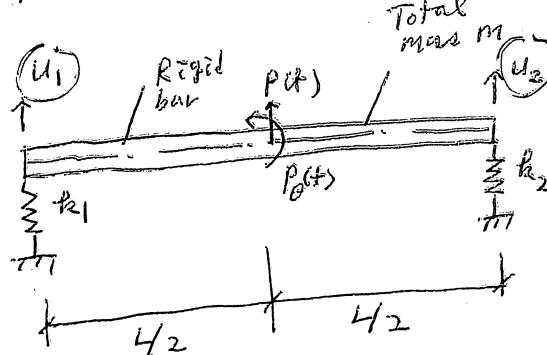


①

Example 9.2  $\frac{P_t}{L}$   $\rightarrow$  transformation  $\frac{u_1, u_2}{k_{11}, k_{22}}$   
 Total mass  $m$



" $u_1, u_2 \rightarrow \frac{P_t}{L}$ "

"운동 방정식을 구하는 데서"

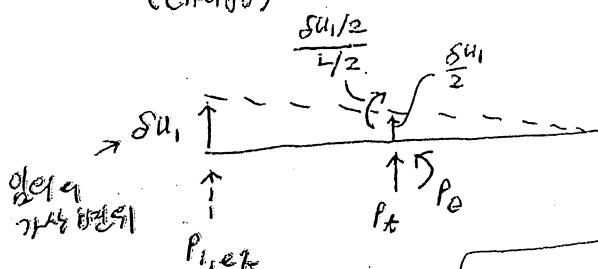
$$f_L + f_R = P(t)$$

(Rigid bar의 주기로 인해  
 2차유연성이 있음, 여기까지는  
이유는 차후로 서술하기로 함)

(solution)

1. Work-Equivalent Nodal Forces  
 (Energy)

위에서  $P_t$  및  $P_\theta$ 가  $u_1, u_2$ 의  
 위치에 작용하는 것으로  
 두 가지 작용력으로 대체됨  
 (가장 일반적인 work-equivalent  
 criterium 사용)



$$P_{1,eq} \times \delta u_1 = P_t \left( \frac{\delta u_1}{2} \right) - (P_\theta \times \frac{\delta u_1}{L})$$

$$(P_{1,eq} - \frac{1}{2}P_t + \frac{P_\theta}{L}) \times \delta u_1 = 0$$

arbitrary

$$\therefore P_{1,eq} = \frac{1}{2}P_t - \frac{1}{L}P_\theta$$

유사한 방정식이 있다.

$$\therefore P_{2,eq} = \frac{1}{2}P_t + \frac{1}{L}P_\theta$$

$$\left. \begin{array}{l} P(t) \\ \hline 2 \times 1 \end{array} \right\} = \begin{bmatrix} \frac{P_t}{2} - \frac{P_\theta}{L} \\ \frac{P_t}{2} + \frac{P_\theta}{L} \end{bmatrix}$$

2. Stiffness matrix

$$\underline{K} = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \quad u_1 = 1 \quad \begin{cases} u_1 = 1 \\ u_2 = 0 \end{cases}$$

Decoupled  $\underline{K}_2$

$$\begin{cases} u_1 = 0 \\ u_2 = 0 \end{cases} \quad \begin{cases} u_1 = 0 \\ u_2 = 1 \end{cases}$$

$k_{11} = k_{22}$

$$\begin{cases} u_1 = 0 \\ u_2 = 0 \end{cases} \quad \begin{cases} u_1 = 1 \\ u_2 = 1 \end{cases}$$

$k_{12} = k_{21}$

3. Mass matrix  $\underline{M} \leftarrow$  가로하중을 mass coefficient approach 적용하여 계산  
Influence

$$\underline{M} = \begin{bmatrix} m & & \\ \frac{m}{3} & & \\ & & \frac{m}{3} \\ & & \frac{m}{3} \end{bmatrix}$$

Coupled M

$$\begin{aligned} \dot{u}_i &= 1.0 & \left(\frac{m}{L} \times \frac{x}{L} \times 1.0\right) / \text{unit length} \\ M_{11} &= \frac{m}{L} \times \frac{Lx}{2} \\ &= \frac{m}{2} \times \frac{2}{3} \\ &= \frac{m}{3} \\ M_{12} &= \frac{m}{L} \times \frac{Lx}{2} \\ &= \frac{m}{2} \times \frac{1}{3} \\ &= \frac{m}{6} \end{aligned}$$

이사하거니,  $M_{12} = \frac{m}{6}$ ,  $M_{22} = \frac{m}{3}$

Note: 본래 질량인 경우와는  $\underline{M}$ 는 coupled matrix가 되어짐.

다른 물체의 운동 방정식은,

$$\begin{bmatrix} \frac{m}{3} & \frac{m}{6} \\ \frac{m}{6} & \frac{m}{3} \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} k_{11} & 0 \\ 0 & k_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{Pt}{2} - \frac{P_0}{L} \\ \frac{Pt}{2} + \frac{P_0}{L} \end{bmatrix} \quad \dots (*)$$

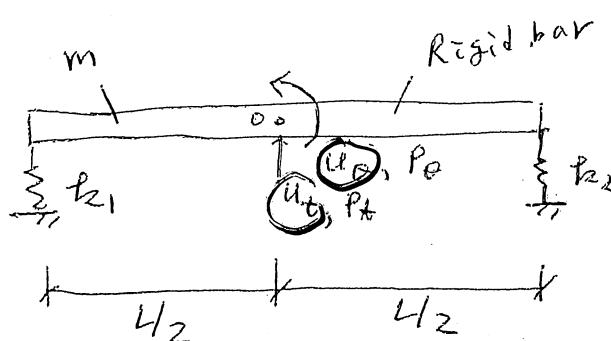
Example 9.3 例.

† A different choice of D.O.F.

(가로하중에 걸친 중심에서의 병진 및 회전 방정식은?)

(solution)

1. Work-Equivalent Nodal Forces  $\rightarrow$  틀



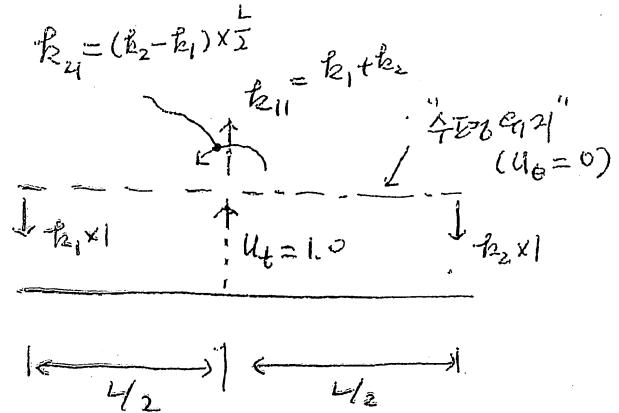
$$\underline{P}(t) = \begin{bmatrix} P_t \\ \dots \\ P_\theta \end{bmatrix}$$

(3)

2. Stiffness matrix  
Stiffness  
Influence  
coefficient  
approach

$$\underline{k} = \begin{bmatrix} k_1 + k_2 & \frac{(k_2 - k_1)L}{2} \\ \frac{(k_2 - k_1)L}{2} & \frac{L^2(k_1 + k_2)}{4} \end{bmatrix}$$

Coupled

↑  
symm. & Positive definite  
( $k_{12} > 0$ )

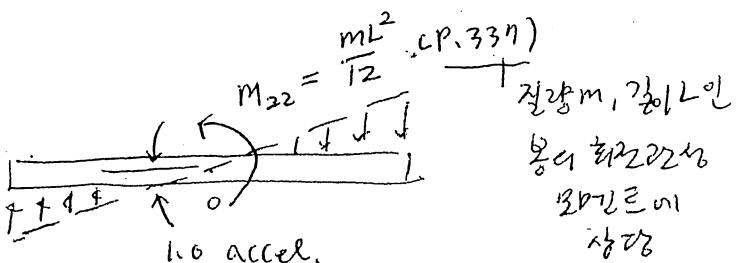
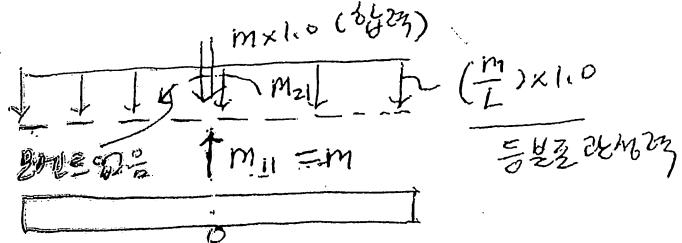
3. Mass matrix

Mass coefficient approach  
Influence

$$\underline{m} = \begin{bmatrix} m & 0 \\ 0 & \frac{mL^2}{12} \end{bmatrix}$$

1.0  
accel.

Decoupled



$$(\sum Y_T = 0, \text{ thereby } m_{12} = 0)$$

자연주수 2 3 4 5 6 7,

$$\left[ \begin{array}{cc|c} m & 0 & \\ 0 & \frac{mL^2}{12} & \end{array} \right] \begin{bmatrix} \ddot{u}_t \\ \ddot{u}_θ \end{bmatrix} + \left[ \begin{array}{cc} k_1 + k_2 & \frac{(k_2 - k_1)L}{2} \\ \frac{(k_2 - k_1)L}{2} & \frac{L^2(k_1 + k_2)}{4} \end{array} \right] \begin{bmatrix} u_t \\ u_θ \end{bmatrix} = \begin{bmatrix} P_t \\ P_θ \end{bmatrix} \quad \text{--- (**)}$$

(4)

Note : (\*\*) 이는 "차동변환" on entity (\*) 대신 유리  $\frac{u_1}{u_2}$  를 쓰시

두 차동변수 서로 다른 경우

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & -1/2 \\ 1 & 1/2 \end{bmatrix} \begin{bmatrix} u_t \\ u_o \end{bmatrix}$$

$\uparrow \quad \uparrow$   
 $u_t = 1.0 \quad u_t = 0$   
 $u_o = 1.0 \quad u_o = 0$

$$\begin{array}{c} \text{--- --- --- --- ---} \\ | \quad | \quad | \quad | \quad | \\ \hline | \quad | \quad | \quad | \quad | \\ \hline \text{--- --- --- --- ---} \end{array}$$

$$\begin{array}{c} \text{--- --- --- --- ---} \\ | \quad | \quad | \quad | \quad | \\ \hline | \quad | \quad | \quad | \quad | \\ \hline \text{--- --- --- --- ---} \end{array}$$

$$\begin{array}{c} \text{--- --- --- --- ---} \\ | \quad | \quad | \quad | \quad | \\ \hline | \quad | \quad | \quad | \quad | \\ \hline \text{--- --- --- --- ---} \end{array}$$

$$\begin{array}{c} \text{--- --- --- --- ---} \\ | \quad | \quad | \quad | \quad | \\ \hline | \quad | \quad | \quad | \quad | \\ \hline \text{--- --- --- --- ---} \end{array}$$

or  $\underline{u} = \underline{\alpha} \underline{\bar{u}}$   
 Displ. transformation matrix on  $\bar{u}$   
 (Force transformation)

$$\underline{m} \ddot{\underline{u}} + \underline{k} \underline{u} = \underline{p} \quad \text{--- (*) (in matrix form)}$$

$$\underline{\underline{m}} \underline{\alpha} \underline{\bar{u}} + \underline{\underline{k}} \underline{\alpha} \underline{\bar{u}} = \underline{p} \quad \text{--- (**)}$$

\$\begin{array}{c} \text{--- --- --- --- ---} \\ | \quad | \quad | \quad | \quad | \\ \hline | \quad | \quad | \quad | \quad | \\ \hline \text{--- --- --- --- ---} \end{array}\$

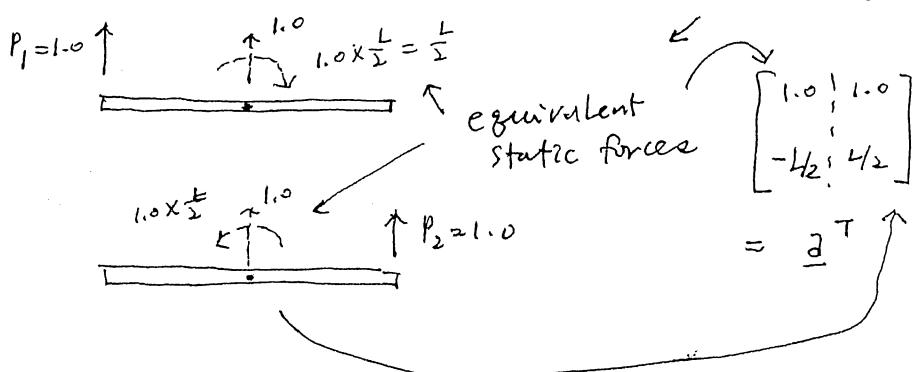
(\*) 대상변수에  $\underline{\alpha}^T$ 를 "적용" ← congruent transformation  
 ( $\underline{\alpha}^T$  : Force transformation matrix,  
 力矩 변환 행렬)

$$\underline{\alpha}^T \underline{m} \underline{\alpha} \underline{\bar{u}} + \underline{\alpha}^T \underline{k} \underline{\alpha} \underline{\bar{u}} = \underline{\bar{p}}$$

$$\underline{\bar{m}} \ddot{\underline{\bar{u}}} + \underline{\bar{k}} \underline{\bar{u}} = \underline{\bar{p}} \quad \text{--- (***)}$$

$$\underline{\bar{p}} = \underline{\alpha}^T \underline{p}$$

( $\underline{\alpha}^T$ 의 특성)



"pdf" upload

(1)

## 9.5 Unsymmetric-plan Bldg = Ground Motion

↳ Equations governing coupled lateral-torsional motion

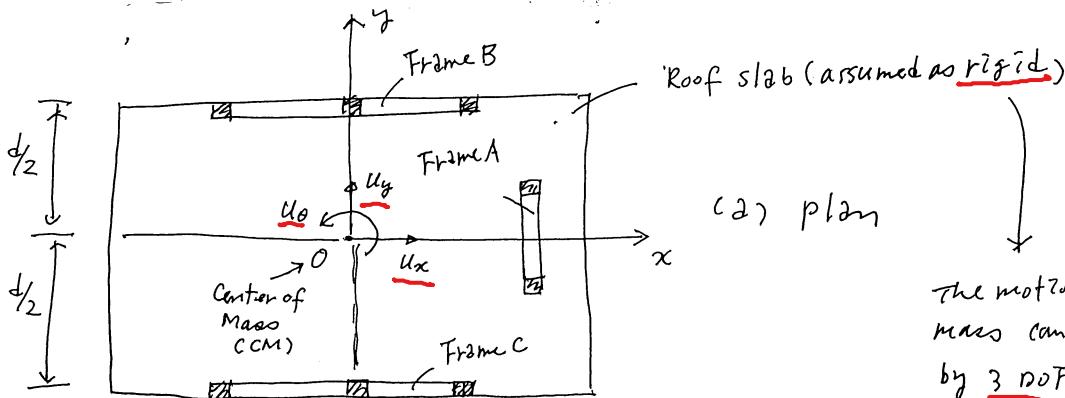
↳ for one-story systems & multi-story buildings

### 9.5.1 One-story, Two-Way Unsymmetric System

9.5.2 " one-way " "

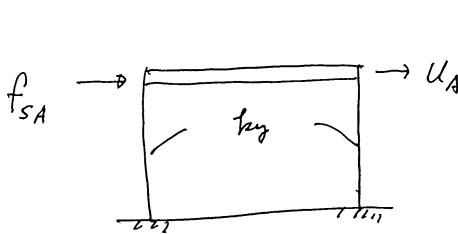
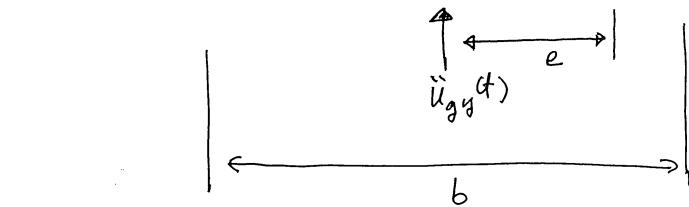
9.5.3 " symmetric system

9.5.4 Multi-story One-Way Unsymmetric system

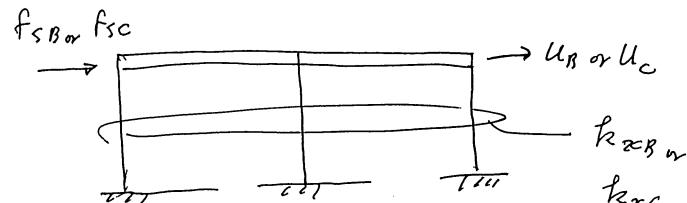


The motion of the roof mass can be described by 3 DOFs at the CM of the roof ;

$u_x, u_y$  and  $u_\theta$



(b) frame A



(c) frame C  
or

Figs. 9.5.1 System considered  
One-story

May be determined by stiffness influence method or direct stiffness method

coeff.

Force-displ. relation

$$k_y u = f_s \text{ or}$$

$$\begin{bmatrix} k_{xx} & k_{xy} & k_{xg} \\ k_{yx} & k_{yy} & k_{yg} \\ k_{xg} & k_{yg} & k_{gg} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ u_\theta \end{bmatrix} = \begin{bmatrix} f_{sx} \\ f_{sy} \\ f_{s\theta} \end{bmatrix}$$

--- (1)

Global stiff. mat.

The lateral stiffness of each frame:

KIND of  
"element" stiff. mat.

$$f_{SA} = k_y u_A$$

--- (2)

Recall

Static

condensation

method in section 9.3

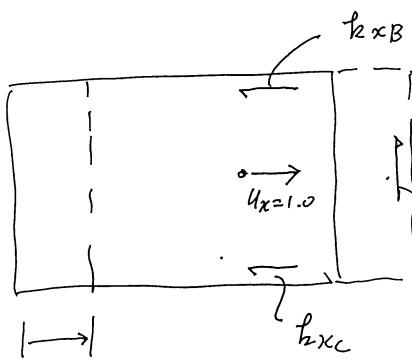
$$f_{SB} = k_{XB} u_B, \quad f_{SC} = k_{XC} u_C \quad --- (3)$$

(2) in 2nd direct stiffness method diagram  
element stiff mat.  $\rightarrow$  assemble  $\rightarrow$

only  
4 rows

Evaluation of stiffness matrix based on stiffness influence coefficient method

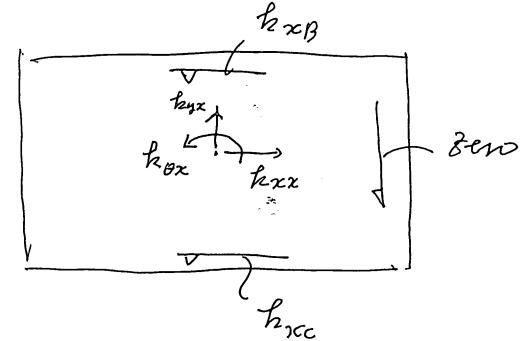
(a)  $u_x = 1.0$  and  $u_y = u_\theta = 0 \rightarrow$  the 1st column of  $\underline{k}$



1.0

"the effect of  $u_x = 1.0$  is  
easy to see on  $k_{XB}$  and  $k_{XC}$   
 $k_{YX}$ "

(orthogonal)  
-explanatory

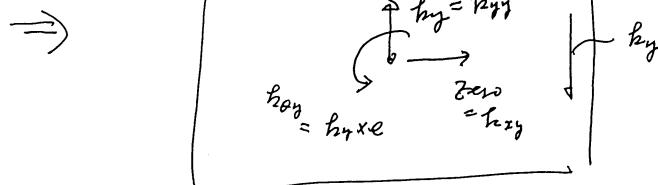
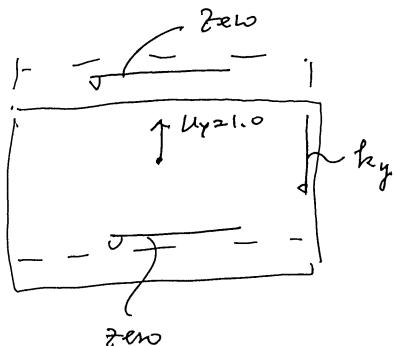


$$k_{XX} = k_{XB} + k_{XC}$$

$$k_{YX} = 0$$

$$k_{BX} = (k_{XC} \times \frac{d}{2} - k_X \times \frac{d}{2})$$

(b)  $u_y = 1.0$  and  $u_x = u_\theta = 0$



$$\therefore \underline{k} =$$

$$\begin{bmatrix} k_{XB} + k_{XC} \\ 0 \\ \frac{d}{2}(k_{XC} - k_{XB}) \end{bmatrix}$$

$$\begin{bmatrix} k_y \\ k_{YX} \\ k_{YZ} \end{bmatrix}$$

$$\begin{bmatrix} k_{YY} \\ k_{YX} \\ k_{YZ} \end{bmatrix}$$

--- (4)

HWT, 3502

## Inertia forces

since the selected global d.o.f.s are located at the CM,

$$f_{Ix} = m \times \ddot{u}_x^t ; f_{Iy} = m \times \ddot{u}_y^t ; f_{I\theta} = I_o \ddot{u}_\theta^t \quad \dots \dots (11)$$

where  $m$  = the diaphragm mass (translational mass)

$$I_o = \frac{m}{12} (b^2 + d^2) \text{ about } O$$

$\ddot{u}_x^t, \ddot{u}_y^t, \ddot{u}_\theta^t$  = the total (absolute) accelerations  
at the CM

or  $f_I = m \cdot \ddot{u}^t = \begin{bmatrix} m & & \\ & m & \\ & & I_o \end{bmatrix} \begin{Bmatrix} \ddot{u}_x^t \\ \ddot{u}_y^t \\ \ddot{u}_\theta^t \end{Bmatrix} \quad \dots \dots (12)$

## Equations of motion

$$\rightarrow m \ddot{y}^t + k_y y = 0 \quad \dots \dots (13)$$

Note  $\begin{Bmatrix} \ddot{u}_x^t \\ \ddot{u}_y^t \\ \ddot{u}_\theta^t \end{Bmatrix} = \begin{Bmatrix} \ddot{u}_x \\ \ddot{u}_y \\ \ddot{u}_\theta \end{Bmatrix} + \begin{Bmatrix} \ddot{u}_{gx} \\ \ddot{u}_{gy} \\ \ddot{u}_{g\theta} \end{Bmatrix}$  or  $\ddot{y}^t = \ddot{y} + \ddot{u}_g \quad \dots \dots (14)$

$$\begin{bmatrix} m & & \\ & m & \\ & & I_o \end{bmatrix} \ddot{y} + \underbrace{\left( \begin{matrix} k \\ - \end{matrix} \right)}_{\text{Uncoupled}} y = - \begin{Bmatrix} m \ddot{u}_{gx} \\ m \ddot{u}_{gy} \\ I_o \ddot{u}_\theta \end{Bmatrix} \quad \dots \dots (15)$$

Uncoupled

$$\underbrace{k}_{\text{Coupled}} = \begin{bmatrix} k_{xx} & 0 & k_{x\theta} \\ 0 & k_{yy} & k_{y\theta} \\ k_{x\theta} & k_{y\theta} & k_{\theta\theta} \end{bmatrix}$$

Thus the response of the system to the  $y$  (ad  $y$ ) component of ground motion is not restricted to lateral displ. in the  $x$  (ad  $y$ ) direction, but will produce lateral motion in the transverse direction,  $y$  (and  $x$ ), and torsion of the roof diaphragms about the vertical axis.

### 9.5.2 One-story, One-way Unsymm. Systems

$\rightarrow$  2 special case of the system in Fig. 9.5-1

Let  $k_{x_B} = k_{x_C} = k_x \rightarrow$  symm. about the x-axis  
but not for the y-axis.

Then, Eq (9.5.15) is simplified to,

$\checkmark$  例題 (HW#)

$$\left[ \begin{array}{c|c} m & \\ \hline m & I_0 \end{array} \right] \begin{pmatrix} \ddot{u}_x \\ \ddot{u}_y \\ \ddot{u}_\theta \end{pmatrix} + \left[ \begin{array}{c|c|c} 2k_x & 0 & 0 \\ \hline 0 & -k_y & -k_\theta \\ 0 & e^{j\omega t} & e^{j\omega t} + \frac{j^2}{2} k_x \end{array} \right] \begin{pmatrix} u_x \\ u_y \\ u_\theta \end{pmatrix} = -m \ddot{u}_y = \left[ \begin{array}{c} m \ddot{u}_{gx} \\ \hline m \ddot{u}_{gy} \\ 0 \end{array} \right] \quad \text{--- (17)}$$

partitioning

The first of the eqs,

$$m \ddot{u}_x + (2k_x) u_x = -m \ddot{u}_{gx} \quad \text{--- (18)}$$

$\Rightarrow$  ( $u_y, u_\theta$  are decoupled in Eq. 18)

After overwriting  $u_x$  in Eq. 18

new eqs

The remaining two eqs,

$$\left[ \begin{array}{c|c} m & 0 \\ \hline 0 & I_0 \end{array} \right] \begin{pmatrix} \ddot{u}_y \\ \ddot{u}_\theta \end{pmatrix} + \left[ \begin{array}{cc} k_{yy} & e^{j\omega t} \\ e^{-j\omega t} & k_{\theta\theta} \end{array} \right] \begin{pmatrix} u_y \\ u_\theta \end{pmatrix} = - \left[ \begin{array}{c|c} m & 0 \\ \hline 0 & I_0 \end{array} \right] \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ddot{u}_{gy} \quad \text{--- (19)}$$

$u_y, u_\theta$  are coupled

Note:

$$\left[ \begin{array}{c|c} m \ddot{u}_y \\ \hline I_0 \ddot{u}_\theta \end{array} \right] + \left[ \begin{array}{c|c} k_{yy} u_y + e^{j\omega t} u_\theta \\ e^{-j\omega t} u_y + k_{\theta\theta} u_\theta \end{array} \right] = \left[ \begin{array}{c|c} -m \ddot{u}_{gy} \\ 0 \end{array} \right]$$

$$m \ddot{u}_y + k_{yy} u_y + e^{j\omega t} u_\theta = -m \ddot{u}_{gy}$$

$$I_0 \ddot{u}_\theta + e^{-j\omega t} u_y + k_{\theta\theta} u_\theta = 0 \rightarrow u_y = \boxed{\phantom{0}}$$

(5)

### 9.5.3 One-story symm system

↳ A further specialization to the system in Fig. 9.5.1

Let  $k_{x_B} = k_{x_C} = k_x$ , and  $e = 0$

Frame A is located at  
the CM

Then Eq. (9.5.15) reduces to,

$$\begin{bmatrix} m & m \\ m & I_0 \end{bmatrix} \ddot{y} + \begin{bmatrix} 2k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & (d/2)k_x \end{bmatrix} y = - \begin{bmatrix} m \ddot{u}_{gx} \\ m \ddot{u}_{gy} \\ I_0 \ddot{u}_{go} \end{bmatrix}$$

Decoupled, diagonal matrix

$\left\{ \begin{array}{l} \text{(1)} \quad x \text{ has } \frac{\partial u_x}{\partial z} = 2 \frac{\partial u_x}{\partial z} \text{ and } u_x \text{ has } \frac{\partial^2 u_x}{\partial z^2} = 0 \\ \text{(2)} \quad y \text{ " " } u_y \text{ " " } \\ \text{(3)} \quad \text{The system would experience no torsion} \\ \text{unless the base motion includes rotation about a vertical axis.} \end{array} \right.$

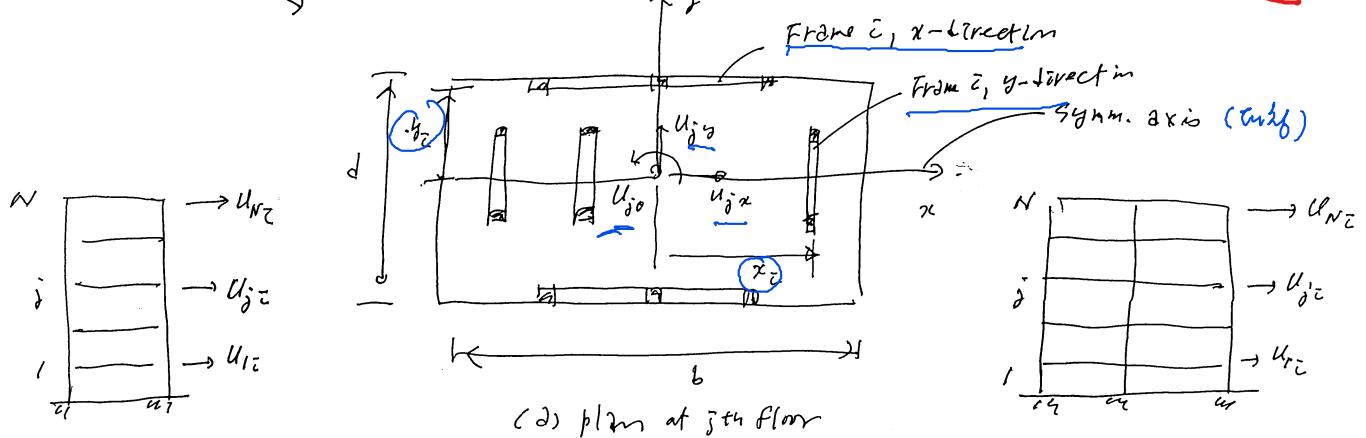
自明.

### 9.5.4 Multi-story One-way unsymm. Systems

(prob. 9.14)  
(10.24)  
(prob. 12.24-15)

(B.34)  
 $\frac{3}{16} k$   
 $\frac{1}{16} k$   
(13.9)  
 $\frac{1}{16} k$

A multi-story system similar to a one-story system symmetric about the  $x$ -axis but unsymm about the  $y$ -axis; the centre of mass O of all floor diaphragms lie on the same vertical axis. changes



(b) frame  $i$  in the  $y$ -dir.

(c) frame  $i$  in the  $x$ -dir

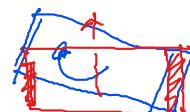
As suggested by the earlier formulation for a one-story system, the x-dir. translational motion due to the x-component of ground motion can be determined by planar analysis of the building.  $\rightarrow$  Eg. of motions on section 9.4.2 (b) (c) (d) (e)

The Eg. of motion governing the response of the system subjected to the y-dir ground motion  $\leftarrow$  2-D plane frame analysis

$\hookrightarrow$  Coupled lateral-torsional motion described by 2N DOFs

$$\underline{u} = \begin{bmatrix} \underline{u}_y \\ \underline{u}_\theta \end{bmatrix} \text{ where } \underline{u}_y = [u_{1y}, u_{2y}, \dots, u_{Ny}]^T$$

$$2N \times 1 \qquad \qquad \qquad \underline{u}_\theta = [u_{1\theta}, u_{2\theta}, \dots, u_{N\theta}]^T$$



Conceptual formulation of the system stiffness matrix w.r.t. the global DOFs  $\underline{u}$  per the direct stiffness method

Step 1: Determine the lateral stiffness matrix for each frame

$$\begin{array}{ll} \xrightarrow{\text{2nd x-dir frame}} & f_{x_i} = k_{x_i} u_{x_i} \quad \left. \begin{array}{l} \text{static condensation} \\ \text{needed in general} \end{array} \right. \\ \xrightarrow{\text{2nd y-dir frame}} & f_{y_i} = k_{y_i} u_{y_i} \end{array}$$

Step 2: Merging the two using (global coordinates)

$$u_{x_i} = \frac{\partial}{\partial x_i} u \quad \text{and} \quad u_{y_i} = \frac{\partial}{\partial y_i} u$$

$$\frac{\partial}{\partial x_i} = \begin{bmatrix} 0 & -y_i I \\ N \times N & N \times N \end{bmatrix} \quad \text{and} \quad \frac{\partial}{\partial y_i} = \begin{bmatrix} I & x_i I \\ N \times N & N \times N \end{bmatrix}$$

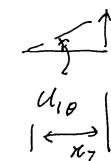
ex)  $N=2$

objectively  
decouple

$$\begin{bmatrix} u_{1x} \\ u_{2x} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{1y} \\ u_{2y} \end{bmatrix} + \begin{bmatrix} 0 & -y_1 I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{1\theta} \\ u_{2\theta} \end{bmatrix}$$

$$(u_{1y})_i = (u_{1y})_i + (u_{1\theta}) \times x_i$$

global  
y-displ  
at node i



Step 3: Transform the "local  $\underline{k}_c$ " to the "global" "ith frame  $\underline{k}$ "

(17)

$$\underline{k} = \underline{k}_c^T \underline{x}_c \underline{x}_c^T \underline{k}_c \text{ or } \underline{k} = \underline{k}_c^T \underline{x}_c \underline{x}_c^T \underline{k}_c$$

$2N \times 2N$        $2N \times N$        $N \times N$        $N \times 2N$

$x_i^T \underline{k}_c \underline{x}_c$

Step 4:

$$\begin{aligned} \underline{k} &= \sum_c \underline{k}_c \\ &= \left[ \begin{array}{c} \underline{k}_{yy} \\ \underline{k}_{y\theta} \\ \underline{k}_{\theta y} \\ \underline{k}_{\theta\theta} \end{array} \right] \quad \sum \underline{k}_{yz} \\ &= \sum (x_i^2 k_{yy} + \theta_i^2 k_{\theta\theta}) \end{aligned}$$

H.W.

( $\underline{u}_y$   $\underline{\theta}$ )

\*  $\underline{u}_y$   $\underline{\theta}$

show

$\underline{u}_y$

Note:

$$\underline{x}_c^T \underline{k}_c \underline{x}_c = \begin{bmatrix} 0 \\ -g_i I \\ -g_i I \end{bmatrix} \underline{k}_c \underline{x}_c \quad \begin{bmatrix} 0 \\ g_i I \\ g_i I \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -g_i \underline{k}_c \underline{x}_c \end{bmatrix} \begin{bmatrix} 0 & -g_i I \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \underline{k}_c \underline{x}_c \end{bmatrix} \times \begin{bmatrix} \underline{u}_y \\ \underline{u}_\theta \end{bmatrix}$$

Partial eqns!

The equation of motion of the building subjected to ground motion

$$\begin{bmatrix} m & I_0 \\ I_0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{u}_y \\ \ddot{u}_\theta \end{bmatrix} + \begin{bmatrix} \sum k_{yy} & \sum x_i k_{yz} \\ \sum x_i k_{yz} & \sum (x_i^2 k_{yy} + \theta_i^2 k_{\theta\theta}) \end{bmatrix} \begin{bmatrix} \ddot{u}_y \\ \ddot{u}_\theta \end{bmatrix} = - \begin{bmatrix} m \\ I_0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \ddot{u}_g$$

--- (2B)

$I_0 = r^2 m$   
 $r^2 = \frac{1}{2}(b^2 + l^2)$  for rectangular floors

9.6 Symmetric-plan buildings subjected torsional excitation

$\ddot{u}_y$   $\ddot{u}_\theta$  zero  $\rightarrow$  decoupled

Self-evident,

$$(r^2 m) \ddot{u}_\theta + k_{\theta\theta} u_\theta = -r^2 m \sin \ddot{u}_g \quad \dots (29)$$

of mass  $I_0$

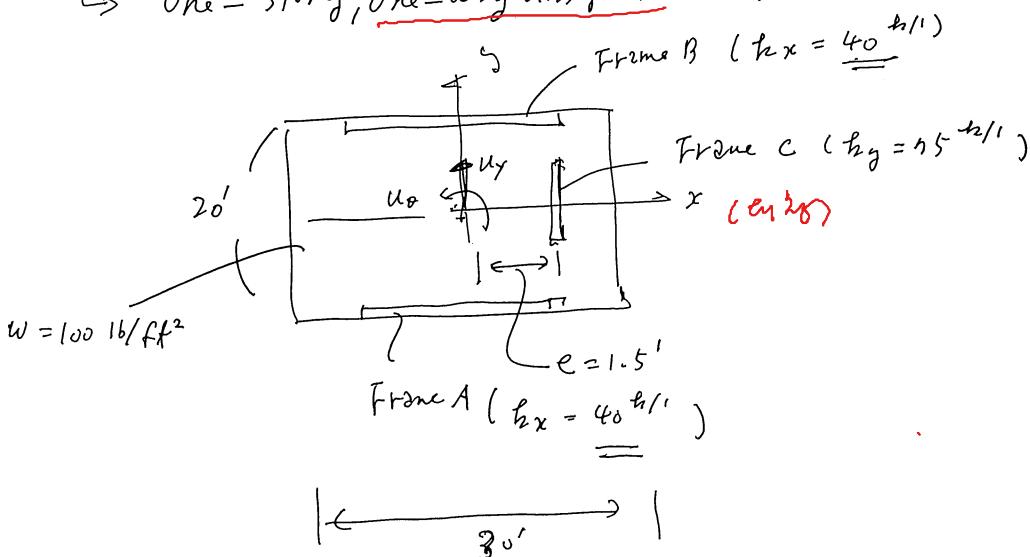
not directly measured,  
( $\ddot{u}_y$   $\ddot{u}_\theta$ )

Accidental torsion  
 $\ddot{u}_y$   $\ddot{u}_\theta$ )

(8)

Ex. 10-6

One-story, One-way Unsymm Case.



Natural periods and mode shapes?

$$\begin{aligned} (\text{Sol}) \quad \bar{\omega} &= \sqrt{20 \times 20 \times 100} = 60 \text{ rad/s} \\ m &= \bar{\omega}/g = 1.863 \text{ ft-s}^2/\text{lb} \\ I_b &= \frac{m(b^2 + l^2)}{12} = 202. \underline{\underline{b}} \text{ ft-s}^2 \end{aligned}$$



= Decoupled

Motion in the  $x$ -dir.

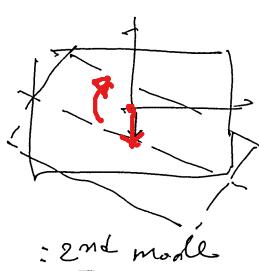
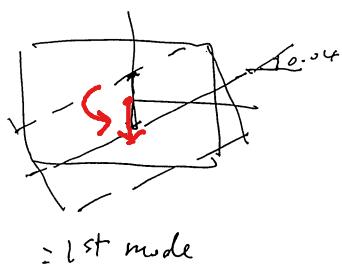
$$m u_x + 2k_x u_x = 0 \rightarrow \omega_x = \sqrt{\frac{2k_x}{m}} = \underline{\underline{6.55 \text{ rad/s}}} \quad \text{translational mode}$$

Motion coupled ( $u_y$  and  $u_\theta$ )  $\rightarrow$  (1st mode)

$$\underline{u} = \begin{bmatrix} u_y \\ u_\theta \end{bmatrix} \quad \underline{k} = \begin{bmatrix} k_{yy} & e k_{yz} \\ e k_{yz} & k_{zz} = 1.5 \times 15 \\ - & - \\ " & e^2 k_{yz} + \frac{d^2 k_x}{2} \\ - & = 81.69 \end{bmatrix} \quad \Rightarrow \quad [\underline{k} - \omega_m^2 \underline{m}] \underline{\phi} = 0 \quad \underline{\phi}_1 = \begin{bmatrix} -0.52 \\ 0.04 \end{bmatrix}$$

$$\underline{m} = \begin{bmatrix} 1.863 & 0 \\ 0 & 202 \end{bmatrix}$$

$$\omega_1 = \underline{\underline{5.9 \text{ rad/s}}} \quad \omega_2 = \underline{\underline{6.8 \text{ rad/s}}} \quad \underline{\phi}_2 = \begin{bmatrix} -0.51 \\ -0.05 \end{bmatrix}$$



(9)

Ex. 10.7 → Consider a special case of the system of Ex. 10.6 in which frame A is located at the CM (i.e.,  $\ell = 0$ ).  
 ↓  
 Change in eigen properties?

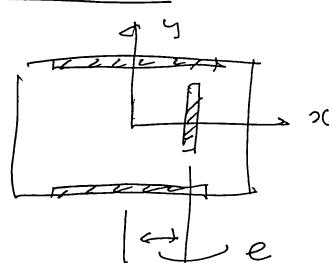
(Solution) Three decoupled equations,

$$\left( \begin{array}{l} (1) m\ddot{u}_x + 2k_x u_x = 0 \rightarrow \omega_x = 6.55 \text{ rad/s} \\ (2) m\ddot{u}_y + k_y u_y = 0 \rightarrow \omega_y = 6.344 \text{ rad/s} \\ (3) I_0 \ddot{u}_\theta + \frac{J_2}{2} K_\theta = 0 \rightarrow \omega_\theta = 6.3 \text{ rad/s} \end{array} \right)$$

### 13.3 EQ analysis of multi-story bldgs with unsymm. plan

Symm about  $x$ -axis

but not about the  $y$ -axis



(for example)

The general procedure developed in section 13.1 is applicable to unsymm. buildings because of Eq. (1) above. It is of the same form as  $m\ddot{u} + c\dot{u} + k u = P_{\text{eff}}$

$$P_{\text{eff}}(t) = - \left( \begin{bmatrix} m & 0 \\ 0 & r^2 m \end{bmatrix} \ddot{u}_{\theta}, t \right) \equiv - \underline{S} \ddot{u}_{\theta}(t)$$

$$\underline{\phi}_m = \begin{bmatrix} \phi_{ym} \\ \phi_{\theta m} \end{bmatrix}$$

Concept of  
modal  
expansion

$$\checkmark \quad \underline{S} = \sum_{j=1}^{2N} S_m = \sum \Gamma_m \begin{bmatrix} m \phi_{ym} \\ r^2 m \phi_{\theta m} \end{bmatrix}$$

$$\checkmark \quad \Gamma_m = \frac{L_m^4}{M_m} ; \quad L_m^4 = [\underline{\phi}_{ym}^\top, \underline{\phi}_{\theta m}^\top] \begin{bmatrix} m & 0 \\ 0 & r^2 m \end{bmatrix} = \underline{\phi}_{ym}^\top m \underline{I} = \sum_{j=1}^N m_j \underline{\phi}_{jm}^\top \underline{\phi}_{ym}$$

$$\checkmark \quad M_m = \underline{\phi}_m^\top \begin{bmatrix} m & 0 \\ 0 & r^2 m \end{bmatrix} \underline{\phi}_m = \underline{\phi}_{ym}^\top m \underline{\phi}_{ym} + r^2 \underline{\phi}_{\theta m}^\top m \underline{\phi}_{\theta m}$$

$$\checkmark \quad S_m = \begin{bmatrix} S_{ym} \\ S_{\theta m} \end{bmatrix} = \Gamma_m \begin{bmatrix} m \phi_{ym} \\ r^2 m \phi_{\theta m} \end{bmatrix} \rightarrow S_{ym} = \Gamma_m m_j \phi_{jm} \quad \text{Why?} \\ S_{\theta m} = \Gamma_m r^2 m_j \phi_{\theta jm}$$

10

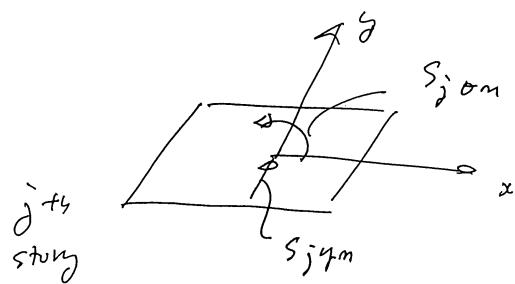
$$\left( \underline{1}^T x \right) \begin{bmatrix} m & 1 \\ \theta \\ \vdots \end{bmatrix} = \left( \underline{1}^T x \right) \sum_{m=1}^{2N} \Gamma_m \begin{bmatrix} m & \phi_{ym} \\ r^2 m & \phi_{\theta m} \end{bmatrix}$$

4 It can be shown,

$$\sum_{m=1}^{2N} M_m^k = \sum_{j=1}^N M_j^k + \sum_{m=1}^{2N} I_{mN}^k = 0$$

Where  $M_m^* = \frac{(L_m^*)^2}{M_m}$ ,  $Z_{om}^* = r^2 R_m \mathbf{1}^T \underline{m} \underline{\phi}_{om}$

## Modal responses



$\therefore$  Modal forces due to  
 $\underbrace{\text{The } n^{\text{th}}}$  "unit" input

$$\text{Equivalent static force } f_{eq} = \begin{bmatrix} f_{ym}(t) \\ f_{om}(t) \end{bmatrix} = \begin{bmatrix} S_{ym} \\ S_{om} \end{bmatrix} \times A_m(t)$$

Table 13.3.1 Model static responses

Date: 2026

Response	Modal static response $T_m^{st}$
$\ddot{V}_b$	$T_{bm}^{st} = \sum_{j=1}^N S_j y_m = \Gamma_m L_m^{-1} = M_m^*$
$T_b$	$T_{bm}^{st} = \sum_{j=1}^N S_j \theta_m = I_m^*$
$u_{j\theta}$	$u_{j\theta}^{st} = (\Gamma_m / \omega_n^2) \times \phi_{j\theta m}$

Note:  $V_{bm}(t) = M_m^* A_m(t)$

$$T_{bm}(t) = Z_m^* A_m(t), \quad M_{bm}(t) = h_m^* M_m^* A_m(t) \text{ etc.}$$

(Expression for Absolute acceleration of  $j$ th floor?) 13.428 31 of 3