$+\frac{1}{2}(244+488)\times 10\times\frac{2}{3}\times\frac{10}{3} - 244\times 10\times\frac{1}{2}\times\frac{10}{3} = 344\times 100$$

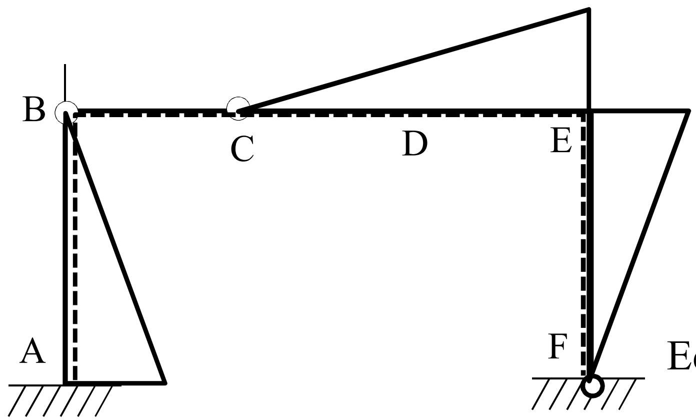
Last PH @ E



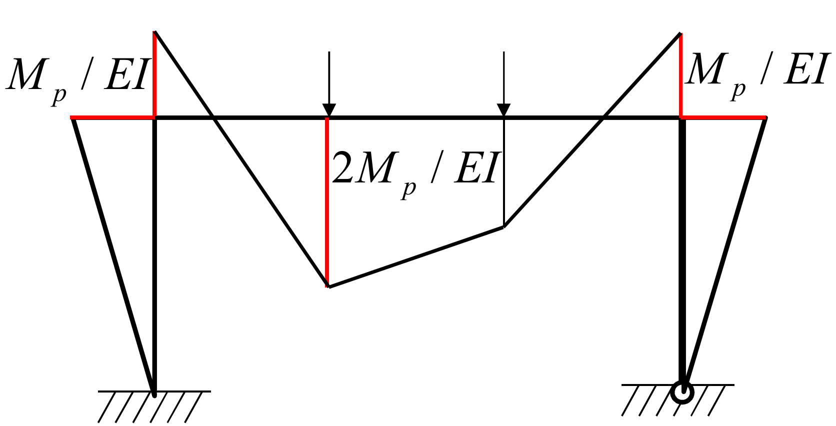


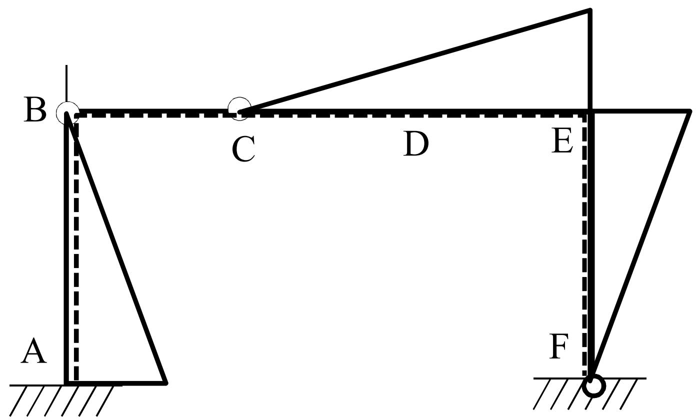
Displacement system

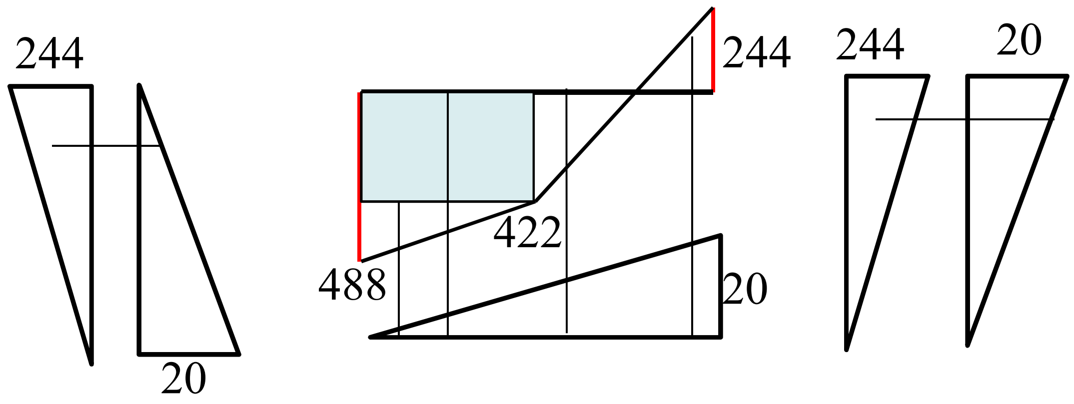




Equilibrium system







$$\delta_{CE}/EI = -\frac{1}{2}\times 15\times 244\times\frac{1}{3}\times 20$$

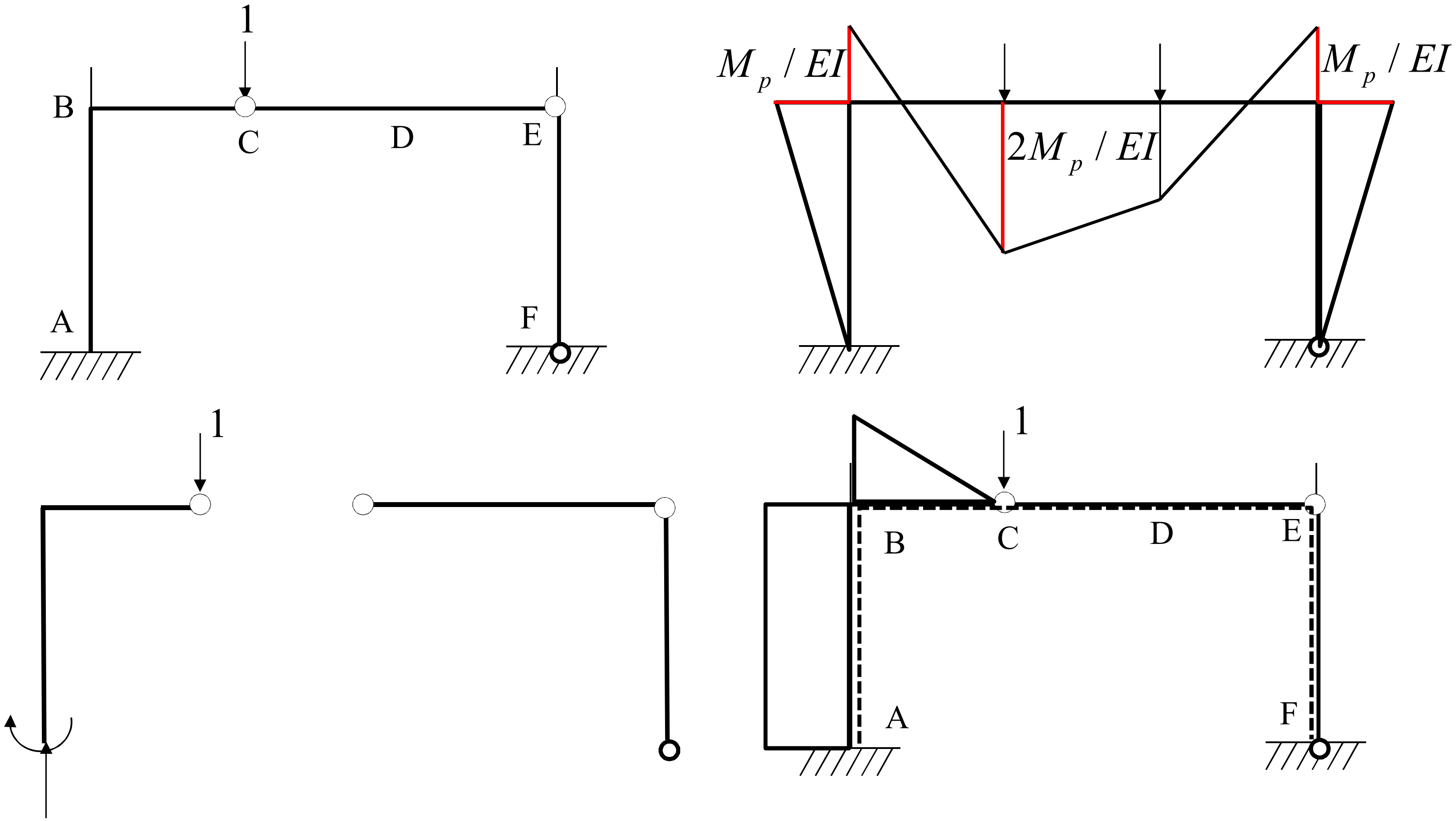
$$-10\times 422\times\frac{1}{2}\times 10-\frac{1}{2}\times 10\times 66\times\frac{1}{3}\times 10$$

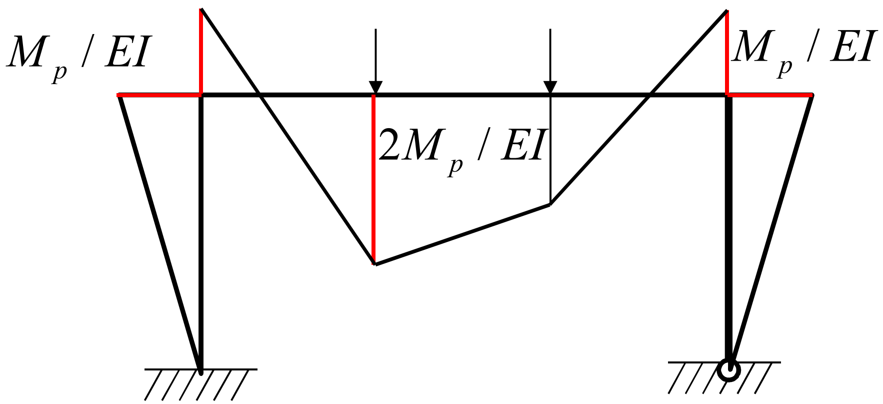
$$-\frac{1}{2}\times 10\times 666\times\left(10+\frac{10}{3}\right)+10\times 244\times\left(10+\frac{10}{2}\right)$$

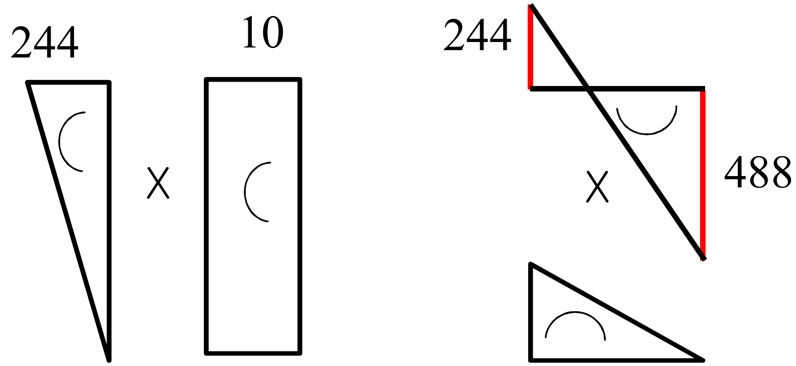
$$+\frac{1}{2}\times 15\times 244\times\frac{2}{3}\times 20=\frac{1080}{6}\times 100=180\times 10^{2}$$

$$\delta_{CE}=\frac{-31.1\times 10^{6}}{EI}(\text{in})$$

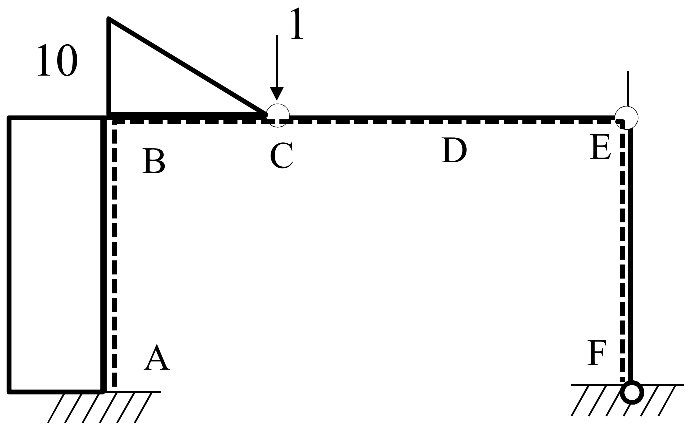
Last PH @ B







Last plastic hinge @B

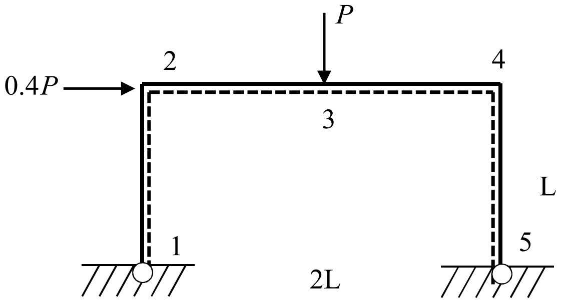


$$EI\delta_{CB} = \frac{1}{2}\times 15\times 244\times 10 - \frac{1}{2}\times 10\times 732\times \frac{1}{3}\times 10 + 10\times 244\times \frac{10}{2}$$

$$= 18300$$

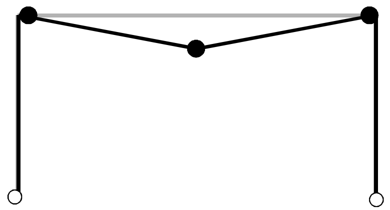
$$\delta_{CB} = \frac{31.6}{EI}\times 10^6\,(\text{in})$$

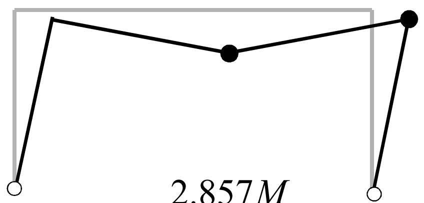
6.7.2



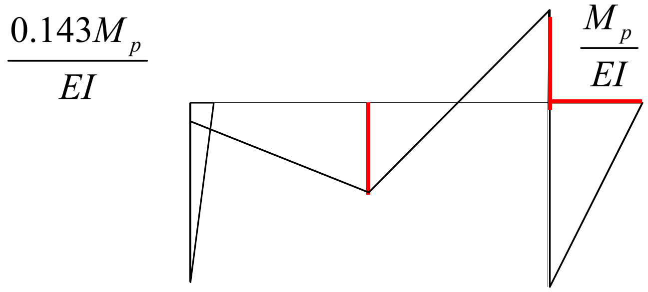
Determine the collapse load
Vertical displacement @3
Horizontal displacement @4

Possible PH=3
Number of redundancy=1
Independent mechanism 3-1=2

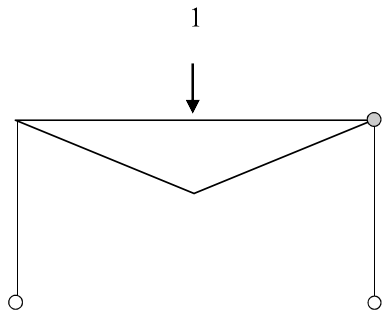


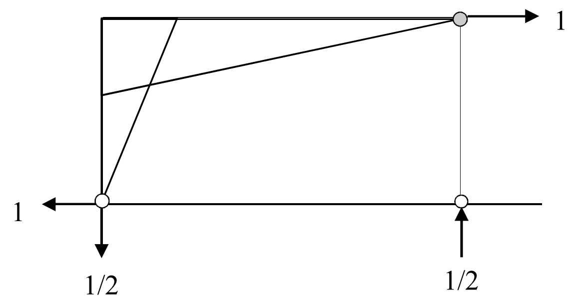


$$P_u = \frac{2.857 M_p}{L}$$

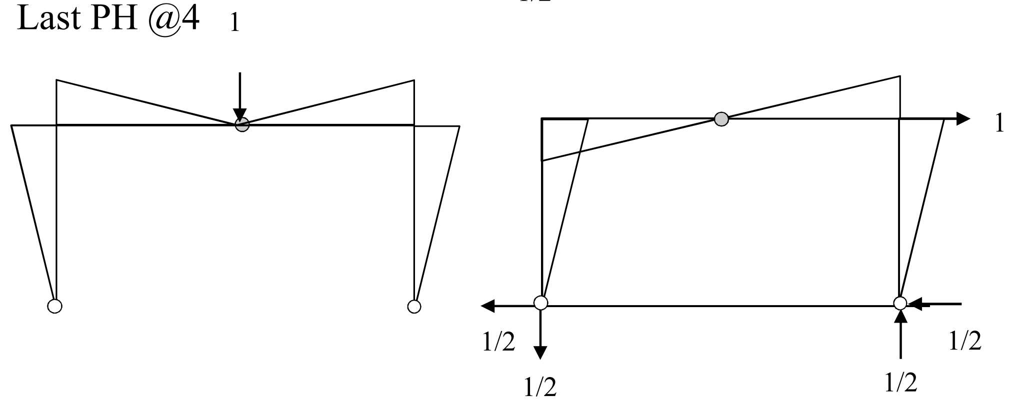


Last PH @3



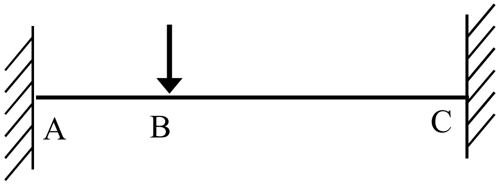


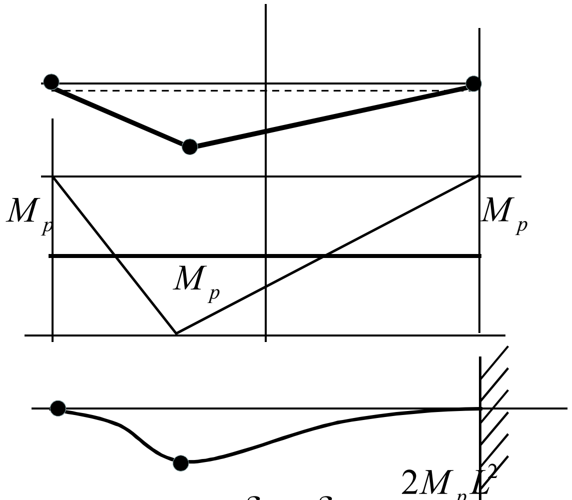
Last PH @4



6.9 Rotational Capacity

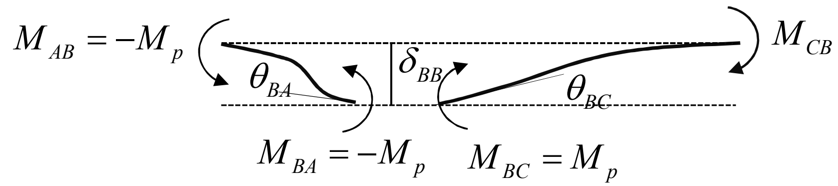
Use of slope deflection Equation





$$\delta_B = \delta_{BC} = \frac{2M_p L^2}{27EI}$$

Last PH at C



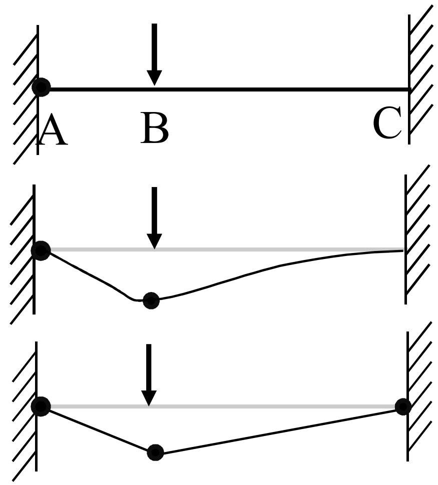
$$H_A = \theta_A = \frac{\delta_{BB}}{L/3} + \frac{L/3}{3EI}\left(M_{AB} - \frac{M_{BA}}{2}\right)$$

$$\begin{cases} \delta_{BB} = \dfrac{2M_p L^2}{27EI} \\ M_{AB} = -M_p \\ M_{BA} = -M_p \end{cases}$$

$$H_{BA} = \theta_{BA} = \frac{\delta_{BB}}{L/3} + \frac{L/3}{3EI}\left(M_{BA} - \frac{M_{AB}}{2}\right) = \frac{M_p L}{6EI}$$

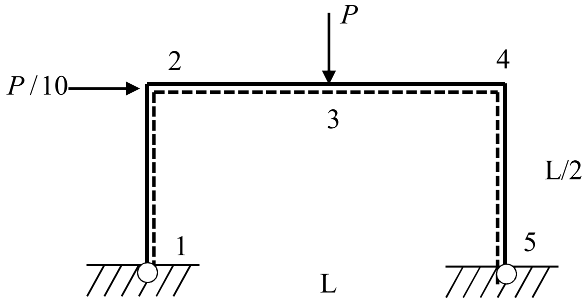
$$H_{BC} = \frac{\delta_{BB}}{2L/3} + \frac{L/3}{3EI}\left(M_{BC} - \frac{M_{CB}}{2}\right) = 0$$

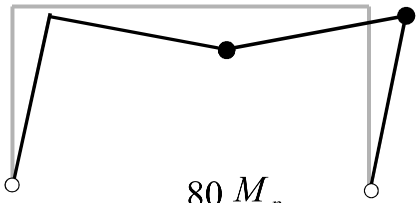
$$H_B = H_{BA} - H_{BC} = \frac{M_p L}{6EI}$$



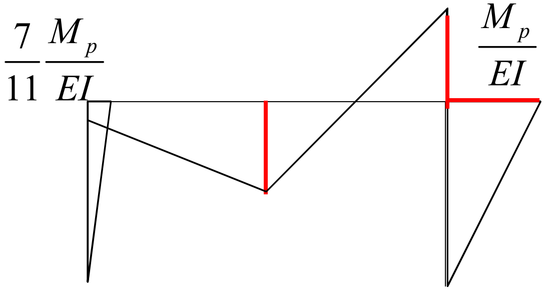
$$\begin{cases} H_A = \dfrac{M_p L}{6EI} \\ H_c = 0 \end{cases}$$

6.10 Rotational Capacity





$$P_u = \frac{80}{11}\frac{M_p}{L}$$



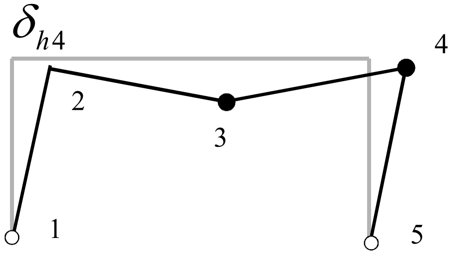
(a) δ_{v3} and δ_{h4}

(b) Required rotation capacity

(c) $P-\Delta$ effect

(a) We need two equations

- Continuity @ 2
- Last plastic hinge location



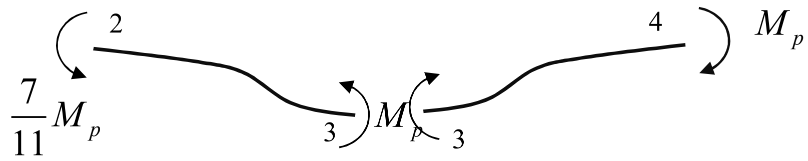
$$\begin{cases} \theta_{21} = 0 + \dfrac{\delta_{h4}}{L/2} + \dfrac{L/2}{3EI}\left(\dfrac{7}{11}M_p - 0\right) \\ \theta_{23} = 0 + \dfrac{\delta_{v3}}{L/2} + \dfrac{L/2}{3EI}\left(-\dfrac{7}{11}M_p + \dfrac{M_p}{2}\right) \end{cases}$$

Since $\theta_{21} = \theta_{23}$

$$\delta_{v3} = \frac{17}{264}\frac{M_p L^2}{EI} + \delta_{h4}$$

For last plastic hinge

Case 1) @3



$$\theta_{32} = 0 + \frac{\delta_{v3}}{L/2} + \frac{L/2}{3EI}\left(-M_p + \frac{7M_p}{2\times 11}\right)$$

$$\theta_{32} = \frac{2\delta_{v3}}{L} - \frac{5}{44}\frac{M_p L}{EI}$$

$$\theta_{34} = 0 - \frac{\delta_{v3}}{L/2} + \frac{L/2}{3EI}\left(M_p - \frac{M_p}{2}\right)$$

$$= -\frac{2\delta_{v3}}{L} + \frac{1}{12}\frac{M_p L}{EI}$$

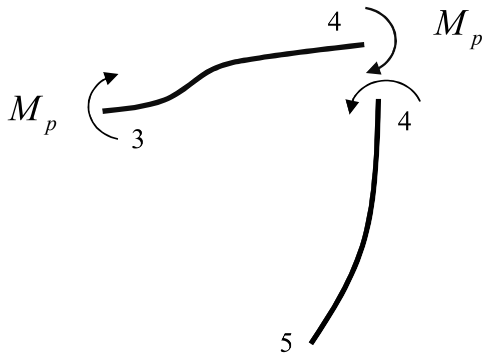
Since $\theta_{32} = \theta_{34}$

$$\frac{2\delta_{v3}}{L} - \frac{5}{44}\frac{M_p L}{EI} = -\frac{2\delta_{v3}}{L} + \frac{1}{12}\frac{M_p L}{EI}$$

$$\delta_{v3} = \frac{13}{264}\frac{M_p L^2}{EI}$$

$$\delta_{h4} = -\frac{1}{66}\frac{M_p L^2}{EI}$$

For last plastic hinge
Case 2) @4



$$\theta_{32} = 0 + \frac{\delta_{v3}}{L/2} + \frac{L/2}{3EI}\left(-M_p + \frac{7M_p}{2\times 11}\right)$$

$$\theta_{32} = \frac{2\delta_{v3}}{L} - \frac{5}{44}\frac{M_p L}{EI}$$

$$\theta_{34} = 0 - \frac{\delta_{v3}}{L/2} + \frac{L/2}{3EI}\left(M_p - \frac{M_p}{2}\right)$$

$$= -\frac{2\delta_{v3}}{L} + \frac{1}{12}\frac{M_p L}{EI}$$

$$\theta_{32} = \theta_{34}$$

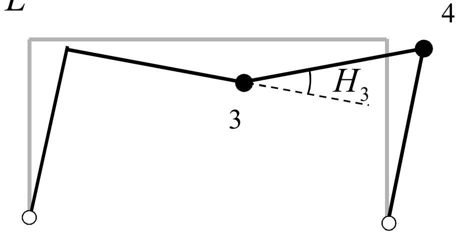
$$\frac{2\delta_{v3}}{L} - \frac{5}{44}\frac{M_p L}{EI} = -\frac{2\delta_{v3}}{L} + \frac{1}{12}\frac{M_p L}{EI}$$

$$\delta_{v3} = \frac{13}{264}\frac{M_p L^2}{EI}$$

$$\delta_{h4} = -\frac{1}{66}\frac{M_p L^2}{EI}$$

(b) Required rotation capacity @3

$$P_u = \frac{80}{11}\frac{M_p}{L}$$



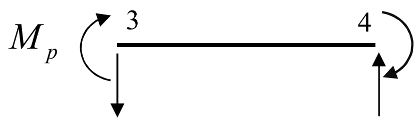
$$H_3 = \theta_{32} - \theta_{34}$$

$$= \frac{2\delta_{v3}}{L} - \frac{5}{44}\frac{M_pL}{EI}$$

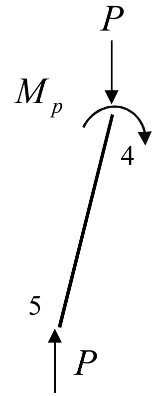
$$+ \frac{2\delta_{v3}}{L} - \frac{1}{12}\frac{M_pL}{EI}$$

$$H_3 = \frac{2}{11}\frac{M_pL}{EI}$$

(c) P-Δ effect



$$P = \frac{2M_p}{L/2} = \frac{4M_p}{L}$$



$$M_{P-\Delta} = P \times \Delta = \frac{4M_p}{L}\delta_{h4} = \frac{4}{33}\frac{M_p}{L}\frac{M_pL^2}{EI}$$

$$\frac{M_{P-\Delta}}{M_p} = \frac{4}{33}\frac{M_pL}{EI} = \frac{4}{33}\left(\frac{F_y}{E}\right)\left(\frac{Z}{I}\right)L$$