445.204

Introduction to Mechanics of Materials

(재료역학개론)

Myoung-Gyu Lee, 이명규

Tel. 880-1711; Email: myounglee@snu.ac.kr

TA: Seong-Hwan Choi, 최성환

Lab: Materials Mechanics lab.

Office: 30-521

Email: cgr1986@snu.ac.kr

Final exam.

- June 15, 2020 (Monday)
- 3:30 pm to 6:15pm
- Eng. Building #33, Rooms: 225,226,228,229,230,231

- Chapters 1 – 7, 10
- About 10 questions with sub-questions
- Excludes optional chapters, but includes one taught in the class
- Absolute evaluation

Classroom of each student will be notified before test.

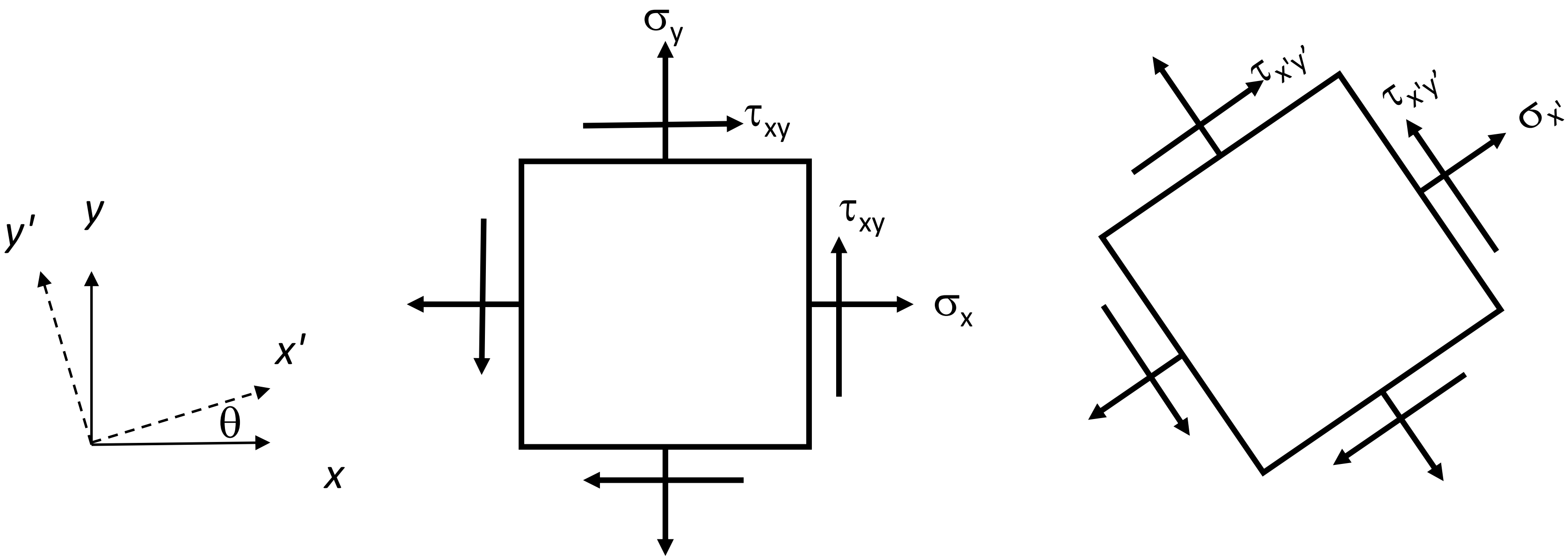
* 수강생이 강의실(기말시험 고사장) 입장 전, 개인방역(손 소독, 마스크) 필수, 체온측정, 문진표 작성 후 문제가 없을 시 입장 가능.(마스크 미 착용시 강의실 입장 불가).
* 수강생 강의실 입장 시 비치되어 있는 손소독 티슈로 책상 및 의자 소독
* 체온 측정 시, 발열증상이 있는 학생은 고사실 입장 불가 (즉시 귀가 조치. 발열 증상이 있는 학생은 선별진료소 혹은 1339로 연락하도록 안내)

Chapter 8

Transformation of stress

Objectives of the chapter

- The most common problems in engineering mechanics involve "transformation of axes"
- Question: stresses are known in x-y plane (or x-y coordinate). Then, what is the stress in the coordinate rotated about θ degrees?
- Stress (and strain) is not coordinate dependent, but they have different components if the coordinates are different!



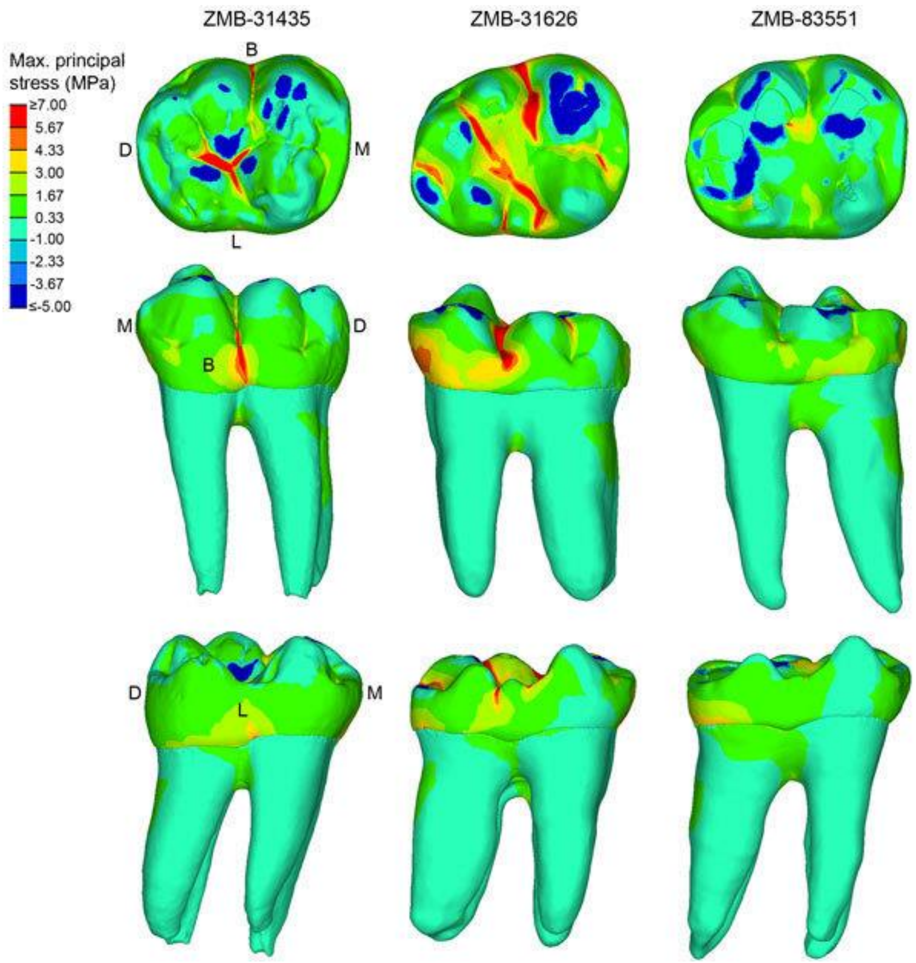
Objectives of the chapter

Two pieces of wood, cut at an angle, and glued together. The wood is being pulled apart by a tensile force *P*.

How do we know if the glued joint can sustain the resultant stress that this force produces?

(Assume that we know the tensile and shear properties of the glue)

Objectives of the chapter



Maximal principal stress distribution observed in three gorilla teeth of an unworn (left), a lightly worn (middle) and a worn (right) condition

Researchers at the Max Planck Institute for Evolutionary Anthropology in Leipzig, Germany, and the Senckenberg Research Institute in Frankfurt am Main, Germany, have conducted stress analyses on gorilla teeth of differing wear stages. Their findings show that different features of the occlusal surface antagonize tensile stresses in the tooth to tooth contact during the chewing process. They further show that tooth wear with its loss of dental tissue and the reduction of the occlusal relief decreases tensile stresses in the tooth. The result, however, is that food processing becomes less effective. Thus, when the condition of the occlusal surface changes during an individual’s lifetime due to tooth wear, the biomechanical requirements on the existing dental material change as well – an evolutionary compromise for longer tooth preservation.

Transformation of Stresses: 2D

Plane stress

- A plane stress condition exists in 2D when the stress in the third direction is not very significant
  - Example: σ_x, σ_y and τ_{xy} may be non-zero, while σ_z, τ_{xz} and τ_{yz} are zero
  - For plane stress problems, however, the strains in all the three directions are non-zero (i.e., ε_x, ε_y and ε_z are non-zero)
- Applications: pressure vessels, thin sheets under stretch
- Special cases of plane stress conditions
  - Uniaxial stress state (ex: $\sigma_y = 0$, $\tau_{xy} = 0$)
  - Pure shear state ($\sigma_x = \sigma_y = 0$)
  - Biaxial stress state ($\tau_{xy} = 0$)

Transformation of Stresses: 2D

Plane stress

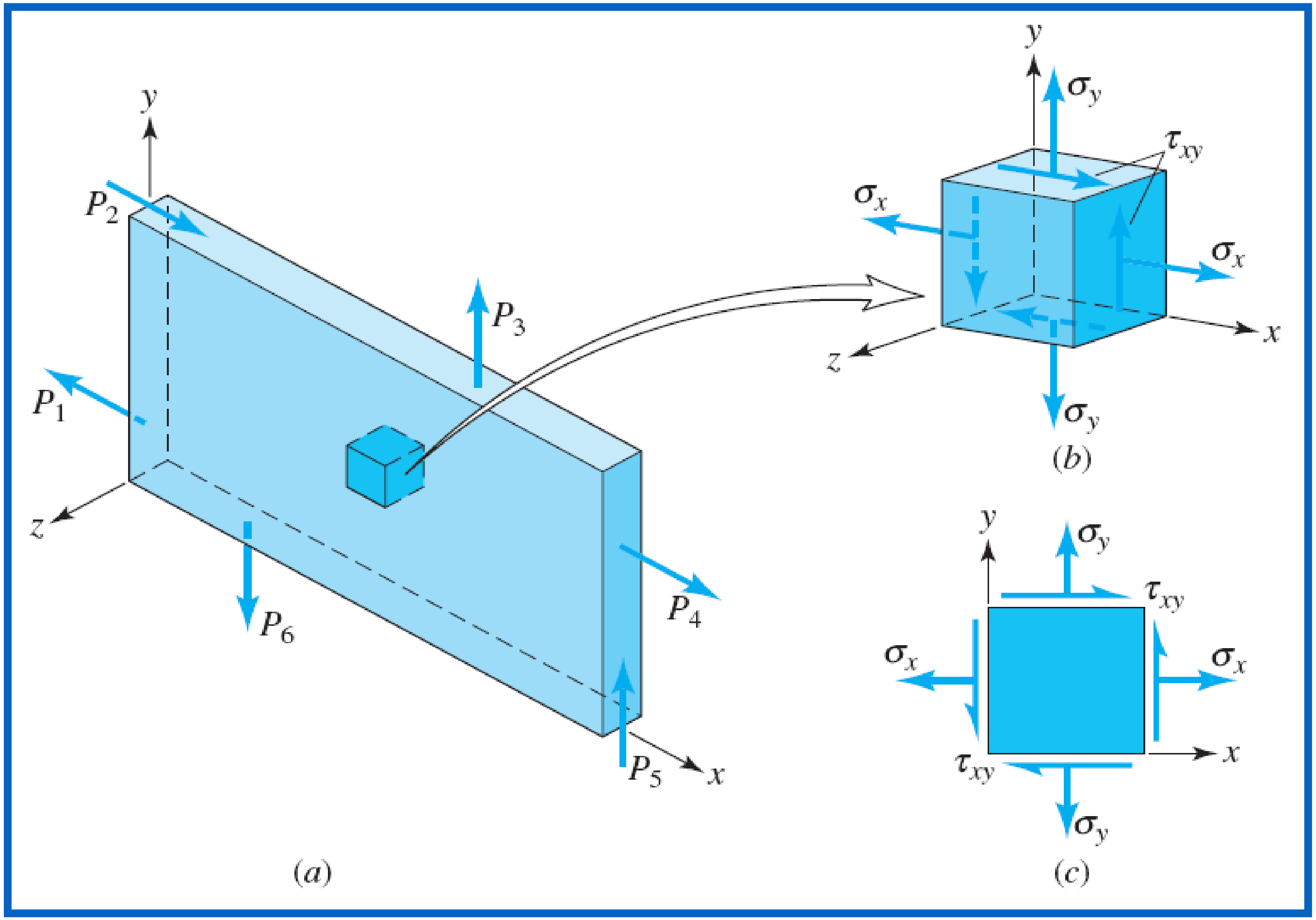
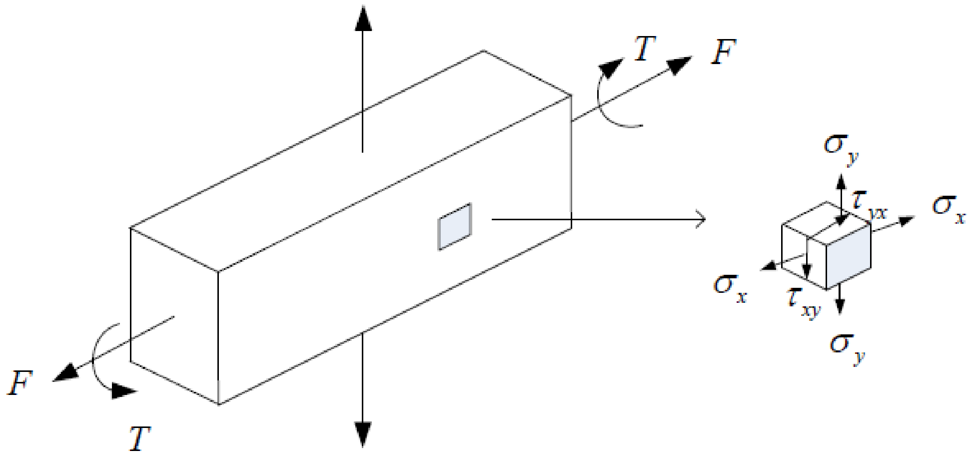
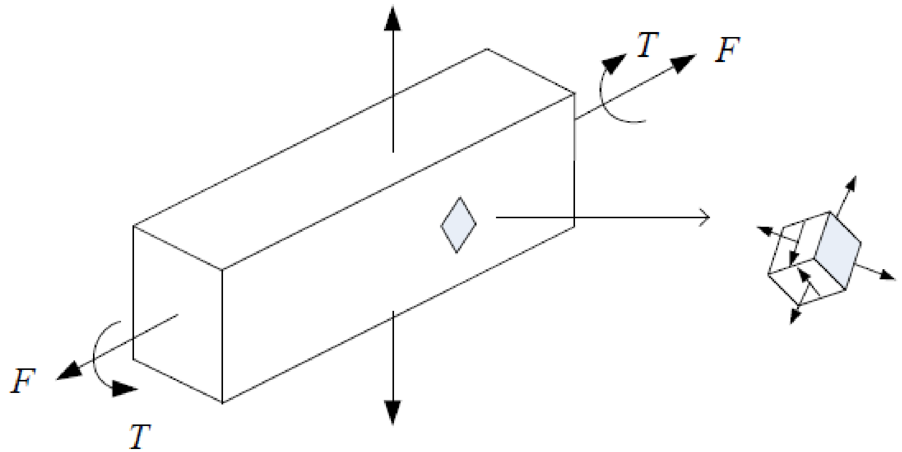


FIGURE 8.1 (a) Thin plate with in-plane loads; (b) element in plane stress; (c) two-dimensional presentation of plane stress.

Transformation of Stresses: 2D

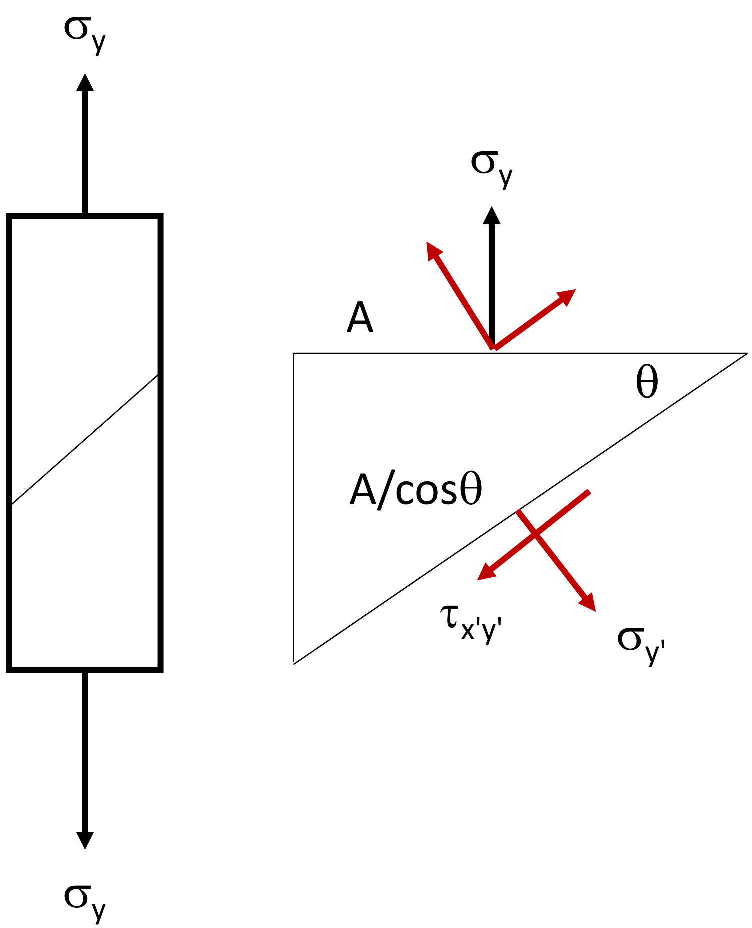
Consideration of static equilibrium





Transformation of Stresses: 2D

1D uniaxial tension



Force equilibrium in the y' direction (normal to the y' plane)

$$(\sigma_y A)\cos\theta - \sigma_{y'}(A/\cos\theta) = 0$$

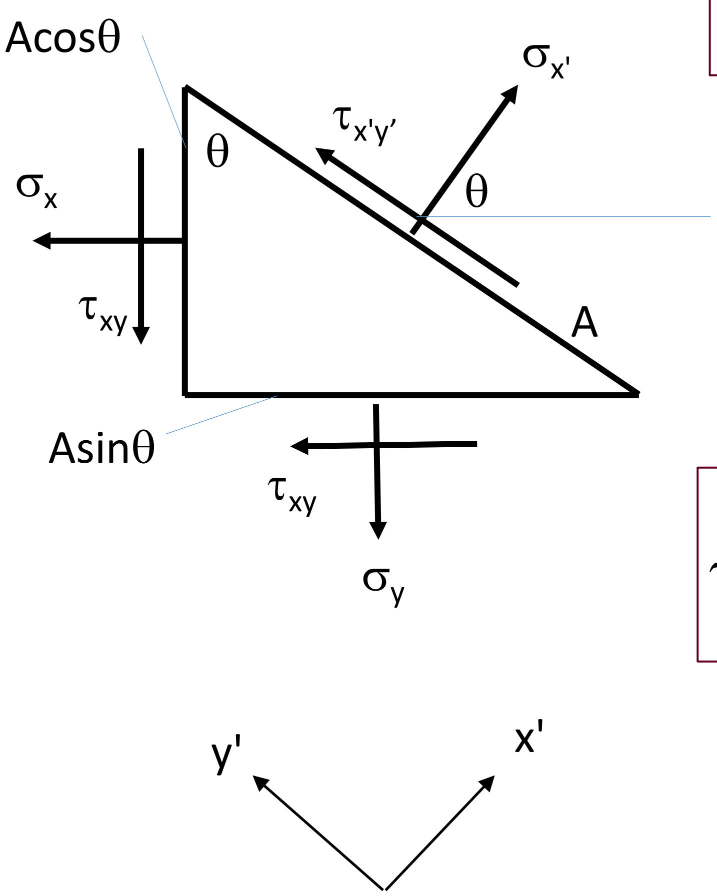
$$\boxed{\sigma_{y'} = \sigma_y \cos^2\theta}$$

Force equilibrium in the tangential direction

$$\boxed{\tau_{x'y'} = \sigma_y \sin\theta \cdot \cos\theta}$$

Transformation of Stresses: 2D, Direct approach

2D plane stress



Force equilibrium in the x’ direction

$$\boxed{\sigma'_x = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta}$$

Force equilibrium in the y’ direction

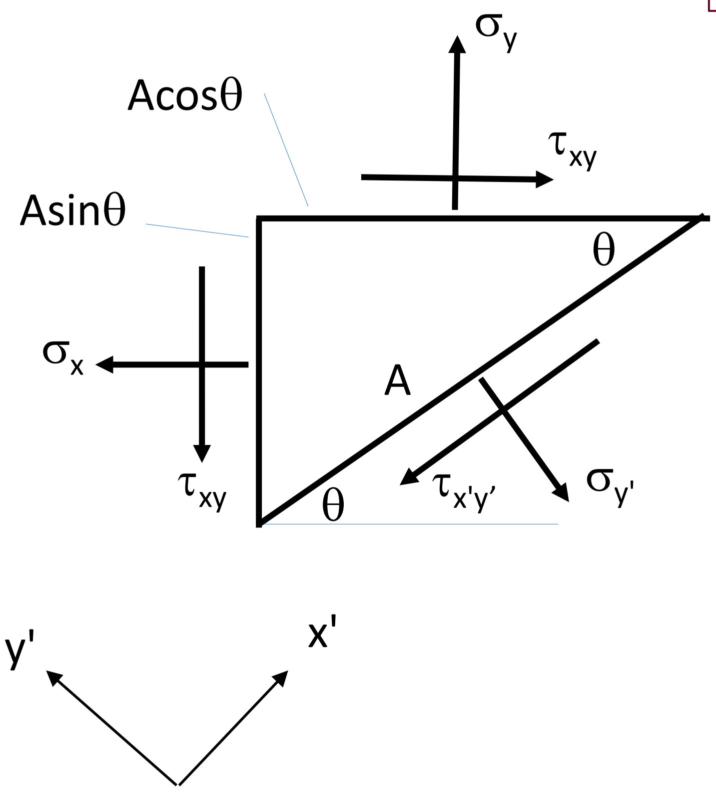
$$\boxed{\tau_{x'y'} = -\frac{\sigma_x}{2}\sin 2\theta + \frac{\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta}$$

Transformation of Stresses: 2D, Direct approach

2D plane stress

<u>Force equilibrium in the y' direction</u>

$$\boxed{\sigma'_y = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cos\theta}$$



Transformation of Stresses: 2D, Direct approach

2D plane stress: summary

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$$

$$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cos\theta$$

$$\tau_{x'y'} = \left(\sigma_y - \sigma_x\right) \sin\theta \cos\theta + \tau_{xy} \left(\cos^2\theta - \sin^2\theta\right)$$

Matrix form

$$\begin{pmatrix} \sigma_{x'} \\ \sigma_{y'} \\ \tau_{x'y'} \end{pmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} \quad \text{or} \quad \boldsymbol{\sigma}' = \mathbf{A}\boldsymbol{\sigma}$$

Transformation of Stresses: 2D

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$$

$$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cos\theta$$

$$\tau_{x'y'} = \left(\sigma_y - \sigma_x\right) \sin\theta \cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$

Trigonometric Identities

$$\sin 2\theta = 2\sin\theta \cos\theta$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

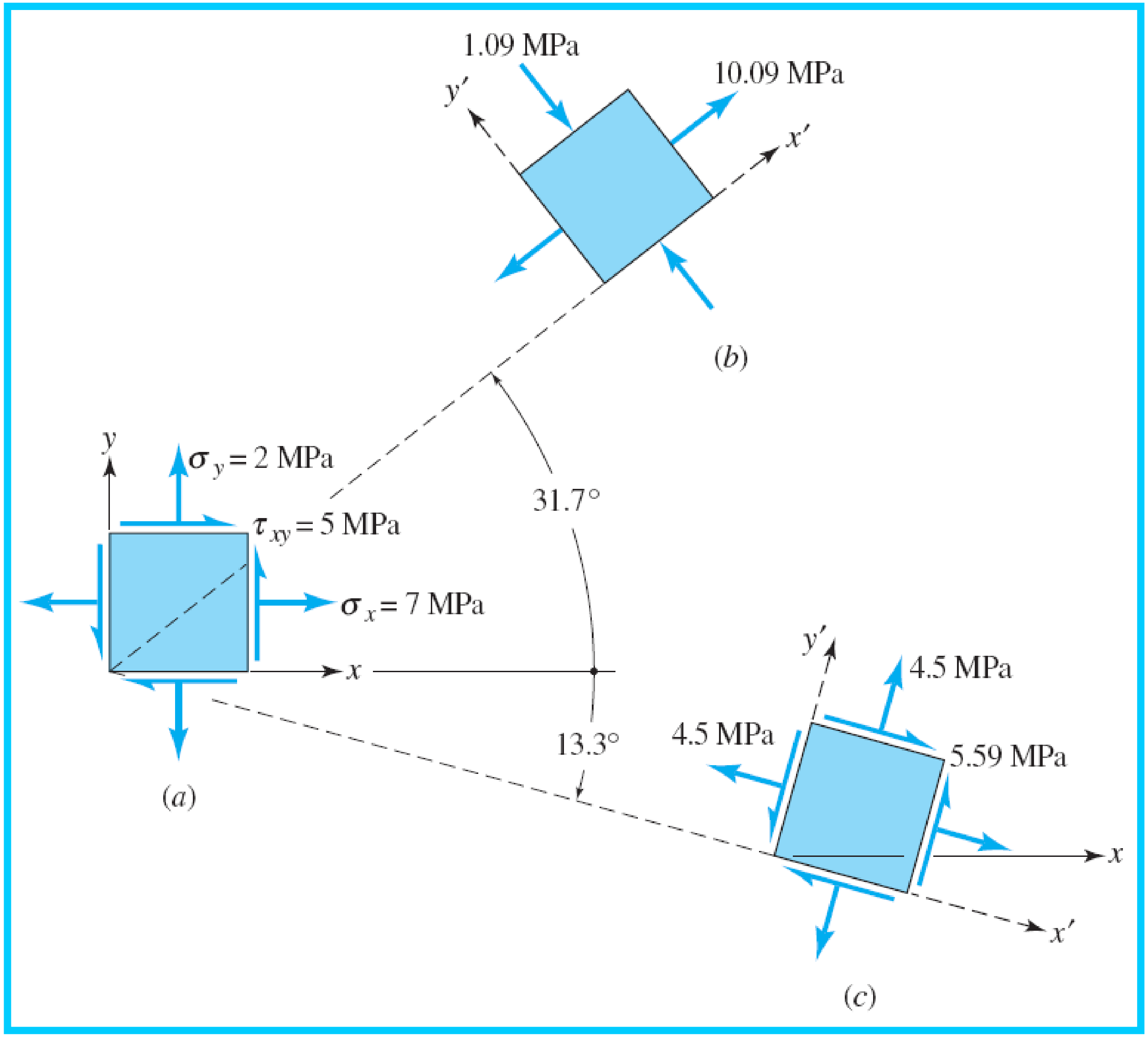
$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

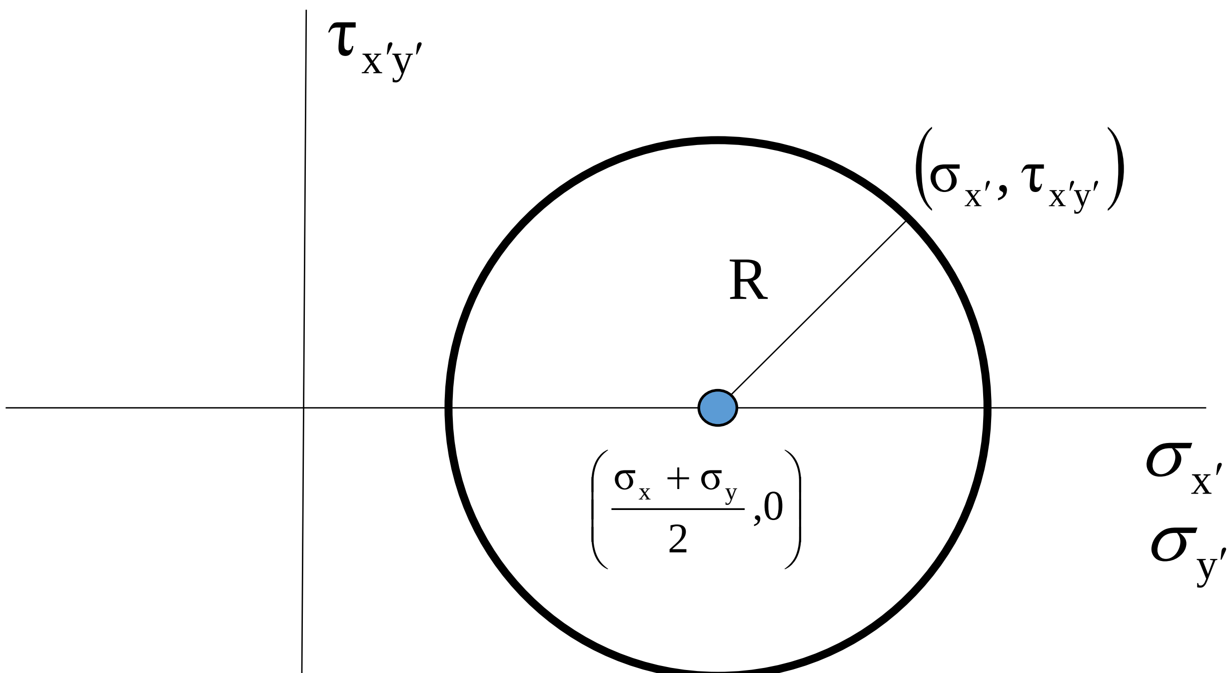
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Transformation of Stresses: Example



Transformation of Stresses: 2D, Mohr Circle

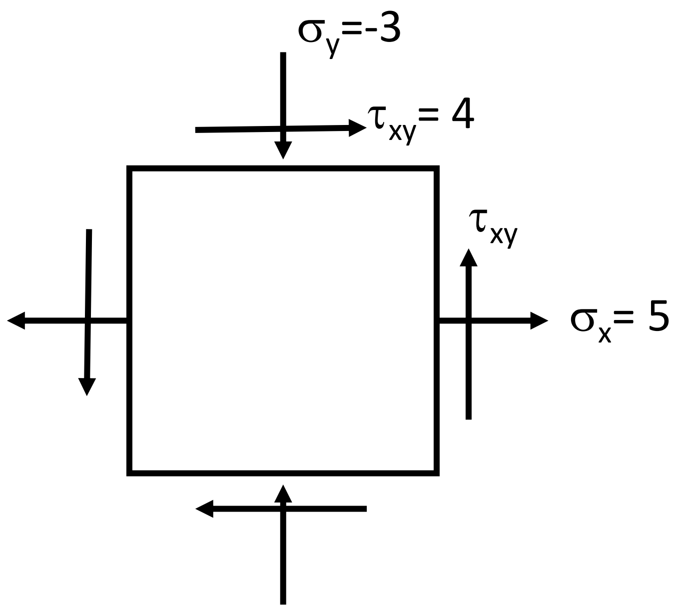
$$\boxed{\left(\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{x'y'}{}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$



$$R = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

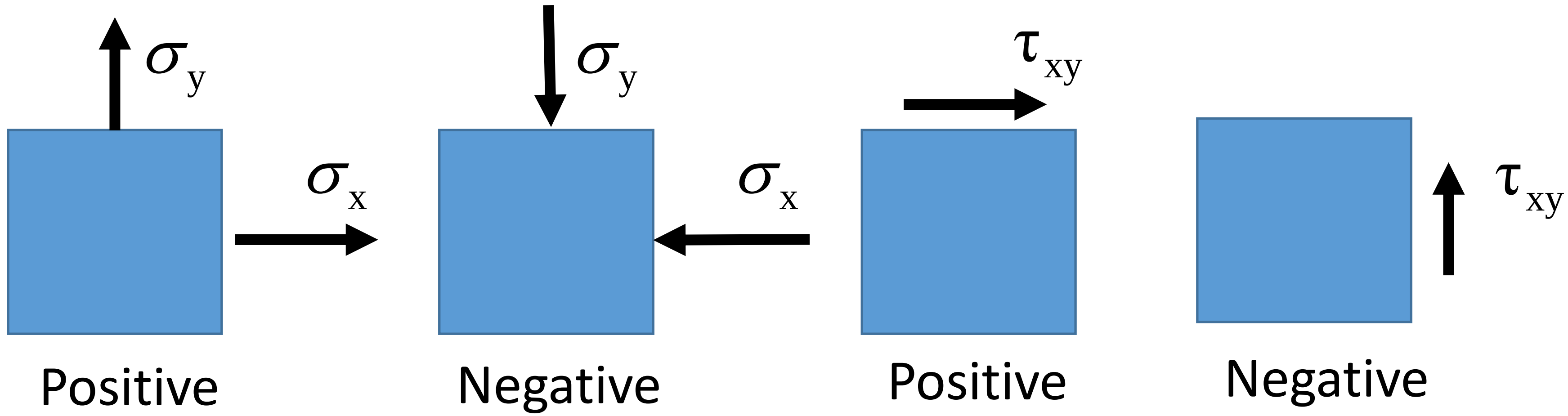
Transformation of Stresses: 2D, Mohr Circle

Steps to draw Mohr's circle: illustration



Step 1:

- Consider a shear stress acting in a clockwise-rotation sense as being positive (+), and counter-clockwise as negative (-)
- The shear stresses on the x and y faces have opposite signs
- The normal stresses are positive in tension and negative in compression as usual



Transformation of Stresses: 2D, Mohr Circle

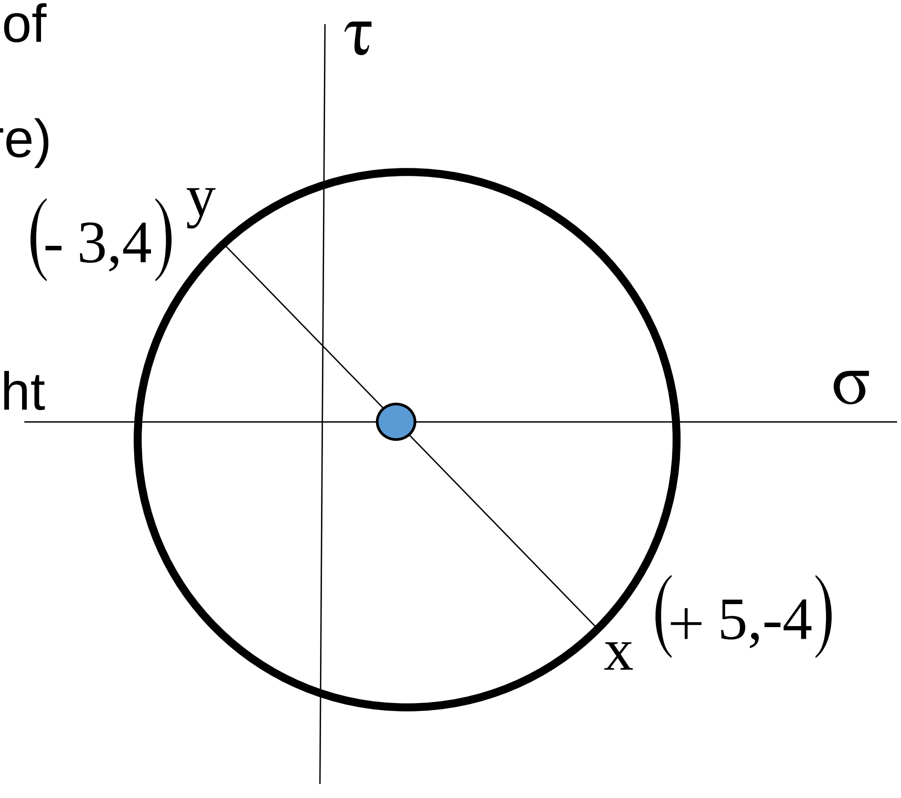
Steps to draw Mohr's circle: illustration

Step 2:

- Construct a graph with t as the ordinate (y axis) and σ as abscissa (x axis).
- Plot the stresses on the x and y faces of the stress as two points on this graph(follow the sign convention before)

Step 3:

- Connect these two points with a straight line
- Draw as circle with the line as a diameter

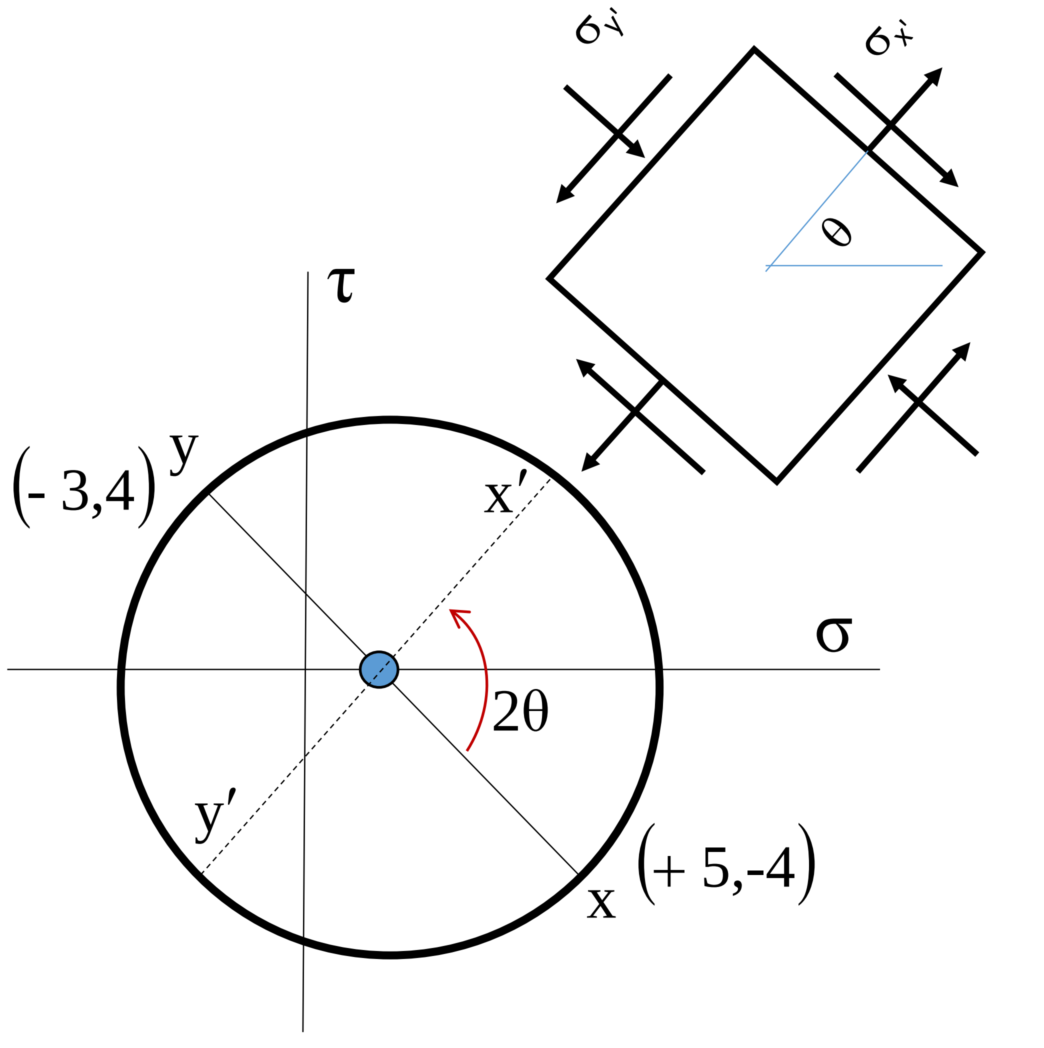


Transformation of Stresses: 2D, Mohr Circle

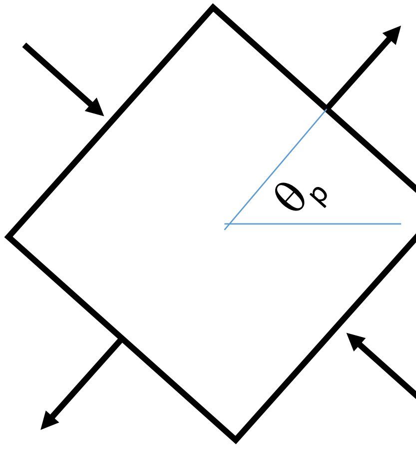
Steps to draw Mohr's circle: illustration

Step 4:

- Determine stresses on a square that has been rotated through an angle θ with respect to the original square
- Rotate the line in the same direction through 2θ. This new end points of the line are labeled as x' and y'.

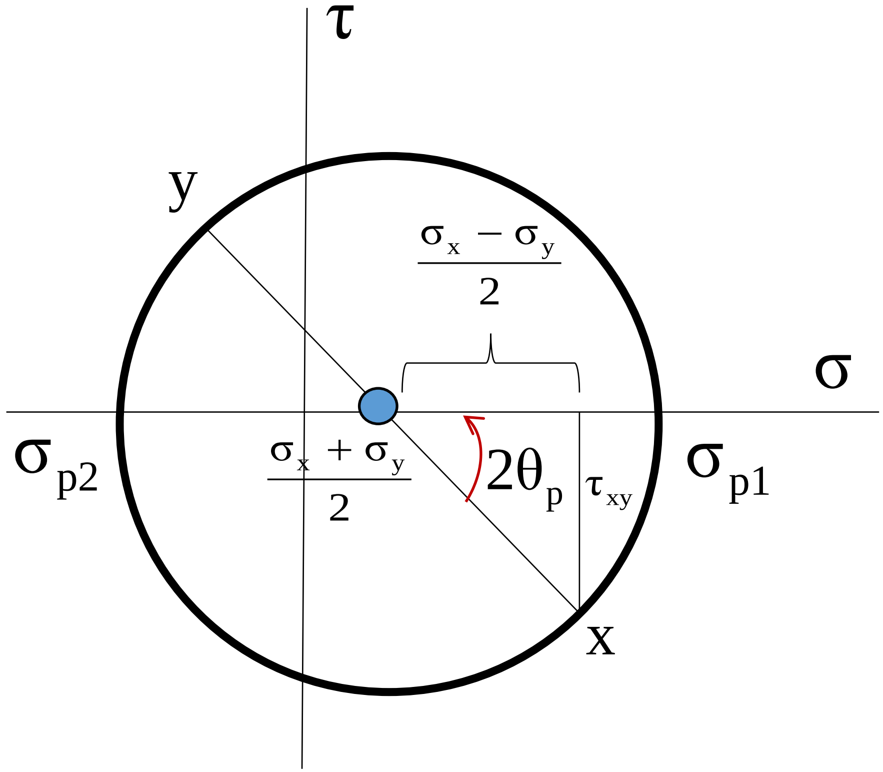


Transformation of Stresses: Principal stress



- Normal stresses become maximum values and the shear stresses are zero

- These normal stresses are called “principal” stresses, σ_{p1} and σ_{p2}



Transformation of Stresses: Principal stress

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

By Pythagorean construction

$$\sigma_{p1,p2} = \frac{(\sigma_x + \sigma_y)}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Principal stresses

$$\tau_{max} = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Maximum shear stress

"The maximum shear are 90 º away from the principal stress points on the Mohr's circle"

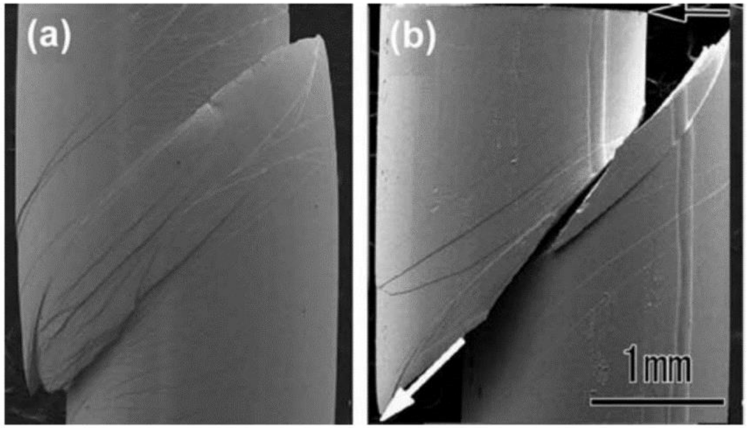
Transformation of Stresses: 2D, Mohr Circle

Maximum shear stress and its plane

Maximum shear stress

$$\tau_{max} = \frac{\sigma_{p1} - \sigma_{p2}}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

" In the tensile specimen, the maximum shear are 45 º away from the loading direction which is the direction of principal stress"



Transformation of Stresses: Invariant

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\Downarrow$$

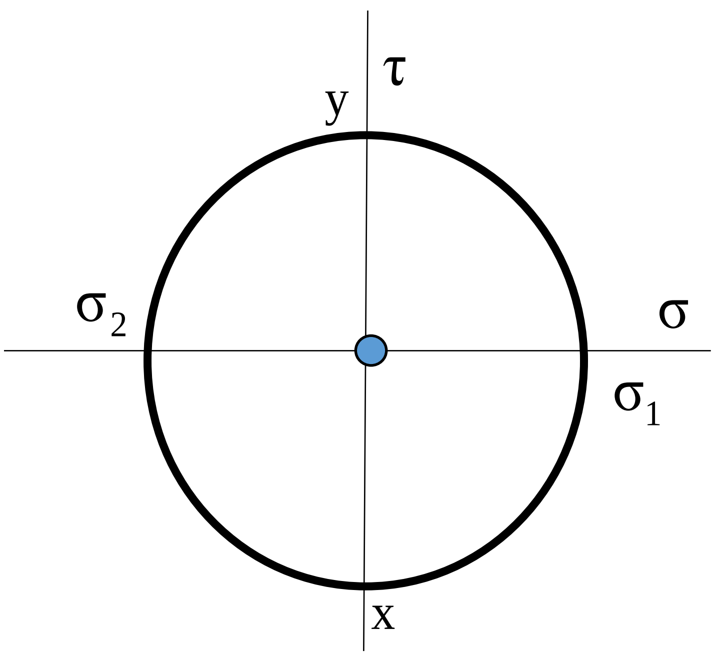
$$\boxed{\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y}$$

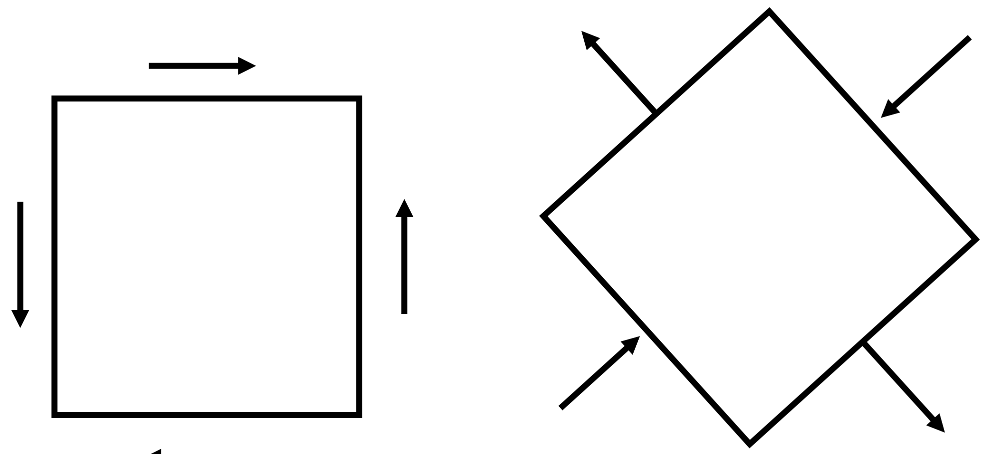
Transformation of Stresses: 2D, Mohr Circle

Pure shear

$$\frac{\sigma_x + \sigma_y}{2} = 0$$

When normal stresses vanish on the plane of maximum shear
Example: the stress state by the simple torsion





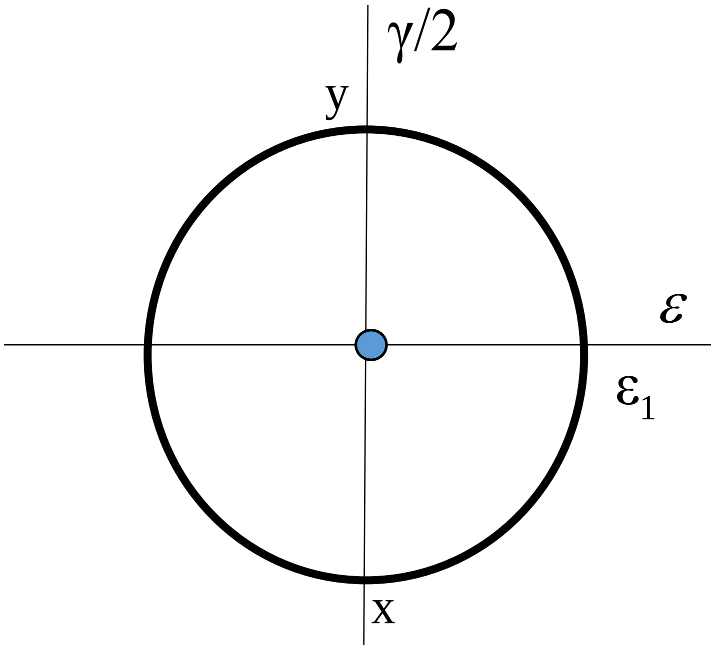
These two stresses are equivalent

Transformation of Stresses: 2D, Mohr Circle

Under Pure shear

$$\tau = G\gamma$$ Hooke's law for shear

$$\varepsilon_1 = \frac{\tau}{2G}$$ From the Mohr circle



Principal strain is related to the principal stress

$$\varepsilon_1 = \frac{1}{E}\left(\sigma_1 - \nu\sigma_2\right)$$

From the Mohr's circle

$$\frac{\tau}{2G} = \frac{1}{E}\left(\tau - \nu(-\tau)\right)$$

$$\boxed{G = \frac{E}{2(1+\nu)}}$$

Thin-Walled Pressure Vessels

- A thin-walled vessel is one in which the distribution of stress is essentially constant
- through the thickness, whereas in thick-walled vessels, the normal stress varies over the wall thickness.

- If the ratio of the wall thickness t to the inner radius r is equal or less than about 1/10 (or $r/t \geq 10$), the vessel is classified as thin-walled. In fact, in thin-walled vessels, there is often no distinction made between the inside and outside radii because they are nearly equal.

Real life examples of cylindrical and spherical pressure vessels (Courtesy CB&I.)

(*a*) (*b*)

Model of a cylindrical pressure vessel and equations

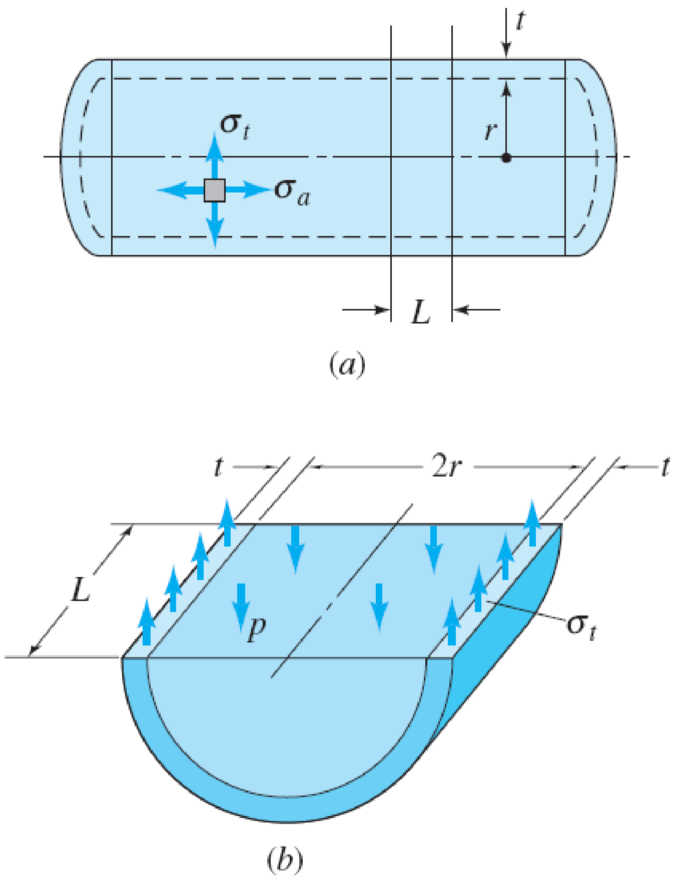


FIGURE 8.18 Thin-walled cylindrical pressure vessel; (a) entire cylinder; (b) free-body diagram of a segment.

tangential stress:

$$\sigma_t = \frac{pr}{t}$$

Model of a cylindrical pressure vessel and equations

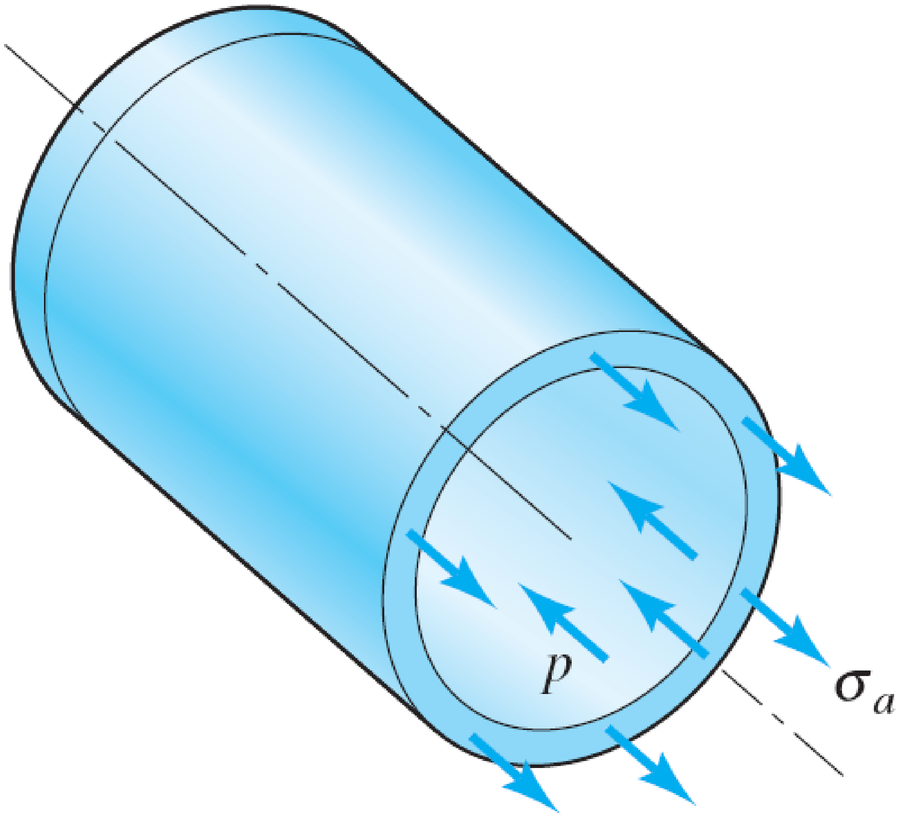


FIGURE 8.19 Free-body diagram of the portion of the vessel shown in Fig. 8.18*a*.

tangential stress:

$$\sigma_t = \frac{pr}{t}$$

axial (longitudinal) stress:

$$\sigma_a = \frac{pr}{2t}$$

Strain due to internal pressure

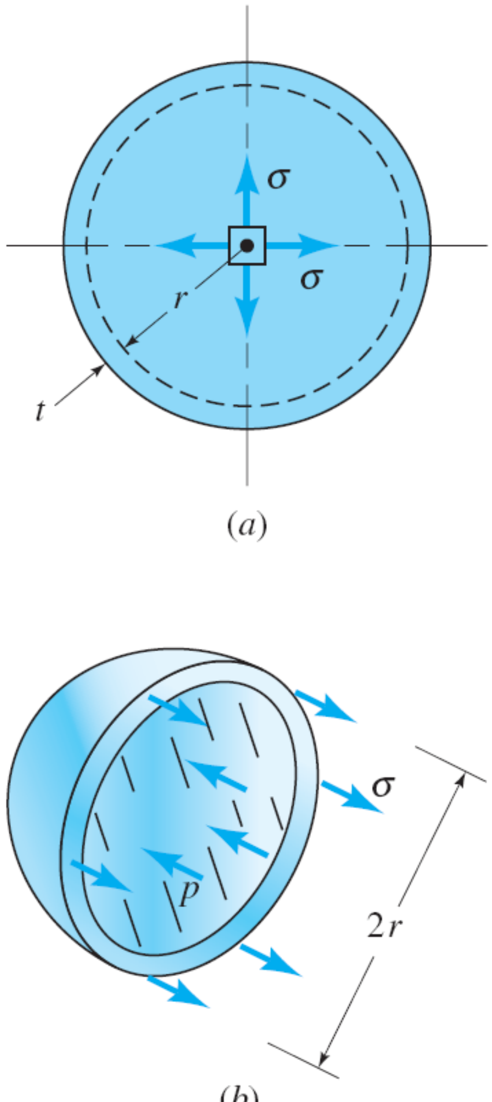
Circumferential strain due to internal pressure:

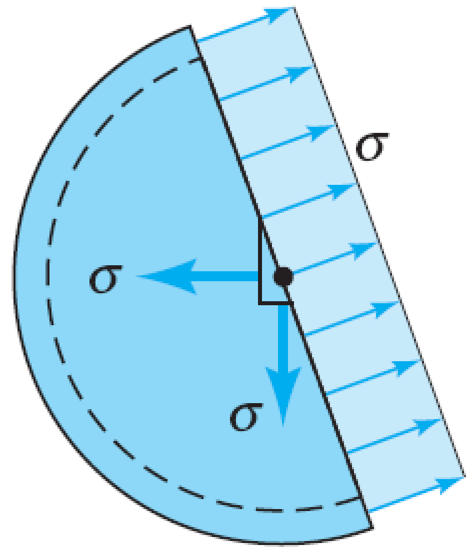
$$\boxed{\begin{gathered}\varepsilon_t = \frac{1}{2\pi r}\left[2\pi(r+\delta_c) - 2\pi r\right] = \frac{\delta_c}{r} \\ \varepsilon_t = \frac{1}{E}(\sigma_t - \nu\sigma_a)\end{gathered}}$$

Extension of the radius of the cylinder:

$$\boxed{\delta_c = \frac{pr^2}{2Et}(2-\nu)}$$

Spherical pressure vessels





(c)

FIGURE 8.20 Thin-walled spherical pressure vessel: (*a*) entire sphere; (*b*, *c*) free-body diagrams of hemisphere.

Spherical pressure vessels

Tangential stress due to internal pressure:

$$\boxed{\sigma = \frac{pr}{2t}}$$

Radial extension of the sphere:

$$\boxed{\delta_s = \frac{pr^2}{2Et}(1 - \nu)}$$

MAXIMUM SHEAR STRESS IN VESSELS

Cylindrical vessel:

$$\boxed{(\tau_{\max})_a = \frac{1}{2}(\sigma_t - 0) = \frac{pr}{2t}}$$

Spherical vessel:

$$\boxed{(\tau_{\max})_a = \frac{1}{2}(\sigma - 0) = \frac{pr}{4t}}$$