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457.646 Topics in Structural Reliability
In-Class Material: Class 02

Recommended reading material:

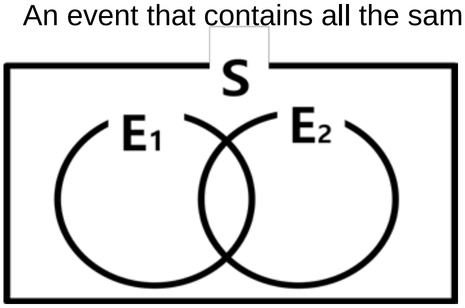
Der Kiureghian, A., and Ditlevsen, O. (2009)

Aleatory or epistemic? Does it matter? *Structural Safety*, 31:105-112

◎ Set Operations → useful for (   ) reliability analysis

① **"Union" of events:** E_1 E_2

- An event that contains all the sample points that are in E_1 E_2



e.g., Concrete mixing

- E_1: shortage of water
- E_2: shortage of sand
- E_3: shortage of gravel
- E_4: shortage of cement

E (concrete can't be produced) $=$

$=$ E_i

e.g., Wind

- E_1: blown off due to pressure
- E_2: missile-like flying objects

$E = E_1$ E_2

e.g., Bridge pier under EQ

- E_1: reaches displacement capacity
- E_2: reaches shear capacity

$E =$ E_1 E_2

※ $A \cup S =$

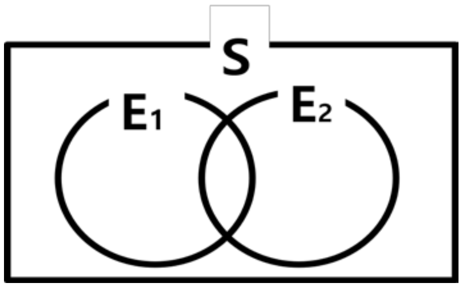
$A \cup \phi =$

$A \cup A =$

If $A \subset B$, then $A \cup B =$

② **"intersection" of events** E_1 E_2 **or**

: an event that contains all the sample points that are both in E_1 E_2



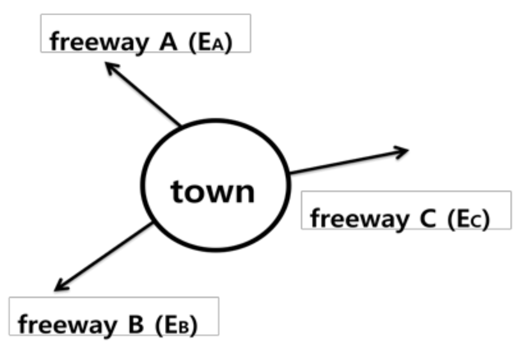
※ $A \cdot S =$

$A \cdot \phi =$

$A \cdot A =$

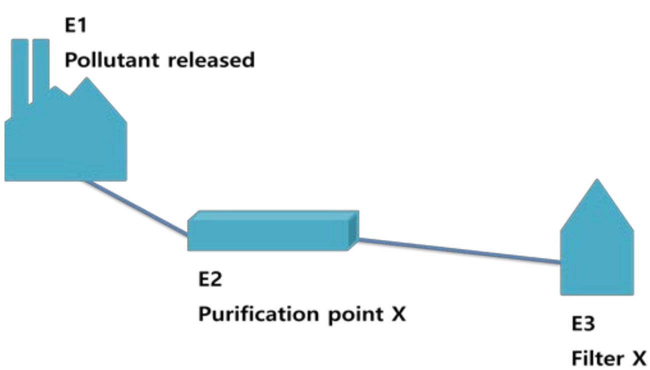
If $A \subset B$, then $AB =$

e.g.,



No evacuation by freeway $E =$

e.g.,



Exposed to pollutant $E =$

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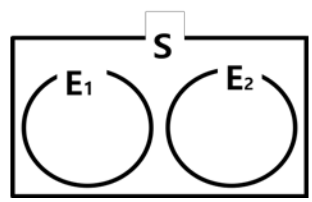
◎ Operation Rules

Commutative Rule	$E_1 \cup E_2 =$ $E_1 E_2 =$
Associative Rule	$(E_1 \cup E_2) \cup E_3 =$ $\qquad\qquad =$ $(E_1 E_2) E_3 =$ $\qquad\qquad =$
Distributive Rule	$(E_1 \cup E_2) E_3 =$ $(E_1 E_2) \cup E_3 =$
De Morgan's Rule	$\overline{(\bigcup_{i=1} E_i)} =$ $\overline{(\bigcap_{i=1} E_i)} =$

◎ Relationship between events

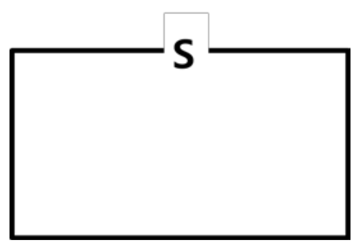
① **Mutually Exclusive events:** $E_1 E_2 =$

- Cannot occur together
- e.g. E_1 and $\overline{E_1}$
- $E_1 \cdots E_n$ and $\overline{E_i}$, $i \in \{1, \cdots, n\}$

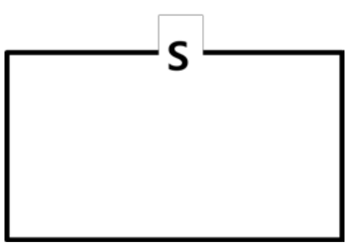


② **Collectively Exhaustive events:** $\bigcup_{i=1}^{n} E_i =$

- The union constitutes the sample space



※ **<u>MECE:</u>**



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2. Mathematics of Probability (measure of likelihood of event)

◎ Four approaches for assigning probability of events

Approach	Description	Example : Prob. (a "Yut" stick shows the flat side)
Notion of **Relative Frequency**	Relative frequency based on empirical data, Prob. = (# of occurrences) / (# of observations)	
On a **Priori** Basis	Derived based on elementary assumptions on likelihood of events	
On **Subjective** Basis	Expert opinion ("degree of belief")	
Based on **Mixed** Information	Mix the information above to assign probability	

◎ **Axioms of Probability**

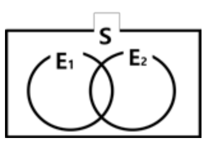
"Axioms": Statements or ideas which people accept as being the foundation of theory

I. $P(E)$ 0
II. $P(S)$ 1
III. M.E $E_1 \& E_2 : P(E_1 \cup E_2) =$

As a result,

① $\le P(E) \le$ ($\because P(S) = P(\quad \cup \quad) = \quad + \quad = \quad$)
② $P(\phi) =$ ($\because P(S \cup \phi) = \quad + \quad = \quad$)
③ $P(\overline{E}) =$ ($\because P(E \cup \overline{E}) = \quad$)
④ $P(E_1 \cup E_2) = P(E_1) \quad P(E_2) \quad P(E_1E_2)$

"Addition Rule"



- Venn Diagram
- Formal Proof

$$P(E_1 \cup E_2) = P(E_1 \cup \overline{E}_1 E_2) = P\left(E_1\right) + P(\overline{E}_1 E_2)$$
$$P(E_2) = P(E_1E_2) + P\left(\overline{E}_1 E_2\right)$$

"Inclusion-Exclusion Rule"

$$P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} P(E_i) - \sum\sum P(E_iE_j) + \sum\sum\sum P(E_iE_jE_k) + \cdots + (-1)^{n-1} \times P(E_1 \cdots E_n)$$

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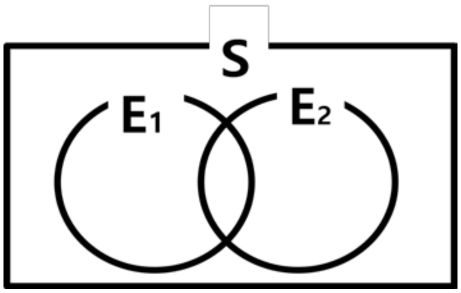
◎ Conditional Probability & Statistical Independence

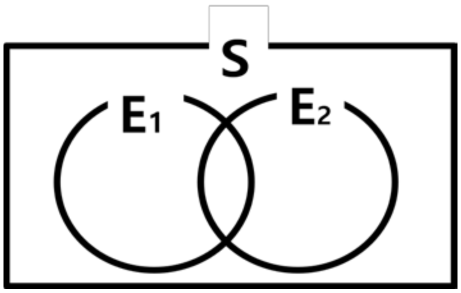
① Conditional Probability

- C.P of given

$P(E_1 | E_2) \equiv$



" "
occurred
⇒



② $P(E_1 | S) =$

③ **"Multiplication Rule":** $P(E_1 E_2) =$

($\because$ $P(E_1 | E_2) =$)

- $P(E_1 E_2 E_3) =$

- $P(E_1 \cdots E_n) =$

④ All the other prob. rules should be applicable to conditional probabilities as long as all the prob. are defined within the same space

- $P(E_1 \cup E_2 | E_3) =$

- $P(E_1 E_2 | E_3) =$

- $P(\overline{E_1} | E_3) =$

⑤ **Statistical Independence:** The occurrence of one event does not affect the likelihood of the other event

- $P(E_1 | E_2) =$

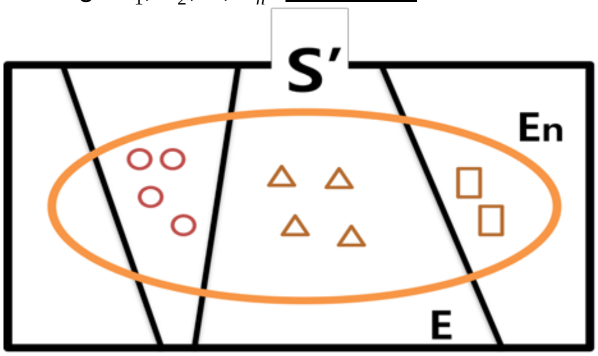
- $P(E_2 | E_1) =$

- $P(E_1 E_2) =$

cf. Mutually Exclusive $P(E_1 E_2) = 0$

◎ Total Prob. Theorem

Setting: $E_1, E_2, ..., E_n$:________ events



$P(E)$ → Not easy to get directly

$P(E \mid E_i)$ → Easier to get

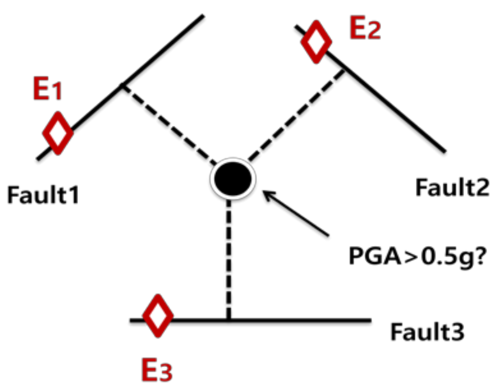
$$P(E) = \sum_{i=1}^{n}$$

Proof:

<u>Examples:</u>

(1) Seismic hazard analysis:

$$P(E) =$$



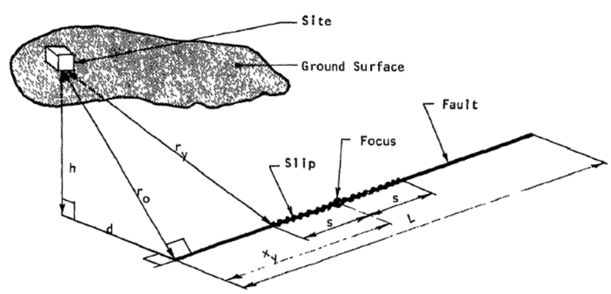


FIG. 3.1 TYPE 1 SOURCE (BASIC CASE)

Der Kiureghian, A. (1976). *A line source-model for seismic risk analysis*, Ph.D. dissertation, University of Illinois at Urbana-Champaign, Urbana, USA.

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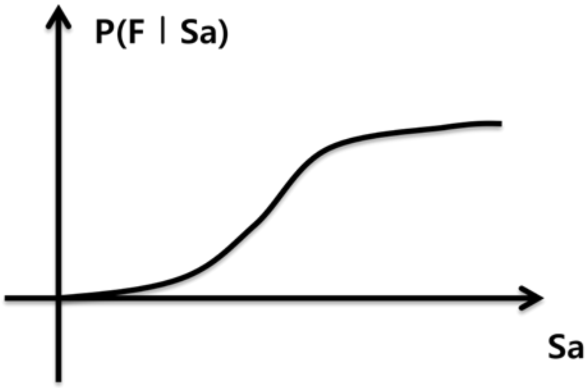
(2) Probability of structural failure under an uncertain input intensity: Fragility



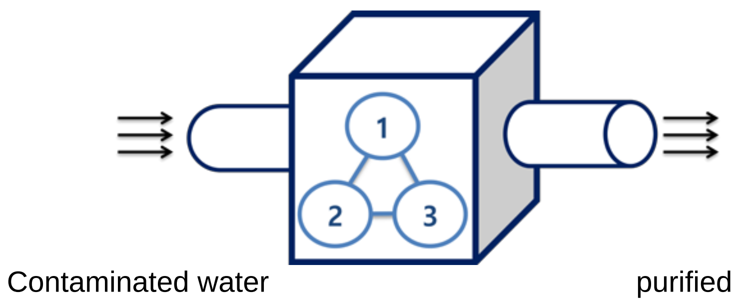
$$P(F) = \sum$$

◎ Bayes Theorem

$$P(E_i | E) = \frac{P(E | E_i)}{}$$

- Decision making
- Parameter estimation
- Inference

Example)



Measure of cleanness, X (0 : contaminated ~ 100 : clean)

	$P(E_i)$	$P(X \le 20 \| E_i)$
1	0.1	0.9
2	0.3	0.2
3	0.6	0.01

$X \le 20$ $\Rightarrow$ Which one failed?

$P(E_i | X \le 20) =$