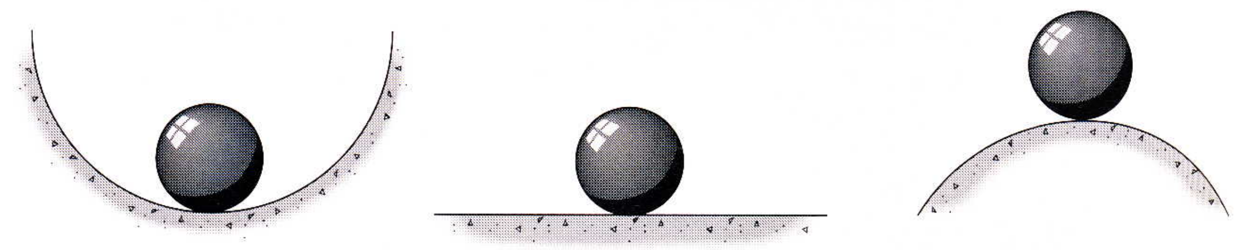
9. Analyzing Columns

9.1 Introduction

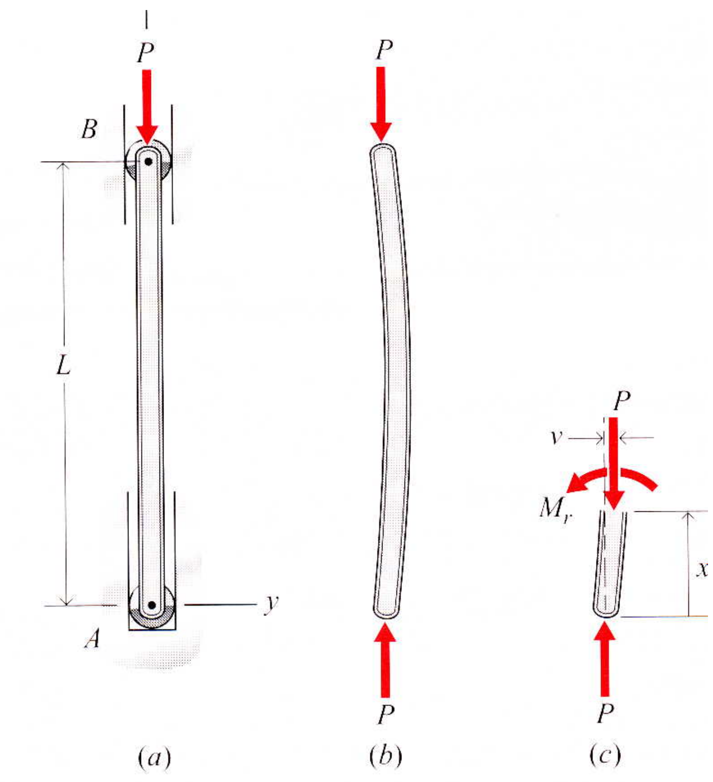
- Buckling: the sudden large deformation of a structure due to a slight increase of an existing compressive load
- Buckling of a column is caused not by failure of the column material but by deterioration of what was a stable state of equilibrium to an unstable one.
- Three equilibrium states of beam deflection:



(a) (b) (c)

9.2 Buckling of long, straight columns

- The minimum axial compressive load for which a pin-ended column will experience lateral deflections



$$EI\frac{d^2v}{dx^2} = M_r = -Pv \quad \rightarrow \quad \frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$

(homogeneous 2nd order linear differential equation)

$$v = A\sin px + B\cos px$$

$$\left(-p^2 + \frac{P}{EI}\right)(A\sin px + B\cos px) = 0$$

$$\therefore p^2 = \frac{P}{EI}$$

$$B.C.: \ v = 0 \ \ at \ \ x = 0 \ \ or \ \ x = L$$

$$\rightarrow v = A\sin pL = 0 \rightarrow pL = n\pi \ (n = 1, 2, 3 \ldots)$$

$$P_{cr} = \frac{\pi^2 EI}{L^2}: \quad Euler\ buckling\ load$$

9.2 Buckling of long, straight columns

When the radius of gyration about the axis of bending is set r

$$I = Ar^2 \,, \;\; \frac{P_{cr}}{A} = \frac{\pi^2 E}{\left(L/r\right)^2} = \sigma_{cr}$$

$$L / r = slenderness\ ratio$$

$$Euleur\ buckling\ condition: \quad L/r > 140\left(steel\ \ columns\right)$$

$$Short\ compression\ \ condition: \quad L/r < 40\left(steel\ \ columns\right)$$

9.2 Buckling of long, straight columns

- Example Problem 9-1

An 8-ft long pin-ended timber column [$E = 1.9(10^6)$ psi and $\sigma_e = 6{,}400$ psi] has a 2 × 4 in. rectangular cross section.

- Slenderness ratio

- Euler buckling load

- The ratio of the axial stress under the action of the buckling load to the elastic strength σ_e of the material

9.2 Buckling of long, straight columns

- Example Problem 9-2

A 12-ft long pin-ended column is made of 6061-T6 aluminum alloy [E = 10(10^6) psi and σ_y = 40,000 psi]. The column has a hollow circular cross section with an outside/inside diameter of 5 in./4in.

- The smallest slenderness ratio for which the Euler buckling load equation is valid

- Critical buckling load

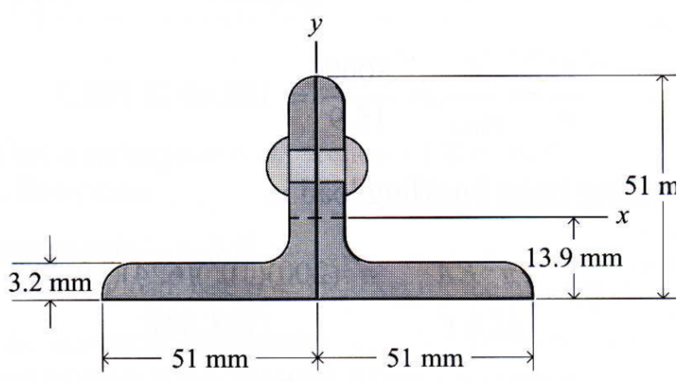
9.2 Buckling of long, straight columns

- Example Problem 9-3

Two 51 × 51 × 3.2 mm structural angles (gyration radius: r_z = 10.1 mm, r_x = 15.9 mm A = 312 mm^2, E = 200 GPa) 3m long will be used as a pin-ended column. Determine the slenderness ratio and the Euler buckling load if

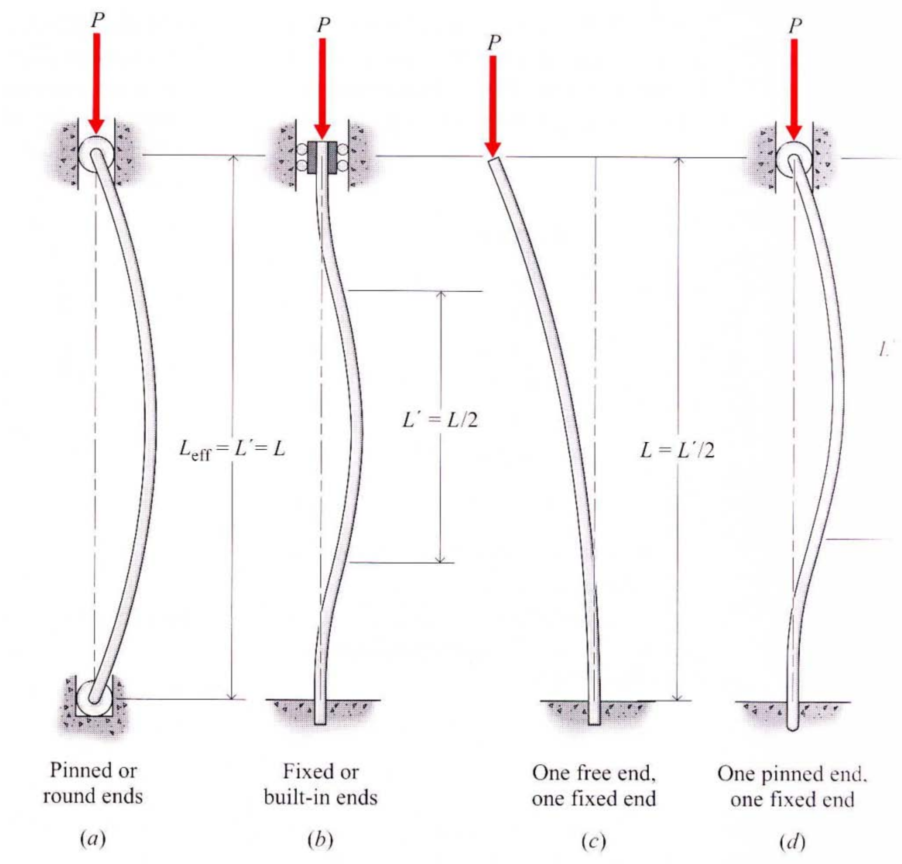
- The two angles are not connected, and each acts as an independent member.

- The two angles are fastened together to act as a unit.



9.3 Effects of different idealized end conditions

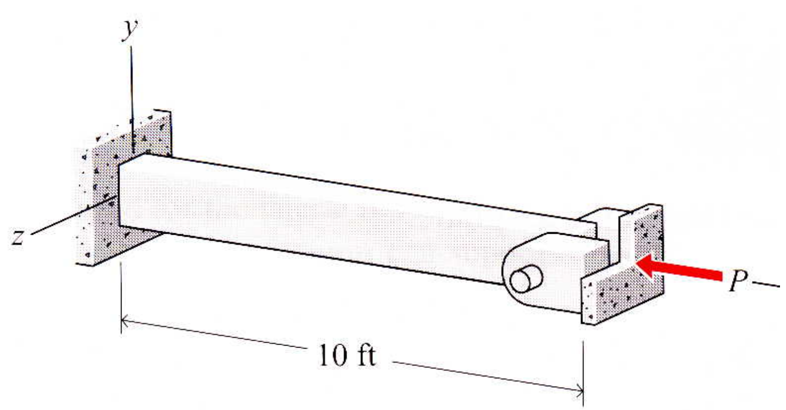
- The effective column length can be used for different end conditions: the distance between successive inflection points or points of zero moments.



- Example Problem 9-4

A 10-ft long structural steel column [E = 29,000 ksi] must support an axial compressive load P. The column has a 1 × 2-in. rectangular cross section.

- Max. safe load for the column if a factor of safety of 2 with respect to failure by buckling is specified.



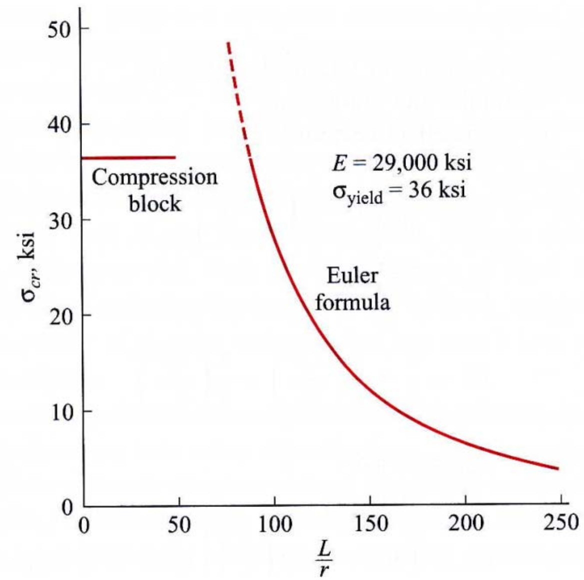
9.4 Empirical column formulas-Centric loading

- Euler's formula is valid when the axial compressive stress for a column is less than the yield strength.

$$Euleur\ buckling\ condition: \quad L/r > 140\,(steel\ \ columns)$$

$$Short\ compression\ \ condition: \quad L/r < 40\,(steel\ \ columns)$$

- Intermediate range: the range between the compression block and the slender ranges



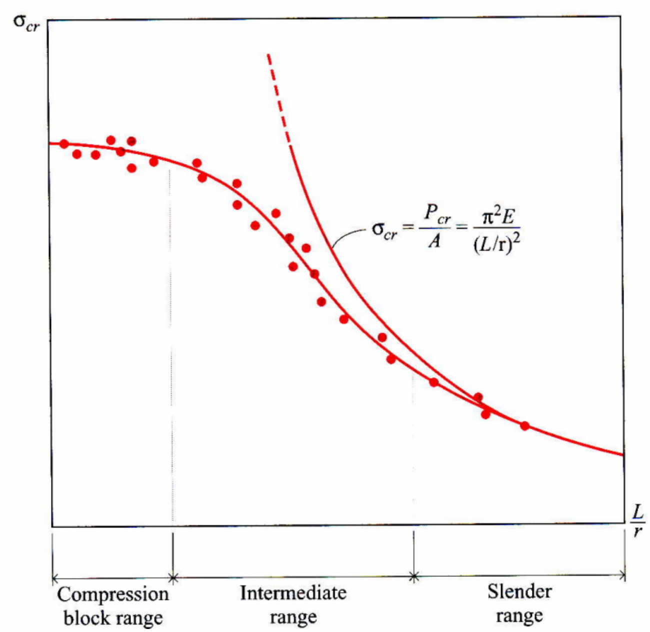


Table 9-1 Some Representative Column Codes for Centric Loading

Code No.	Source	Material	Compression-Block and/or Intermediate-Range Formulas and Limitations (L/r is the effective ratio L'/r)	Slender Range
1	a	Structural steel with a yield point σ_y	$0 \le \frac{L}{r} \le C_c$ $\quad \sigma_{all} = \frac{\sigma_y}{FS}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right]$ $C_c^2 = \frac{2\pi^2 E}{\sigma_y}$ $FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L/r}{C_c}\right) - \frac{1}{8}\left(\frac{L/r}{C_c}\right)^3$	$\frac{L}{r} \ge C_c$ $\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2}$
2	b	2014-T6 (Alclad) Aluminum alloy	$\frac{L}{r} \le 12$ $\quad \sigma_{all} = 28\ \text{ksi}$ $= 193\ \text{MPa}$ $12 \le \frac{L}{r} \le 55$ $\quad \sigma_{all} = \left[30.7 - 0.23\left(\frac{L}{r}\right)\right]\ \text{ksi}$ $= \left[212 - 1.585\left(\frac{L}{r}\right)\right]\ \text{MPa}$	$\frac{L}{r} \ge 55$ $\sigma_{all} = \frac{54{,}000}{(L/r)^2}\ \text{ksi}$ $= \frac{372(10^3)}{(L/r)^2}\ \text{MPa}$
3	b	6061-T6 Aluminum alloy	$\frac{L}{r} \le 9.5$ $\quad \sigma_{all} = 19\ \text{ksi}$ $= 131\ \text{MPa}$ $9.5 \le \frac{L}{r} \le 66$ $\quad \sigma_{all} = \left[20.2 - 0.126\left(\frac{L}{r}\right)\right]\ \text{ksi}$ $= \left[139 - 0.868\left(\frac{L}{r}\right)\right]\ \text{MPa}$	$\frac{L}{r} \ge 66$ $\sigma_{all} = \frac{51{,}000}{(L/r)^2}\ \text{ksi}$ $= \frac{351(10^3)}{(L/r)^2}\ \text{MPa}$
4	c	Timber with a rectangular cross section $b \times d$ where $d < b$	$\frac{L}{d} \le 11$ $\quad \sigma_{all} = F_c{}^*$ $11 \le \frac{L}{d} \le k$ $\quad \sigma_{all} = F_c\left[1 - \frac{1}{3}\left(\frac{L/d}{k}\right)^4\right]$ $k = 0.671\sqrt{E/F_c}$	$k \le \frac{L}{d} \le 50$ $\sigma_{all} = \frac{0.30E}{(L/d)^2}$

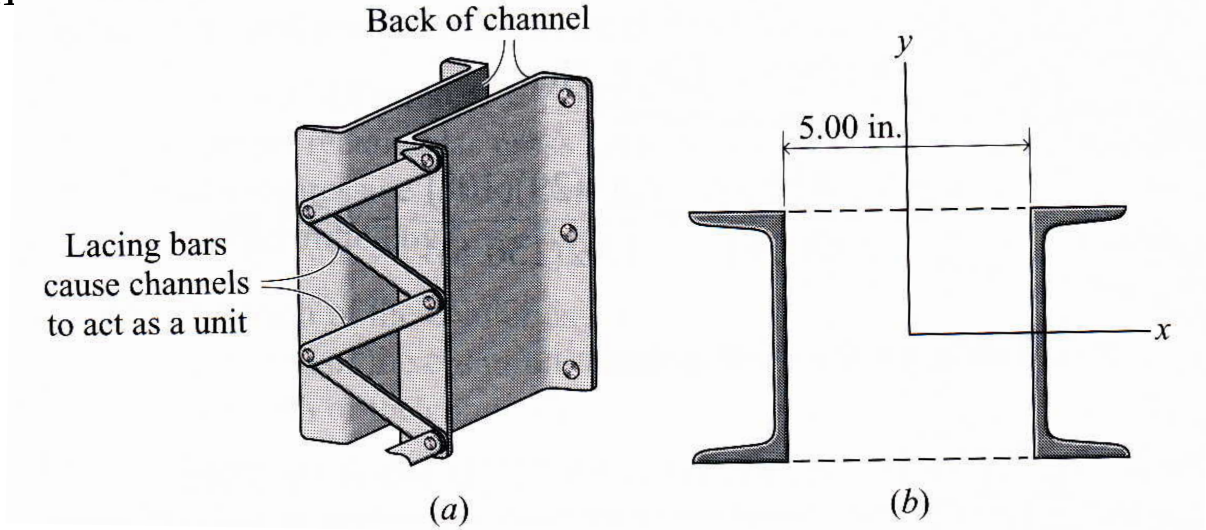
- The strength of materials has little effect on the allowable stress in long columns.

9.4 Empirical column formulas-Centric loading

- Example Problem 9-5

Two structural steel C10 × 25 [r_x = 3.52 in., r_y = 0.676 in., x_c = 0.617 in., A = 7.35 in.^{2} , σ_y = 36 ksi, E = 29,000 ksi] channels are latticed 5 in. back to back to form a column.

- Max. allowable load for effective lengths of 25 ft and 40 ft. (Use Code 1 for structural steel



9.4 Empirical column formulas-Centric loading

- Example Problem 9-6

A Douglas fir [E = 11 GPa, and F_c = 7.6 MPa] timber column with an effective length of 3.5 m has a 150 × 200-mm rectangular cross section.

- Max. compressive load permitted by Code 4

4	c	Timber with a rectangular cross section $b \times d$ where $d < b$	$\frac{L}{d} \le 11$ $\quad \sigma_{all} = F_c{}^*$ $11 \le \frac{L}{d} \le k$ $\quad \sigma_{all} = F_c\left[1 - \frac{1}{3}\left(\frac{L/d}{k}\right)^4\right]$ $k = 0.671\sqrt{E/F_c}$	$k \le \frac{L}{d} \le 50$ $\sigma_{all} = \frac{0.30E}{(L/d)^2}$