6. Discontinuity size

-Assumption: Disc-shaped joints (due to the difficulty in recognizing joint shape and the convenience in mathematical treatment)

Parallel joints

Randomly located joint centers (Poisson process)

- Nomenclature:

s: Joint diameter (size)

l: Joint trace length

$c(s)$: Diameter distribution

$C(s)$: Cumulative probability distribution of joint size

$f(l)$: Joint trace length distribution in an infinite rock exposure

$F(l)$: Cumulative probability distribution of trace length from an infinite sampling window

$g(l)$: Complete trace length distribution from a scanline

$h(l)$: Semi-trace length distribution from a scanline

$H(l)$: Cumulative probability distribution of semi-trace length from a scanline

μ_S: Mean diameter of joints

μ_L: Mean trace length

λ_V: Volumetric frequency of joint centers

▶ **Relation of $c(s)$ and $f(l)$**

① The number of joint traces, N_S, whose centers are located in a rectangular window having an area of A and whose joint diameter is s (ϕ is an acute angle between the joints and sampling window):

$$N_S = V_S\ \lambda_V\ c(s)ds = As\ \sin\phi\lambda_V c(s)ds$$

② The number of joint traces whose centers are located in the sampling window:

$$N_{all} = A \sin\phi\, \lambda_V \int_0^{S_X} s c(s) ds = A \sin\phi\ \lambda_V\ \mu_S$$

③ Proportion of the joint traces whose joint diameter is s:

$$P_S = \frac{N_S}{N_{all}} = \frac{A\ s \sin\ \phi\ \ \lambda_V\ \ c(s)ds}{A \sin\ \phi\ \ \lambda_V\ \ \mu_S} = \frac{s}{\mu_S} c(s)ds$$

④ Proportion of joint traces whose length is l and joint diameter is s:

$$P_{Sl} = P_S P_{l|S} = \frac{s}{\mu_s} c(s) ds \frac{2}{s} dy = \frac{l\, dl\ \ c(s)\ ds}{\mu_s \sqrt{s^2 - l^2}}$$

⑤ Proportion of joint traces in the window whose length is l:

$$P_l = \int_{s=l}^{s=S_X} P_{Sl} = \frac{l\, dl}{\mu_S} \int_l^{S_X} \frac{c(s)}{\sqrt{s^2 - l^2}} ds$$

⑥ PDF of joint trace length, l:

$$f(l) = \frac{P_l}{dl} = \frac{l}{\mu_S} \int_l^{S_X} \frac{c(s)}{\sqrt{s^2 - l^2}} ds$$

⑦ Integratible form:

$$1 - F(l) = \int_l^{l_X} f(l) dl = \int_l^{l_X} \frac{l}{\mu_S} \left(\int_l^{S_X} \frac{c(s)}{\sqrt{s^2 - l^2}} ds \right) dl = \int_l^{S_X} \frac{c(s)}{\mu_S} \left(\int_l^{s} \frac{l}{\sqrt{s^2 - l^2}} dl \right) ds$$

$$= \frac{1}{\mu_S} \int_l^{S_X} \sqrt{s^2 - l^2}\, c(s) ds$$

⑧ Numerical approximation

$$1 - F(l) = \frac{\Delta s}{\mu_S} \sum_{s=l}^{S_X} \sqrt{s^2 - l^2}\, c(s)$$

$$1 - F(S_X) = \frac{\Delta s}{\mu_S}\ \sqrt{S_X^2 - S_X^2}\ \ c(S_X) = 0$$
$$1 - F(S_X - \Delta s) = \frac{\Delta s}{\mu_S}\ \sqrt{S_X^2 - (S_X - \Delta s)^2}\ \ c(S_X)$$
$$1 - F(S_X - 2\Delta s) = \frac{\Delta s}{\mu_S} \left(\sqrt{(S_X - \Delta s)^2 - (S_X - 2\Delta s)^2}\ \ c(S_X - \Delta s) + \sqrt{S_X^2 - (S_X - 2\Delta s)^2}\ \ c(S_X) \right)$$

Rearranging the above with respect to $c(s)$

$$c(S_X) = \frac{\mu_S}{\Delta s} \frac{1 - F(S_X - \Delta s)}{\sqrt{S_X^2 - (S_X - \Delta s)^2}}$$

$$c(S_X - \Delta s) = \frac{\left(\frac{\mu_S}{\Delta s}(1 - F(S_X - 2\Delta s)) - \sqrt{(S_X^2 - (S_X - 2\Delta s)^2} \ c(S_X) \right)}{\sqrt{(S_X - \Delta s)^2 - (S_X - 2\Delta s)^2}}$$

Expressing $S_X \rightarrow N$th , $S_X - \Delta s \rightarrow (N\text{-}1)$th

$$c(N - i) = \frac{\left(\frac{\mu_S}{\Delta s}(1 - F(N - (i+1))) - \sum_{j=0}^{i-1} \sqrt{((S_X - j\Delta s)^2 - (S_X - (i+1)\Delta s)^2} \ c(N - j) \right)}{\sqrt{(S_X - i\Delta s)^2 - (S_X - (i+1)\Delta s)^2}}$$

i: 0~N-1

μ_S: initially set as a non-zero constant such as 1. It can be obtained after $c(s)$ is normalized and completely determined.

Negative value: should be changed into zero.

▶ Relationship between $c(s)$-$g(l)$

① The number of joint traces, N_S, intersecting a scanline whose length is L and joint diameter is s (θ is an acute angle between joint normal and scanline):

$$N_S = V_S \ \lambda_V \ c(s)ds = \frac{\pi}{4} L s^2 \ \cos\theta \, \lambda_V c(s) ds$$

② The number of joint traces intersecting the scanline:

$$N_{all} = \frac{\pi}{4} L \cos \ \theta \ \lambda_V \int_0^{S_X} s^2 c(s) ds = \frac{\pi}{4} L \cos \ \theta \ \lambda_V \ M_{S2}$$

③ Proportion of joint traces of which diameter is s:

$$P_S = \frac{N_S}{N_{all}} = \frac{s^2}{M_{S2}} c(s) ds$$

④ Proportion of joint traces whose length is l and diameter is s:

$$P_{Sl} = P_S P_{l|S} = \frac{s^2}{M_{S2}} c(s) ds \frac{2Ll\,dy\cos\theta}{\frac{\pi}{4} s^2 \cos\theta\, L} = \frac{s^2}{M_{S2}} c(s) ds \frac{2Ll^2 \cos\theta dl}{\frac{\pi}{2} s^2 \cos\theta\, L\sqrt{s^2 - l^2}}$$

$$= \frac{l^2\, dl}{\frac{\pi}{4} M_{s2}\sqrt{s^2 - l^2}} c(s) ds$$

⑤ Proportion of scanline-intersecting joint traces whose length is l:

$$P_l = \int_{s=l}^{s=S_X} P_{Sl} = \frac{4l^2\, dl}{\pi M_{S2}} \int_l^{S_X} \frac{c(s)}{\sqrt{s^2 - l^2}} ds$$

⑥ PDF of joint trace length:

$$g(l) = \frac{P_l}{dl} = \frac{4l^2}{\pi M_{S2}} \int_l^{S_X} \frac{c(s)}{\sqrt{s^2 - l^2}} ds \qquad \text{(p.157, Warburton, 1980)}$$

▶ Estimation of $f(l)$

-Estimation by using the joint trace length distribution from scanlines or sampling windows: $g(l)$, $h(l)$, $f^c(l)$, $f^d(l)$

1) Estimation by using $g(l)$ (Priest & Hudson, 1981)

The probability that a joint trace intersects a scanline in an infinite exposure is proportional to the joint trace length: $g(l) = k\, l\, f(l)$. Because $g(l)$ is PDF,

$$\int_0^{l_X} g(l) dl = k \int_0^{l_X} l\, f(l) dl = k\mu_L = 1$$

$$k = \frac{1}{\mu_L}, \quad g(l) = \frac{l}{\mu_L} f(l)$$

Meanwhile, relationship between $c(s)$ and $f(l)$, and $c(s)$ and $g(l)$ is follows.

$$f(l) = \frac{l}{\mu_S} \int_l^{S_X} \frac{c(s)}{\sqrt{s^2 - l^2}} ds, \quad g(l) = \frac{4l^2}{\pi M_{S2}} \int_l^{S_X} \frac{c(s)}{\sqrt{s^2 - l^2}} ds$$

Because $g(l) = \frac{4l\mu_S}{\pi M_{S2}} f(l)$, we can get the following relation by using the previous definition of $g(l)$

$$\frac{4l\mu_S}{\pi M_{S2}} = \frac{l}{\mu_L}, \ \mu_L = \frac{\pi M_{S2}}{4\mu_S}.$$ (p.159)

※ Relationship among λ_V, λ_A and λ_L

In the process of obtaining the volumetric frequency,

$\lambda_V = \frac{4\lambda_L}{\pi M_{S2}}$ is already known. If we put $\pi M_{S2} = 4\mu_S\mu_L$ from $\mu_L = \frac{\pi M_{S2}}{4\mu_S}$ to this

$\lambda_V = \frac{\lambda_L}{\mu_S\mu_L}$ is obtained. Combining this with $\lambda_V = \frac{\lambda_A}{\mu_S}$ makes

$\lambda_A = \frac{\lambda_L}{\mu_L}$.

2) Estimation by using $h(l)$

① The probability that a joint trace intersecting a scanline has the complete length of m and semi-trace length of l:

$$P_{ml} = g(m)dm\frac{dl}{m} = \frac{f(m)}{\mu_L}dm\,dl$$

② Proportion of joint traces whose semi-trace length is l:

$$P_l = \int_{m=l}^{m=l_x} P_{ml} = \frac{dl}{\mu_L}\int_l^{l_x} f(m)dm = \frac{dl}{\mu_L}(1-F(l))$$

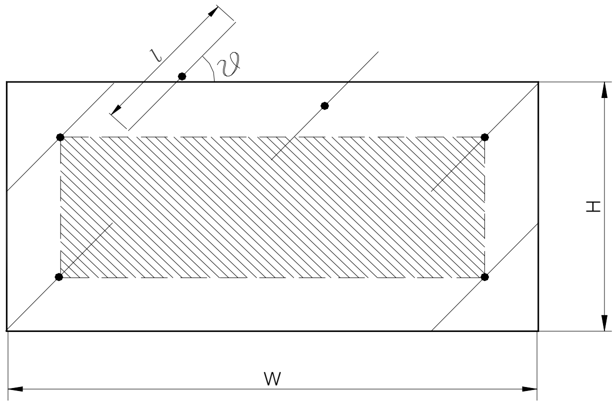
③ Semi-trace length distribution $h(l)$:

$$h(l) = \frac{P_l}{dl} = \frac{1-F(l)}{\mu_L}$$

※ $h(l)$ in case that $f(l)$ follows a negative exponential distribution

If $f(l) = \frac{e^{-l/\mu_L}}{\mu_L}$, $\frac{1-F(l)}{\mu_L} = \frac{e^{-l/\mu_L}}{\mu_L}$ which means $h(l)$ becomes the negative exponential distribution too. In fact, $h(l)$ is always a monotonically decreasing function regardless of $f(l)$. This makes it difficult to estimate $f(l)$ by comparing the sampled $h(l)$ with predicted $h(l)$.

3) Estimation by using $f^c(l)$ (Song & Lee, 2001)



Area in which contained trace centers are located (shaded zone)

$$
\begin{aligned}
A_l^c &= (W - l\cos\theta)(H - l\sin\theta) \\
&= \cos\theta\sin\theta\, l^2 - (W\sin\theta + H\cos\theta)\, l + WH \\
&= a l^2 - b l + c \qquad (\, l < l_X)
\end{aligned}
$$

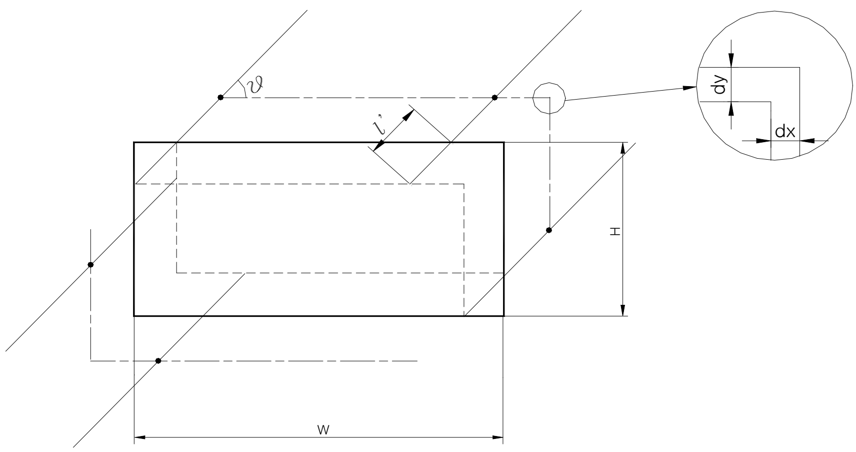
$$N_l^c = \lambda_a A_l^c f(l)\, dl$$

$$f^c(l)dl = \frac{N_l^c}{N_{all}^c} = \frac{\lambda_a}{N_{all}^c} A_l^c f(l)\, dl$$

$$f(l) = \frac{N_{all}^c}{\lambda_a A_l^c} f^c(l)$$

$$\lambda_a = \frac{2N_{all}^c + N_{all}^d}{2WH} \quad \text{(Mauldon, 1998)}$$

4) Estimation by using $f^d(l)$ (Song & Lee, 2001)



Location of dissecting trace centers whose partial length in the sampling window is l' (dash-dot line)

$$
\begin{aligned}
A_{l'}^{d} &= 2(dy(W - l'\cos\theta) + dx(H - l'\sin\theta)) \\
&= 2dl'(\sin\theta(W - l'\cos\theta) + \cos\theta(H - l'\sin\theta)) \\
&= 2dl'(W\sin\theta + H\cos\theta - 2l'\sin\theta\cos\theta)) \\
&= 2dl'(2al' - b)
\end{aligned}
$$

$$
\begin{aligned}
N_{l'}^{d} &= \int_{l'}^{S_X} \lambda_a A_{l'}^{d} f(l)\,dl \\
&= 2\lambda_a(2al' - b)\,dl' \int_{l'}^{S_X} f(l)\,dl
\end{aligned}
$$

$$
f^{d}(l') = \frac{N_{l'}^{d}}{N_{all}^{d}\ dl'} = \frac{2\lambda_a}{N_{all}^{d}}(2al' - b)\int_{l'}^{S_X} f(l)\,dl
$$

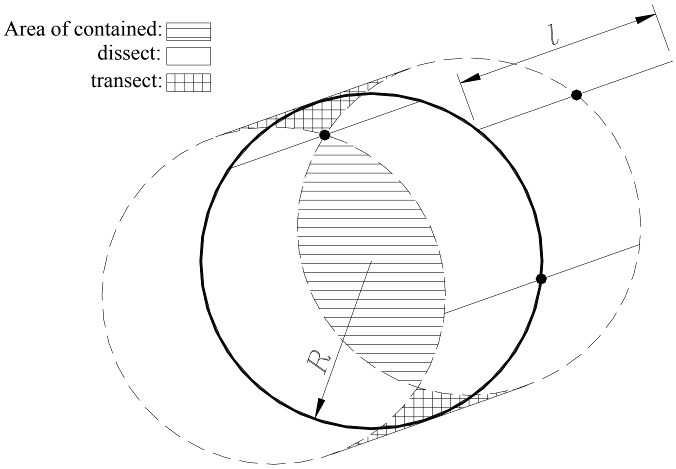
$$
\int_{l'}^{S_X} f(l)dl = 1 - F(l') = \frac{f^{d}(l')\,N_{all}^{d}}{2\lambda_a(2al' - b)}
$$

※ Performance of $f^{c}(l)$, $f^{d}(l)(h(l))$ and $g(l)$ for estimating $f(l)$

Monte Carlo simulation with a virtual rectangular sampling window whose horizontal boundary was used for a scanline making an acute angle of $\theta = 30^\circ$ with joint traces were repeated 20 times and the mean numbers of joint traces were calculated: 12.2, 17.3 and 4 for N_{all}^{c}, N_{all}^{d} and N_{all}^{s}, respectively.

The mean error of $f(l)$ estimated from contained traces, dissecting traces and scanline traces (complete length) were 0.1, 0.6 and 0.3, respectively. This result shows that the contained traces are better than other type of traces for estimating $f(l)$.

※ Contained traces from a circular window



Trace center zone of each trace type in a circular window

$$\dot{A}_l^c = -l\sqrt{R^2 - \left(\frac{l}{2}\right)^2} + 2R^2 \sin^{-1}\left(\frac{\sqrt{R^2 - \left(\frac{l}{2}\right)^2}}{R}\right)$$

$$\dot{f}^c(l) = \frac{\dot{N}_l^c}{\dot{N}_{all}^c\, dl} = \frac{\lambda_a \dot{A}_l^c f(l)}{\dot{N}_{all}^c}$$

$$f(l) = \frac{\dot{N}_{all}^c}{\lambda_a \dot{A}_l^c} \dot{f}^c(l)$$

-Advantage of the circular window: Joint orientation does not affect joint statistics. Circular shape of window is effective in tunnel face mapping.

-Efficiency of contained traces in a circular window for estimating $f(l)$ is same as those in a rectangular window.

▶ Curtailment of long traces - mean trace length estimation

The most important parameter of $f(l)$ is mean value (mean trace length) especially when $f(l)$ follows a negative exponential distribution. Here are introduced various methods to obtain the mean trace length from a finite sampling plane survey.

1) From scanline survey

① When $f(l) = h(l)$ is assumed to be negative exponential: using μ_{iL}

From $H(c) = 1 - e^{\frac{-c}{\mu_L}}$

$$i(l) = \frac{e^{-\frac{l}{\mu_L}}}{\mu_L\left(1 - e^{\frac{-c}{\mu_L}}\right)} \quad \text{.........curtailed distribution}$$

$$\mu_{iL} = \int_0^c l\, i(l) dl = \frac{1}{\mu_L\left(1 - e^{\frac{-c}{\mu_L}}\right)} \int_0^c l e^{-\frac{l}{\mu_L}} dl$$

$$= \mu_L - \frac{c e^{\frac{-c}{\mu_L}}}{1 - e^{\frac{-c}{\mu_L}}}$$

② When $f(l) = h(l)$ is assumed to be negative exponential: using $H(c) = r/n$

From $H(c) = 1 - e^{\frac{-c}{\mu_L}} = r/n$

$$\mu_L = \frac{c}{-\ln(1 - r/n)}$$

We can get μ_L from the gradient of c with respect to $-\ln(1-r/n)$ (Fig.6.6, p172).

③ When $f(l)$ is assumed to be negative exponential: Laslett(1982)'s suggestion

$$\mu_L = \frac{\sum_i^n X_i + \sum_j^m Y_j + \sum_k^p Z_k}{2n+m}$$

where n, m and p are the number of contained, dissecting and transecting traces, respectively, and X_i, Y_j and Z_k are length of each trace type.

④ Without any assumption of $f(l)$: using $h(0) = 1/\mu_L$

From $h(l) = \frac{1-F(l)}{\mu_L}$, $h(l) = 1/\mu_L$ when $l \rightarrow 0$.

That is, when $c \rightarrow 0$ $\quad H(c) = \int_0^c h(l)dl \approx \int_0^c \frac{1}{\mu_L} dl = \frac{c}{\mu_L}$.

From $H(c) = \frac{r}{n} = \frac{c}{\mu_L}$, $\quad \mu_L = \frac{nc}{r}$ $(c \rightarrow 0)$.

We can get μ_L using $\frac{nc}{r}$ at $c = 0$ from the relation between c and $\frac{nc}{r}$ (Fig.6.8, p.176).

2) From window sampling

Without any assumption of $f(l)$: using the point-estimator (Pahl, 1981)

$$\mu_L = \frac{WH(N - N^c + N^t)}{(W\cos\phi + H\sin\phi)(N + N^c - N^t)}$$

which is suggested by Pahl (1981). Even though Pahl's paper does not introduce details of its derivation process we can make the above equation by using the point-estimator of areal frequency as follows.

Following expression has been introduced in process of estimating $f(l)$ from $f^c(l)$.

$$N_l^c = \lambda_a A_l^c f(l)\,dl$$

therefore, $N_{all}^c = \int_0^{l_X} N_l^c = \lambda_a \int_0^{l_X} A_l^c f(l)\,dl.$

Putting $A_l^c = \cos\theta\sin\theta\, l^2 - (W\sin\theta + H\cos\theta)\,l + WH$ to above,

$$N_{all}^c = \lambda_a\left[\cos\theta\sin\theta\, M_{L2} - (W\sin\theta + H\cos\theta)\,\mu_L + WH\right].$$

The same process is applied to transecting traces for $l < l_X$ as follows.

$$A_l^t = l^2\cos\theta\sin\theta$$

$$N_l^t = \lambda_a A_l^t f(l)\,dl$$

$$N_{all}^t = \int_0^{l_X} N_l^t = \lambda_a \int_0^{l_X} A_l^t f(l)\,dl = \lambda_a \cos\theta\sin\theta\, M_{L2}$$

Now $N_{all}^c - N_{all}^t$ can be expressed as

$$N_{all}^c - N_{all}^t = N^c - N^t = -\lambda_a(W\sin\theta + H\cos\theta)\,\mu_L + \lambda_a WH.$$

which can be rearranged with respect to μ_L as follow.

$$\mu_L = \frac{\lambda_a WH - N^c + N^t}{\lambda_a(W\sin\theta + H\cos\theta)},$$

Putting $\lambda_a = \dfrac{N + N^c - N^t}{2\,WH}$ to above makes.

$$\mu_L = \frac{WH(N - N^c + N^t)}{(W\sin\theta + H\cos\theta)(N + N^c - N^t)} = \frac{WH(N - N^c + N^t)}{(W\cos\phi + H\sin\phi)(N + N^c - N^t)}$$

▶ Trimming of short traces

-Trimming cannot be avoided considering joints of micro-crack level.

-In most cases counting joints under trimming level is impossible (truncated).

-It is recommended to set the trimming level as low as possible.

-In many cases of analysis short traces are less influential than long traces.

-Considering both curtailment and trimming in semi-trace length distribution:

$$i(l) = \frac{h(l)}{H(c) - H(t)}$$

▶ Relation between linear frequency and areal frequency

-Case of parallel joints: $\lambda_L = \mu_L \lambda_A$

-Case of randomly oriented joints (Underwood, 1967): $\lambda_L = \frac{2}{\pi}\mu_L \lambda_A$

-General cases: $\frac{2}{\pi}\mu_L \lambda_A < \lambda_L < \mu_L \lambda_A$

▶ Practical determination of discontinuity size

-There are two kinds of method in estimation of joint size: distribution-dependent and distribution-free methods. In the former method, trace length distribution is calculated from an assumed joint size or diameter distribution and is compared with the sampled trace length distribution. This process is repeated after having changed the joint size distribution until the error between sampled and calculated trace length distribution becomes smaller than a threshold value.

-In distribution-free method, no assumption of joint size distribution is required. Since the joint size distribution is directly calculated from the joint trace length distribution it is less time consuming and above all more likely to give a solution with less error than those of distribution-dependent method.

▶ Generation of random fracture networks

-Joint generation space should be always greater than the joint sampling space to remove the end effect (boundary effect). Samaniego & Priest have recommended 4 times greater space for the joint generation space.

-B.4(p.407~411) can be used for the generation of statistical distributions.