# 445.204

# Introduction to Mechanics of Materials

# (재료역학개론)

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# Homework #4

Page 186-188: #5.1, 5.5, 5.7, 5.12, 5.17

Page 198-201: #5.23, 5.24, 5.31, 5.40

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Due by **Mid-night on May 18 (Mon.)**
**Through ETL !**

# Chapter 6

# Shear and Moment in Beams

# Outline

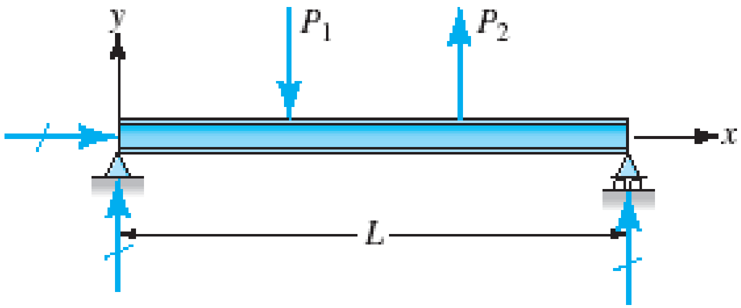
- Classification of Beams
- Calculation of Beam Reactions
- Shear Force and Bending Moment
- Load, Shear, and Moment Relationships
- Shear and Moment Diagrams
- (Optional) Discontinuity Functions

Beams are important members used in bridges (as shown in this picture) and a variety of other structures. The internal shear forces and bending moments in beams resulting from external loadings will be studied in this chapter.
*Paul Raftery/Alamy Limited.*

# What we learn …

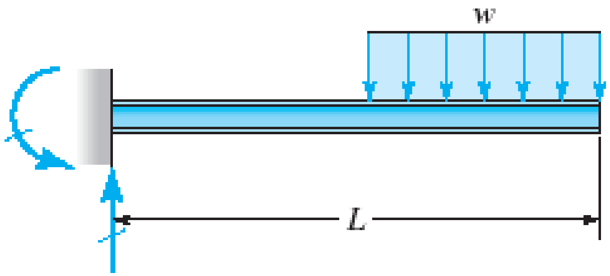
- Analysis of force and moment distributions in beams using statics principles
- What kinds of beams ?
- How to calculate reaction forces at supports
- How to construct 1) shear force and 2) bending moment diagrams

# Types of beams

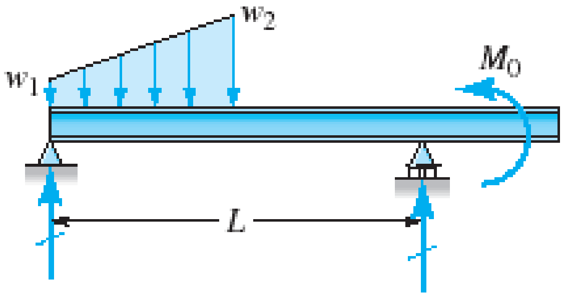



Simple beam

Or simply supported beams




Cantilever beam




Overhang beam

# Types of beams

# Calculation of beam reaction forces

- Linear elastic – Hooke‘s law, unless otherwise specified
- Equations of static equilibrium are used to determine the reaction forces of a loaded beam
- Self-weight of the beam is usually neglected unless otherwise specified.

# Example of cantilever beam

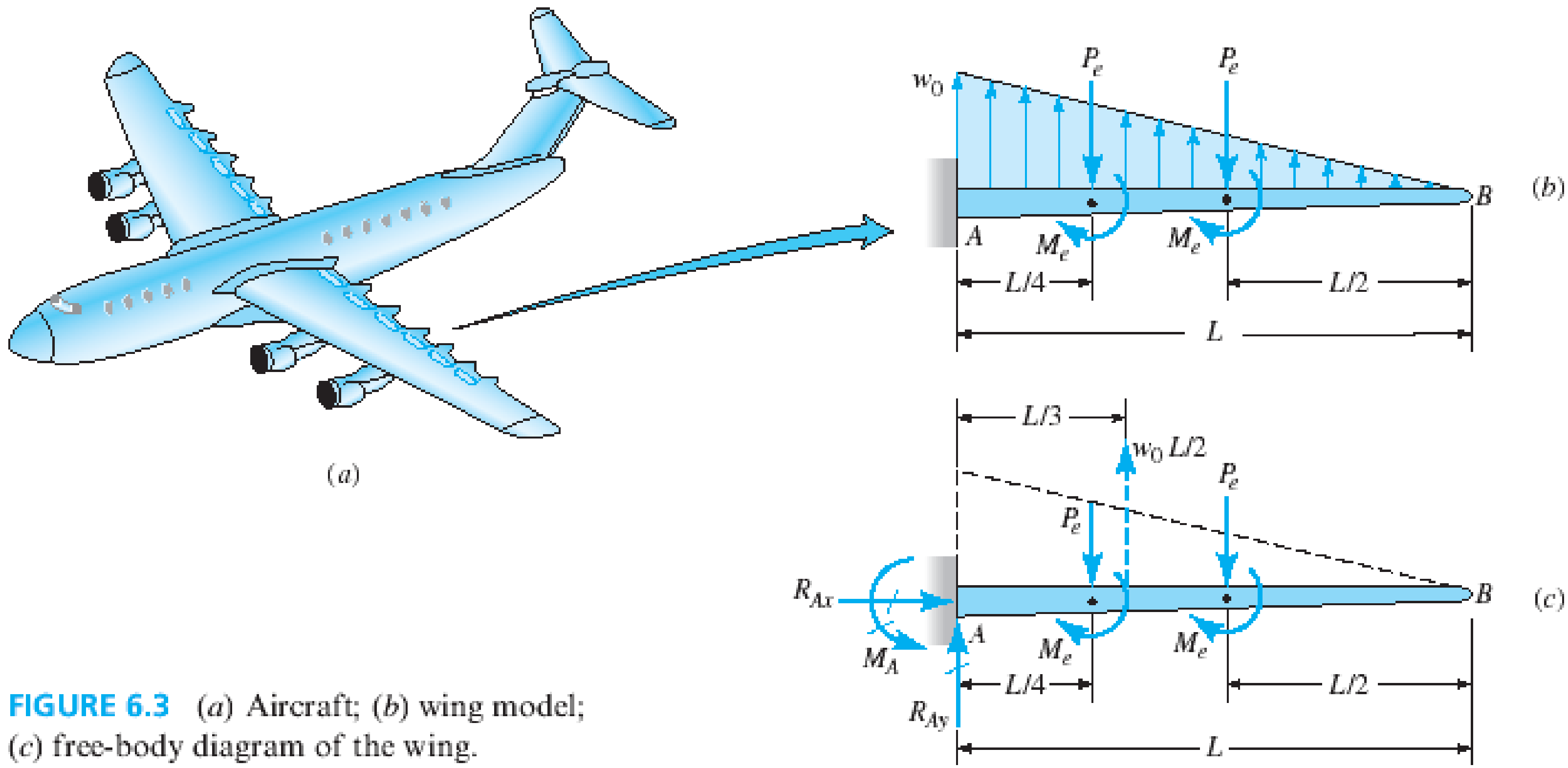
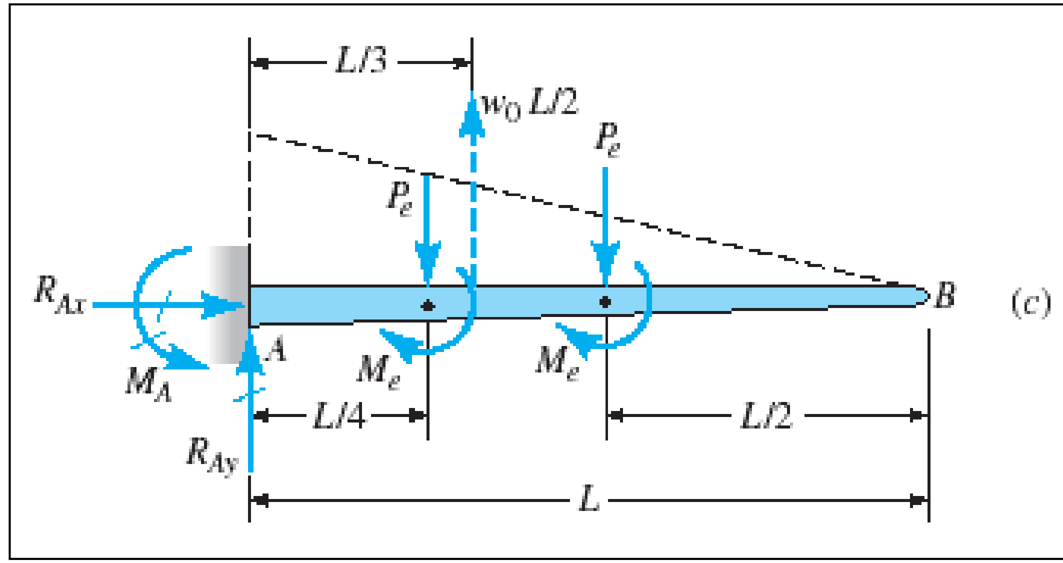



**FIGURE 6.3** (*a*) Aircraft; (*b*) wing model; (*c*) free-body diagram of the wing.




$$\Sigma M_A = \left(\tfrac{1}{2} w_o L\right)\cdot \frac{L}{3} - P_e\left(\frac{L}{4} + \frac{L}{2}\right) - 2M_e + \underline{\underline{M_A}} = 0$$

$$\Sigma F_y = R_{Ay} - 2P_e + \tfrac{1}{2} w_o L = 0$$

$$\therefore \quad M_A = -764 \text{ kip}\cdot\text{in} \circlearrowright \qquad R_{Ay} = -9.2 \text{ kips} \downarrow$$

# Example of overhang beam

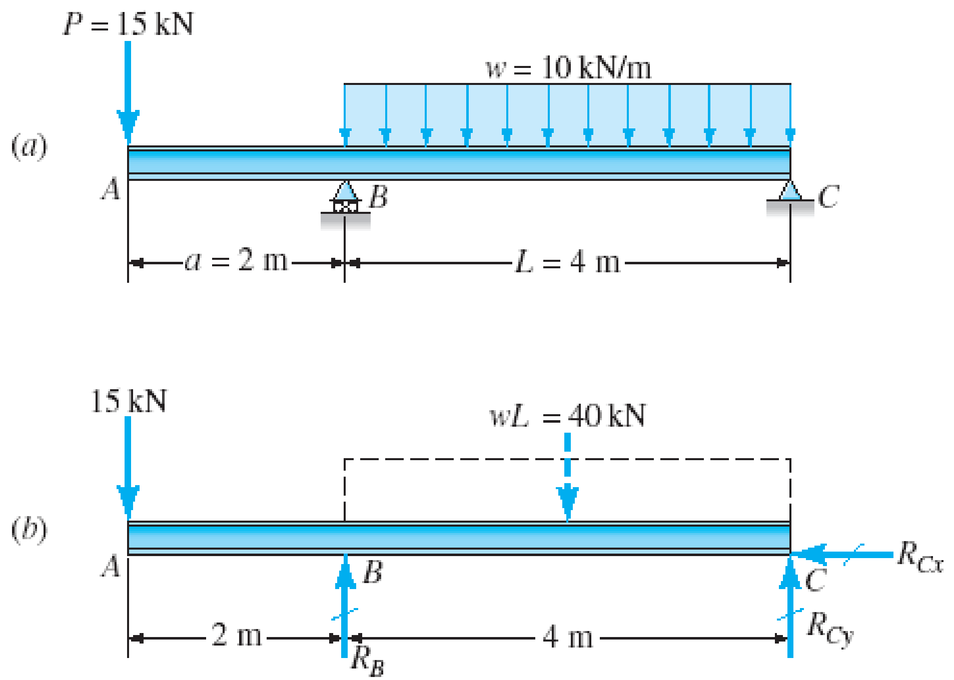
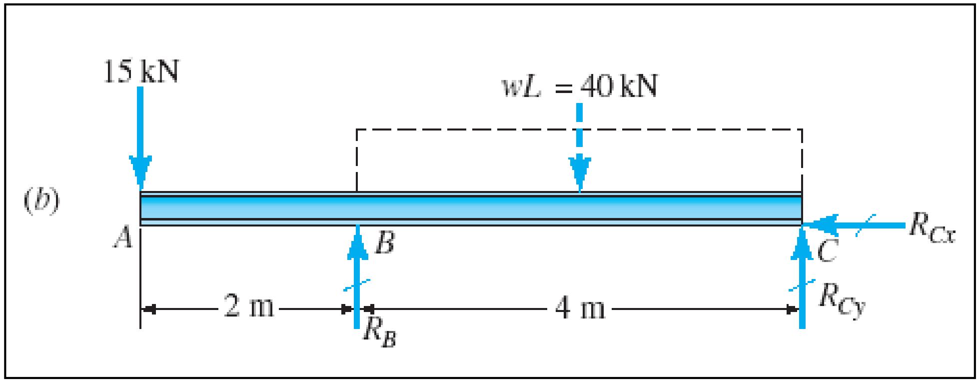



**FIGURE 6.4** (*a*) Beam with an overhang and two loads; (*b*) free-body diagram.




$$\Sigma M_B = 15\cdot(2) - 40\cdot(2) + R_{Cy}\cdot(4) = 0$$

$$\Sigma M_C = 15\cdot(6) - R_B\cdot(4) + 40\cdot(2) = 0$$

$$\therefore R_{Cy} = 12.5 \text{ kN}$$

$$R_B = 42.5 \text{ kN}$$

# Example of simple (simply supported) beam

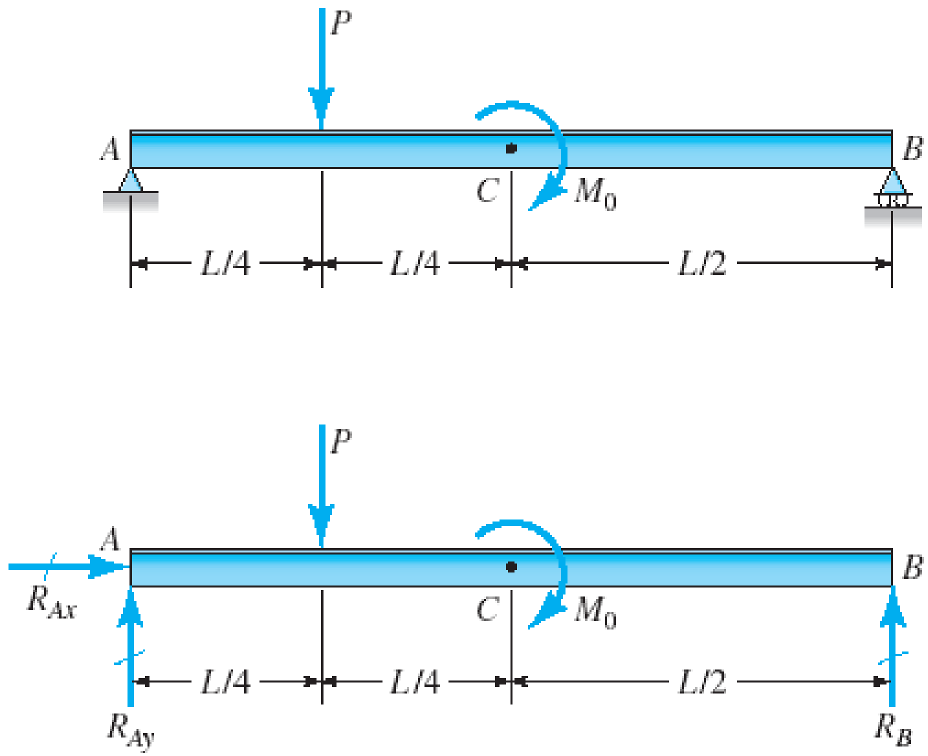
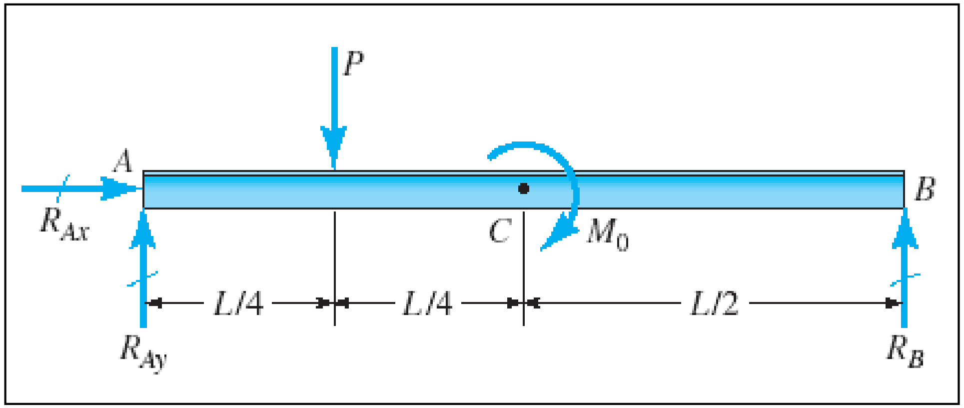



**FIGURE 6.5** (*a*) A simple beam subjected to a concentrated load *P* and moment $M_0$; (*b*) free-body diagram.

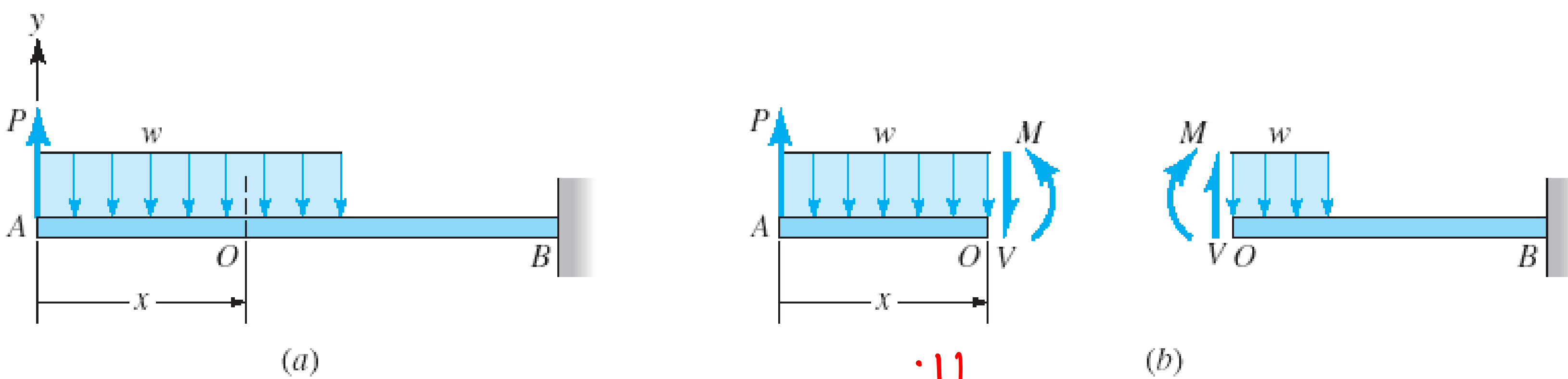



$$\Sigma M_A = R_B \cdot L - M_o - P \cdot \frac{L}{4} = 0$$

$$\Sigma M_B = -R_{Ay} \cdot L + P \cdot \frac{3}{4} L - M_o = 0$$

# Shear force and bending moment

- Under transverse loads in the beam, stress resultants are 1) shear force V and bending moment M



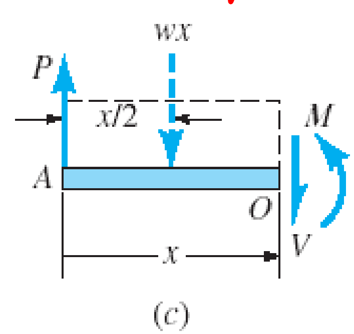




**FIGURE 6.7** Shear force $V$ and bending moment $M$: (*a*) cantilever beam; (*b*) free-body diagrams of parts $AO$ and $BO$; (*c*) free-body diagram of part $AO$ with concentrated loads $P$ and $wx$.

$$\Sigma F_y = P - \omega x - V = 0$$

$$\therefore V = P - \omega x$$

$$\Sigma M_O = -P \cdot x + \omega x \cdot \frac{x}{2} + M = 0$$

$$\therefore M = Px - \frac{1}{2}\omega x^2.$$

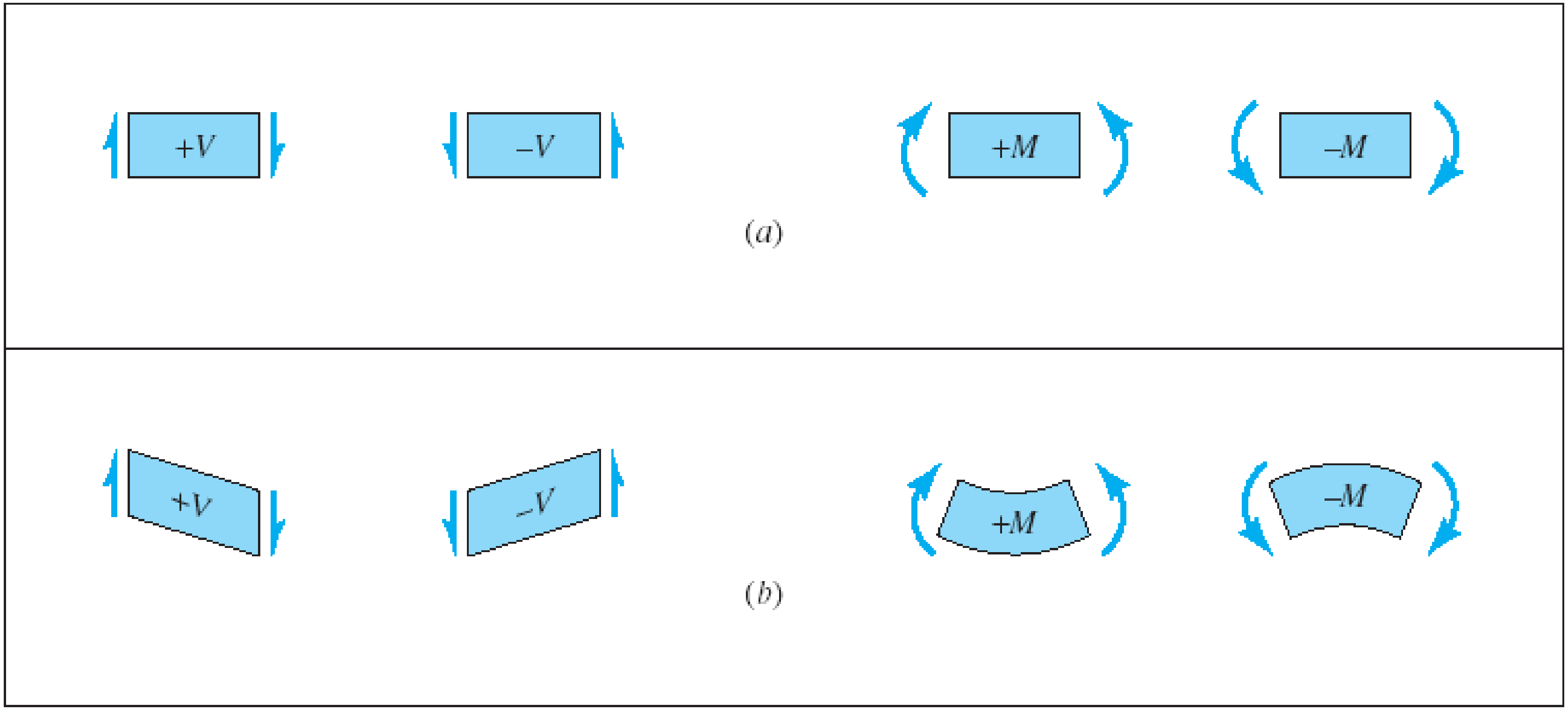
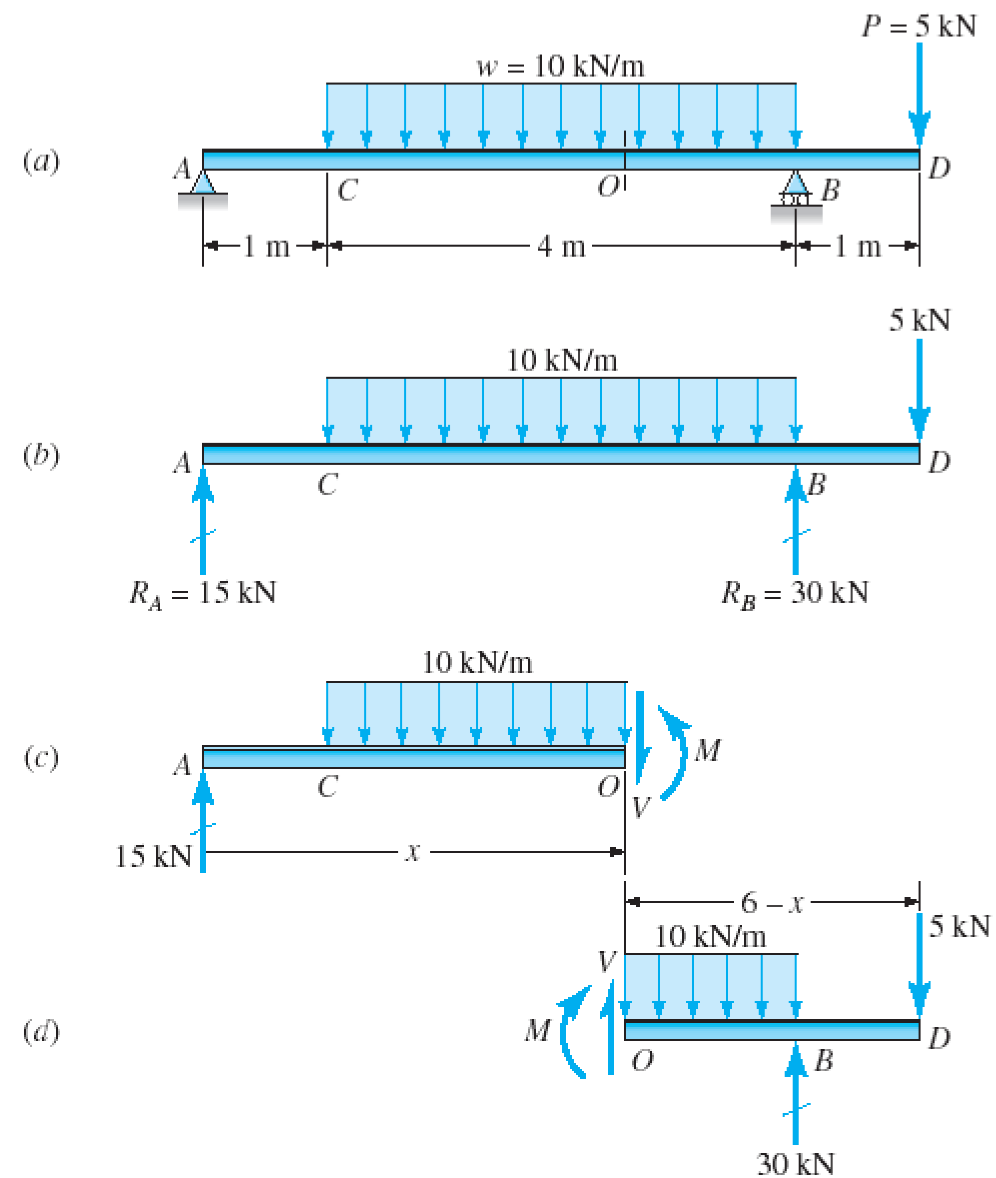



**FIGURE 6.8** Sign convention for beams: (*a*) definitions of positive and negative shear force and bending moment; (*b*) deformations (highly exaggerated) of beam segments caused by shear and moment.



(a)
P = 5 kN
w = 10 kN/m
A
C
O
B
D
1 m
4 m
1 m
(b)
5 kN
10 kN/m
A
C
B
D
R_A = 15 kN
R_B = 30 kN
(c)
10 kN/m
A
C
O
M
V
15 kN
x
(d)
6 – x
10 kN/m
5 kN
V
M
O
B
D
30 kN

(c)

$$\sum F_y = 15 - 10\cdot(x-1) - V = 0$$

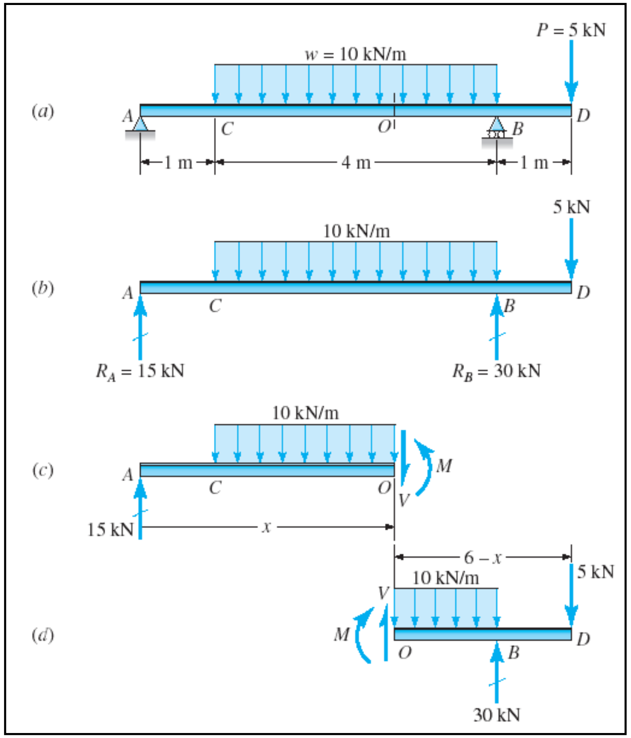
$$\sum M_O = -15\cdot x + \underline{10(x-1)}\cdot\underline{\tfrac{1}{2}(x-1)} + M = 0$$

$$\therefore V = -10x + 25$$

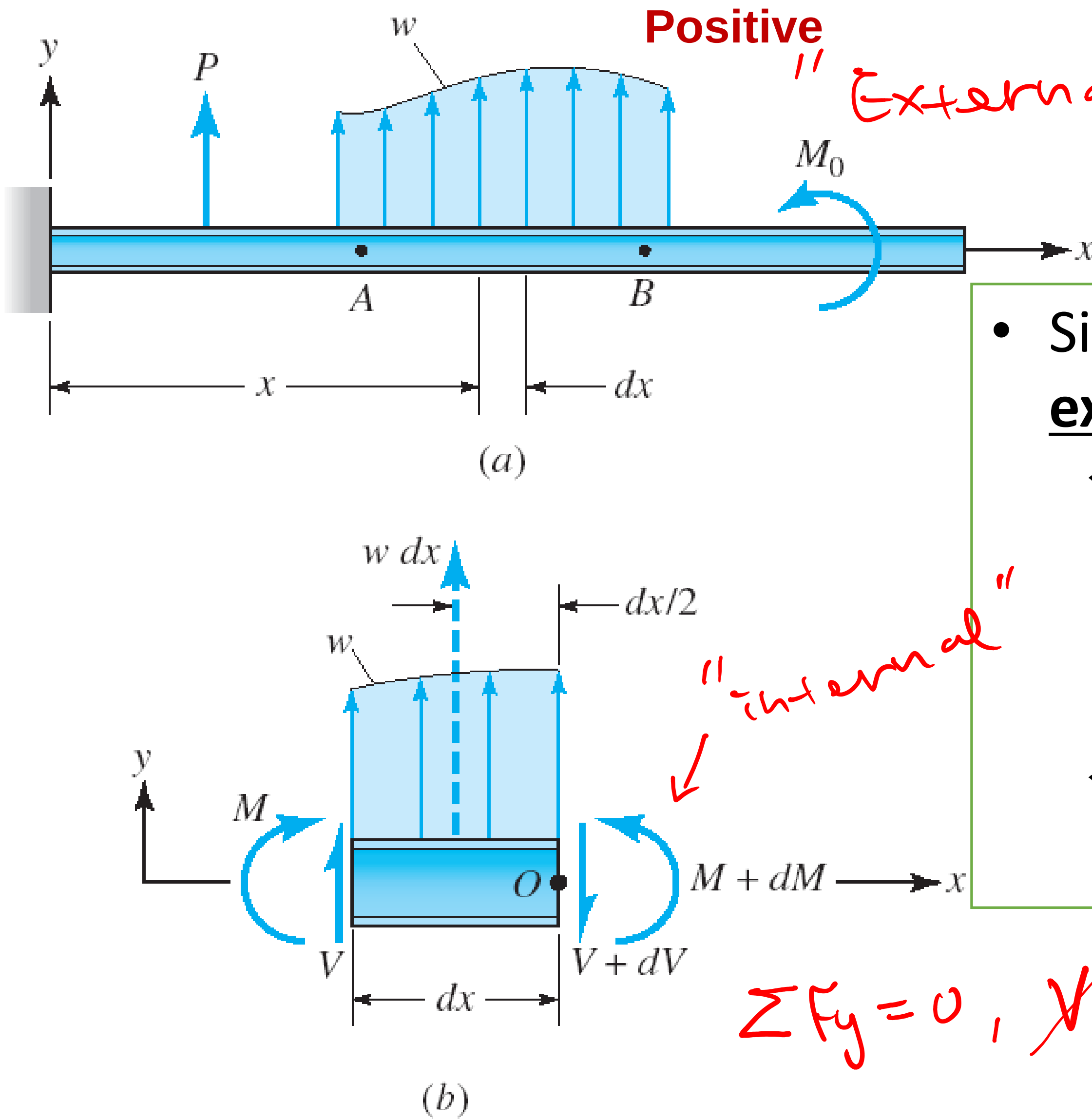
$$M = -5x^2 + 25x - 5$$

at $x = 3.4$ m $\quad\therefore\quad V = -9$ kN $\downarrow$

$M = 22$ kN·m



# Load, shear, and moment relationships




**Note**

- Sign convention for **<u>external loads</u>**
  - ✓ Distributed and concentrated loads: (+) if upward (or +y), (-) if downward
  - ✓ Moment or couple: (+) if counter-clockwise

$\Sigma F_y = 0, \; \cancel{V} - (\cancel{V} + dV) + w \cdot dx = 0$

$\therefore \; \dfrac{dV}{dx} = w.$

# Shear force and load relation

$$\boxed{\frac{dV}{dx} = w}$$

**"Slope of shear force is w"**

if $w = 0 \therefore V = \text{const}.$

* if $w = \text{const}.$ $V = \underline{\text{linear}}$

<u>In general</u>

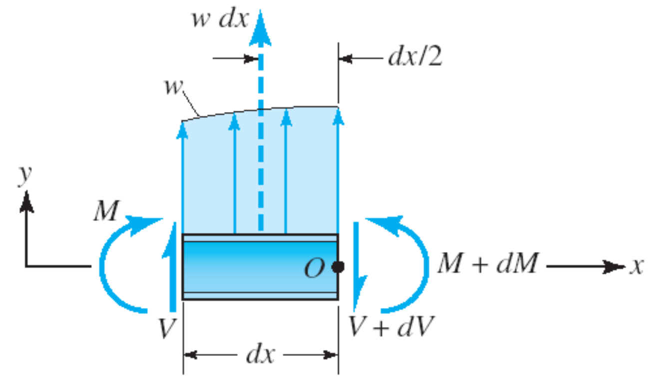
$$\boxed{V_B - V_A = \int_A^B w\,dx}$$

$$\boxed{V_B - V_A = \text{area of the load diagram between } A \text{ and } B}$$

$(\Delta A_{AB})$

$$\therefore \quad \underline{V_B = V_A + \Delta A_{AB}^{(w)}}$$

# Shear force and bending moment relation



(b)

$$\sum M_o = (M + dM) - M - V \cdot dx - (w \cdot dx)\left(\frac{dx}{2}\right) = 0$$

$$\boxed{\frac{dM}{dx} = V}$$

$$\therefore \frac{dM}{dx} = V.$$

$$\boxed{M_B - M_A = \int_A^B V\,dx}$$

$$\boxed{M_B - M_A = \text{area of the shear diagram between } A \text{ and } B}$$

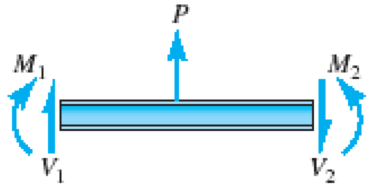
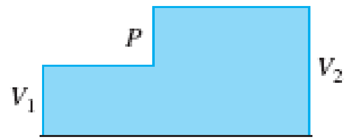
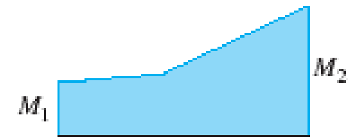
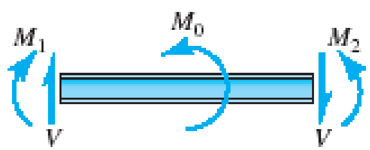
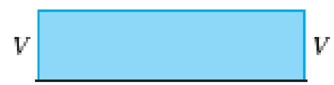
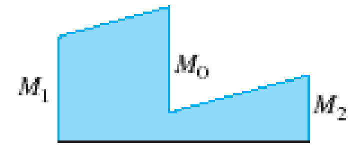
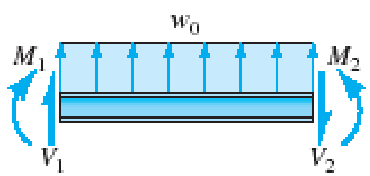
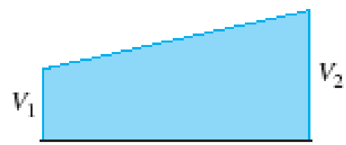
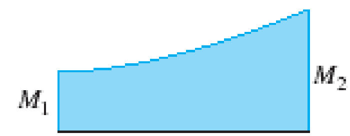
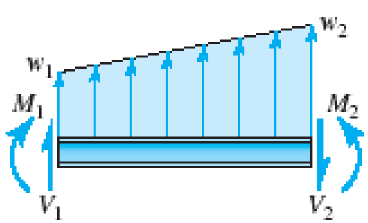
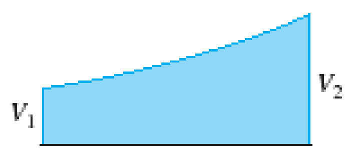
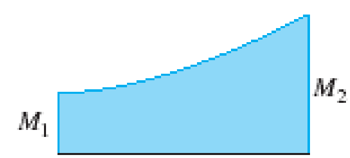
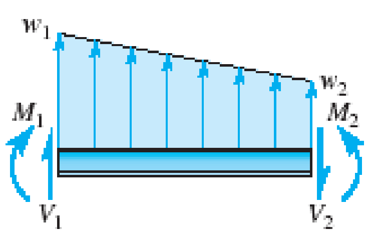
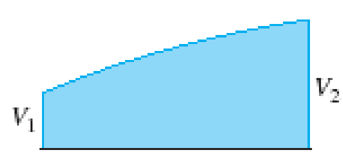
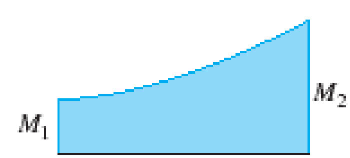
$$M_B = M_A + \Delta A_{AB}^{(V)}$$

# Graphic method

1. Determine the reactions from the free-body diagram, or load diagram, of the entire beam.

2. Determine the value of shear at the change-of-load points, successively summing from the left end of the beam the vertical external forces.

3. Draw the shear diagram obtaining. Locate the points of zero shear. Determine the values of moment at the change-of-load points and at points of zero shear, either continuously summing the external moments from the left end of the beam. Draw the moment diagram.

| | Loading | Shear Diagram $\frac{dV}{dx} = w$ | Moment Diagram $\frac{dM}{dx} = V$ |
|---|---|---|---|
| A |   |   Positive $V$-jump |   Constant slope changes from $V_1$ to $V_2$ |
| B |   |   Zero slope |   Negative $M$-jump |
| C |   |   Constant positive slope |   Positive slope that increases from $V_1$ to $V_2$ |
| D |   |   Positive slope that increases from $w_1$ to $w_2$ |   Positive slope that increases from $V_1$ to $V_2$ |
| E |   |   Positive slope that decreases from $w_1$ to $w_2$ |   Positive slope that increases from $V_1$ to $V_2$ |

# Ex. 6.6.

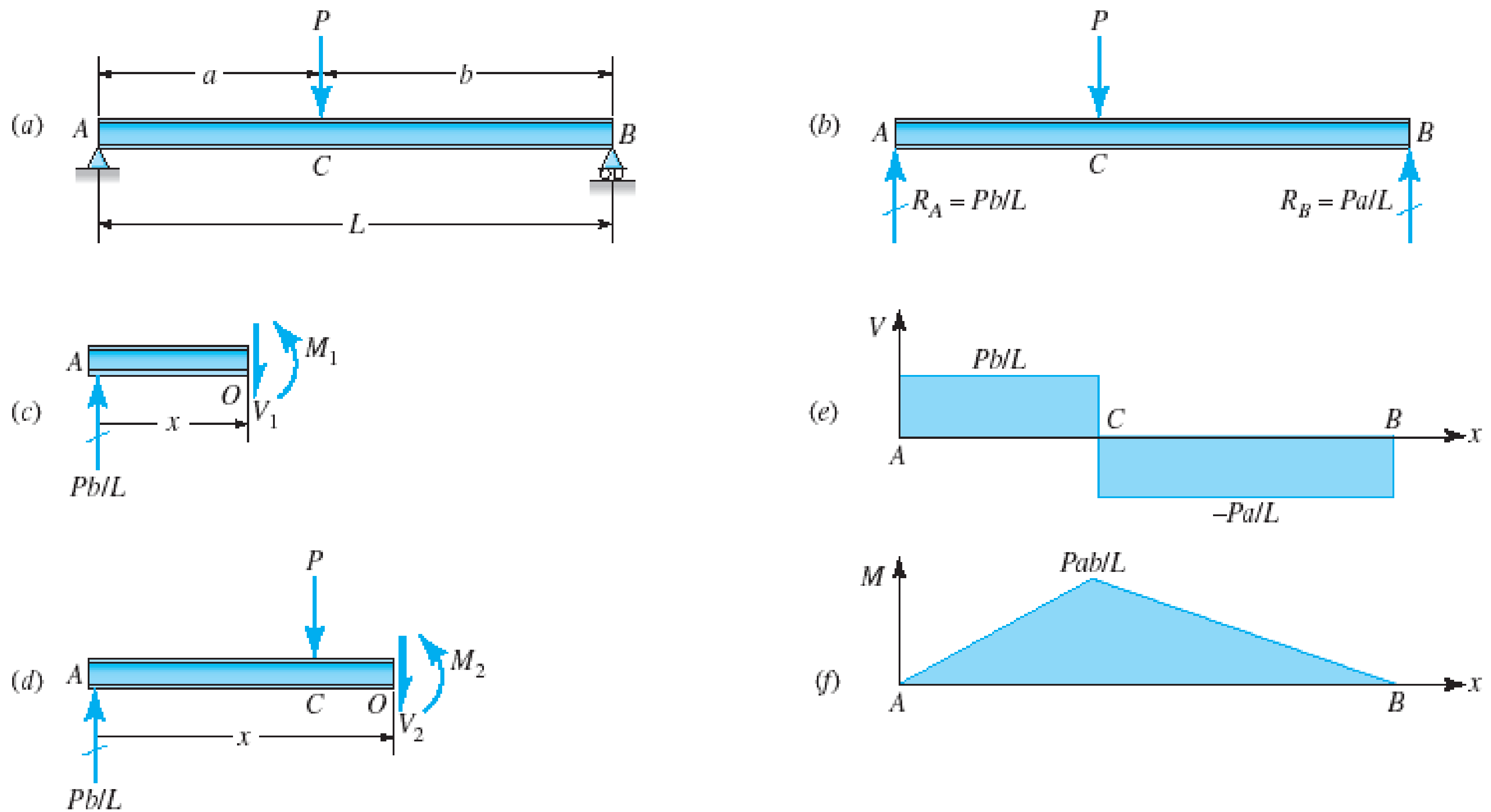



**FIGURE 6.11** Equilibrium approach: (*a*) simply supported beam under a load *P*; (*b*) free-body diagram of entire beam; (*c*, *d*) free-body diagrams of two parts *AO*; (*e*) shear diagram; (*f*) moment diagram.

(i) Reactions

$$R_A = \frac{Pb}{L}, \quad R_B = \frac{Pa}{L}$$

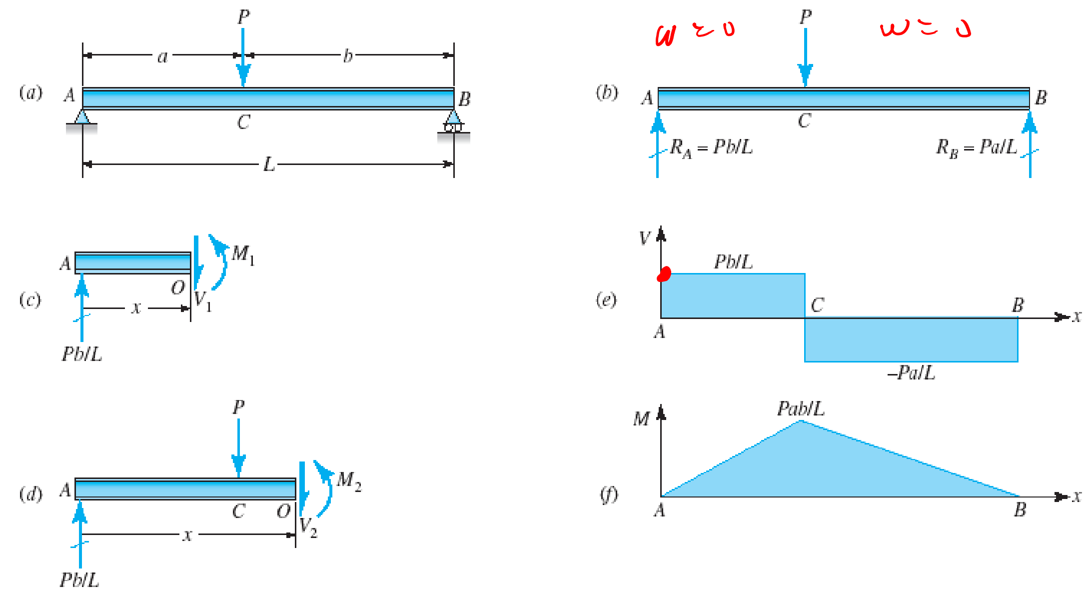



**FIGURE 6.11** Equilibrium approach: (*a*) simply supported beam under a load *P*; (*b*) free-body diagram of entire beam; (*c*, *d*) free-body diagrams of two parts *AO*; (*e*) shear diagram; (*f*) moment diagram.

(ii) $A \rightarrow C$

$$\Sigma F_y = 0, \ \Sigma M_0 = 0$$

$$\therefore \ V_1 = \frac{Pb}{L}, \quad M_1 = \frac{Pb}{L}x$$

(iii) $C \rightarrow B$, $\quad V_2 = \frac{Pb}{L} - P = -\frac{Pa}{L}$

$$M_2 = \frac{Pb}{L}x - P(x-a)$$

# Ex. 6.7.

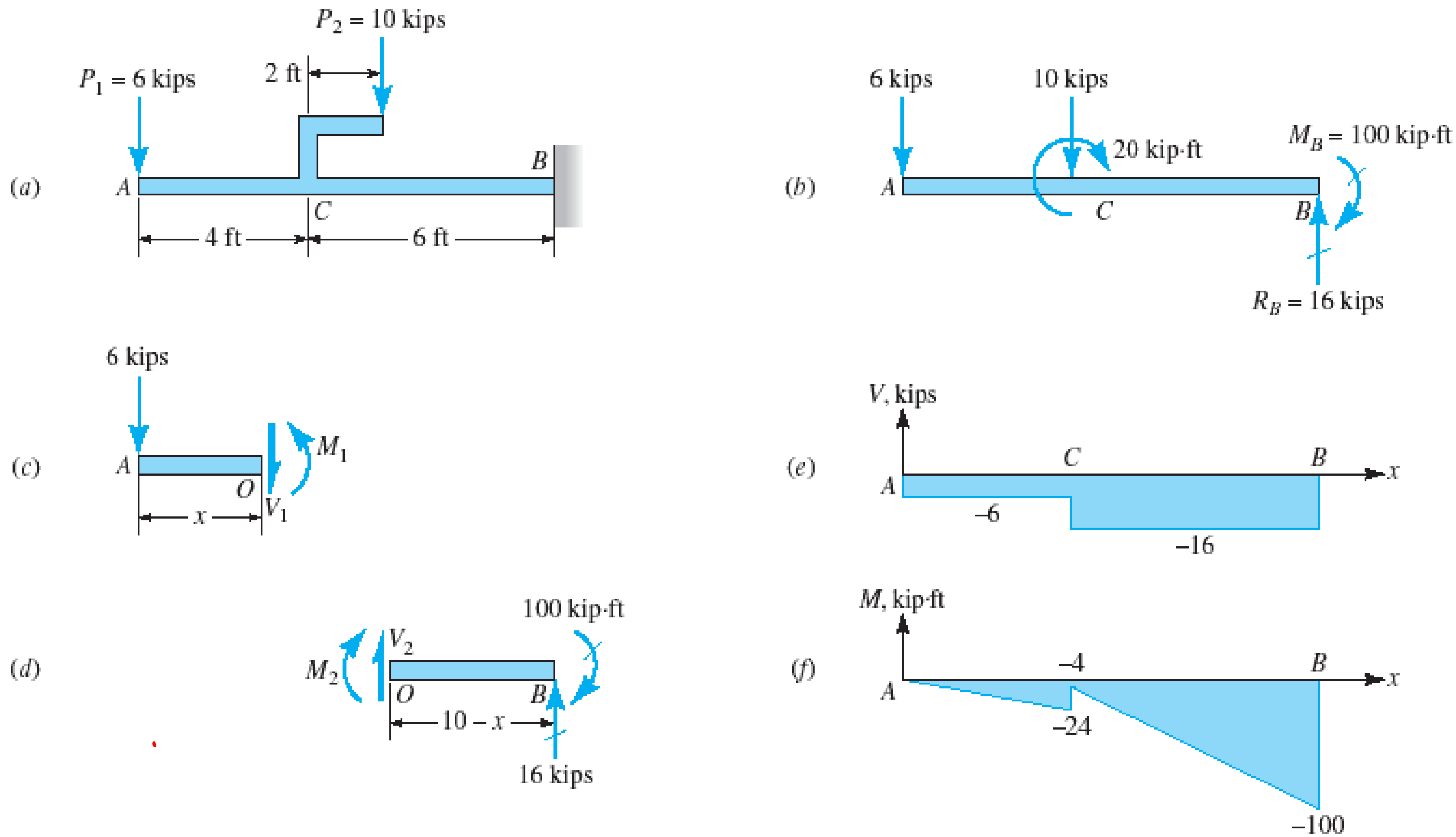



**FIGURE 6.12** Equilibrium approach: (*a*) cantilever beam with two loads; (*b*) free-body diagram of the entire beam; (*c*, *d*) free-body diagrams for parts *AO* and *BO*, respectively; (*e*) shear diagram; (*f*) moment diagram.

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# Ex. 6.9.

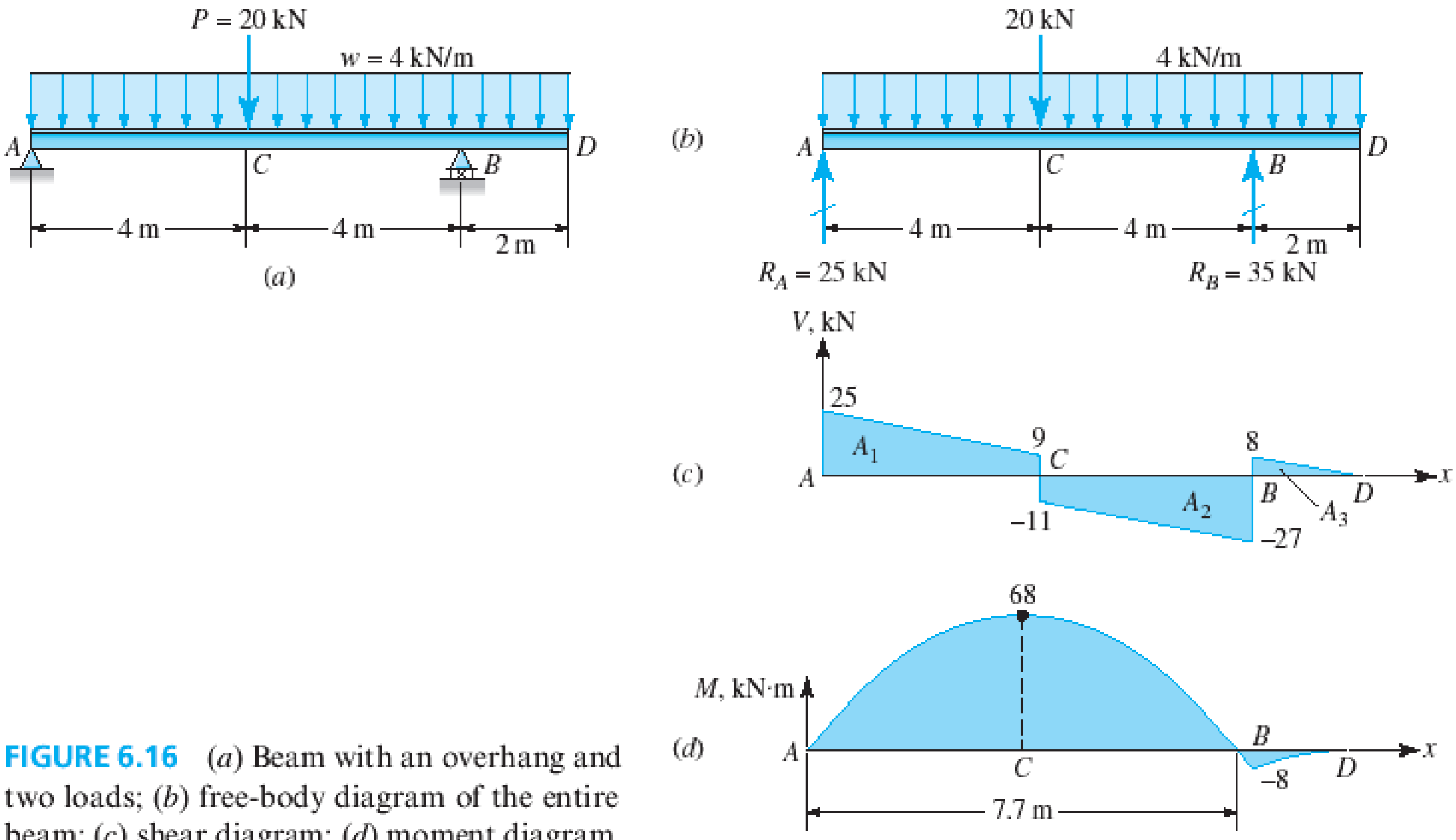


**FIGURE 6.16** (*a*) Beam with an overhang and two loads; (*b*) free-body diagram of the entire beam; (*c*) shear diagram; (*d*) moment diagram.

# Summary

**Shear force and load relation**

$$dV/dx = w$$

$$V_B - V_A = \int_A^B w\,dx$$

$$= \text{area of load diagram between } A \text{ and } B$$

**Moment and shear force relation**

$$dM/dx = V$$

$$M_B - M_A = \int_A^B V\,dx$$

$$= \text{area of shear diagram between } A \text{ and } B$$