Seoul National University
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457.646 Topics in Structural Reliability

In-Class Material: Class 09

III. Structural Reliability (Component) - continued

◎ Joint Probability Distribution Models

⑤ Joint distribution models with marginal & corr. coeff (contd.)

a) Morgenstern: $F_{X_i}(x_i),\ i=1,\ldots,n\ \&\ \alpha_{ij}$ but $|\rho_{ij}| < 0.30$

b) Nataf model (Nataf, 1962)

★ Joint PDF by Nataf model

$$f_{\mathbf{X}}(\mathbf{x}) = f_{\mathbf{Z}}(\mathbf{z}) \cdot \left|\det J_{\mathbf{Z},\mathbf{X}}\right|$$

$$= \varphi_n(\mathbf{z};\mathbf{R}') \underline{\hspace{4cm}}$$

$$J_{\mathbf{Z},\mathbf{X}} = \left[\hspace{6cm} \right]$$

$$\boxed{= \left[\prod_{i=1}^{n} f_{X_i}(x_i)\right] \cdot \underline{\hspace{4cm}}}$$

Note:

$$F_{X_i}(x_i) = \Phi(z_i)$$

$$f_{X_i}(x_i)dx_i = \varphi(z_i)dz_i$$

★ ρ'_{ij} (corr. coeff. b/w Z_i and Z_j)?

$$\rho_{ij} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\underline{\hspace{2cm}}\right)\left(\underline{\hspace{2cm}}\right) f_{X_iX_j}(x_i, x_j)dx_i dx_j$$

$$\therefore \rho_{ij} = \int\int \left(\underline{\hspace{2cm}}\right)\left(\underline{\hspace{2cm}}\right) \varphi_2(z_i, z_j; \rho'_{ij}) \underline{\hspace{3cm}}\ \underline{\hspace{1.5cm}}\ \underline{\hspace{1.5cm}} dz_i dz_j$$

In general, $|\rho'_{ij}| \qquad |\rho_{ij}|$

$\therefore |\rho_{ij}| \le A < 1$ may not cover the whole range of ρ_{ij}

$\rho'_{ij} \cong F \cdot \rho_{ij}$ Liu & ADK (Table 4~6) for pairs of selected distribution types

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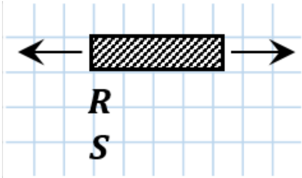
Table 9: Range of ρ_{ij} ~ wider (than Morgenstern)

Later used for transformation of dependent RVs into $\mathbf{U} \sim N(\mathbf{0},\mathbf{I})$

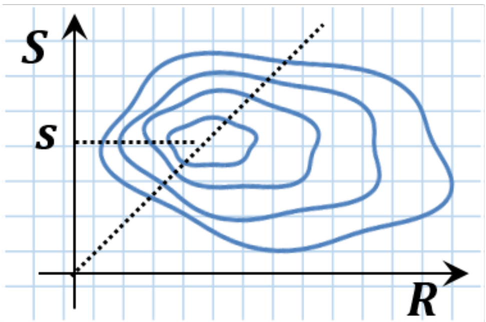
$\mathbf{X}$ $\mathbf{Z}$ $\mathbf{U}$

◎ Elementary Structural Reliability Problem



Describe the failure event in terms of _________ & __________

① **Failure :** $g(\mathbf{x}) = g(\quad,\quad) = \qquad \leq 0$



② **Failure probability :** $P_f = P(\qquad \leq 0)$

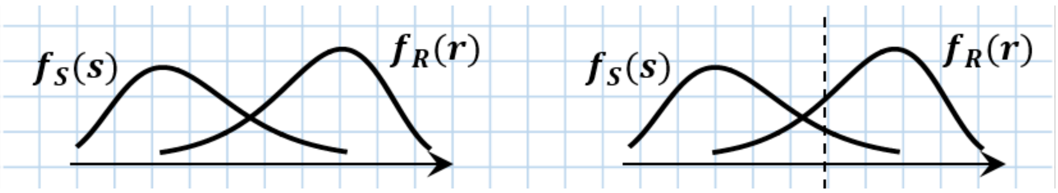
$$P_f = \iint f_{R,S}(r,s)drds$$

$$= \iint f_{R|S}(r|s) \cdot f_s(s)drds$$

$$= \iint f_{R|S}(r|s)\ dr\ f_s(s)ds$$

$$= \int \qquad f_s(s)ds$$

If R&S are s.i $P_f = \int \qquad ds$



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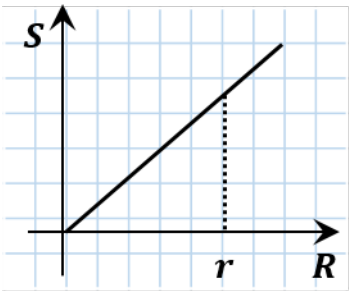
OR

$$P_f = \iint_{r \le s} f_{S|R}(s|r)\, f_R(r)\, ds dr$$

$$= \iint f_{S|R}(s|r) ds f_R(r) dr$$

$$= \int [\qquad\qquad] \, f_R(r) dr$$

$$\text{if s.i} = \int \qquad\qquad dr$$



③ **Reliability Index by "Safety Margin,"** β_{SM}

$$\boxed{M = \qquad\qquad}$$

: Safety Margin

Failure : $\{R - S \le 0\} \quad \Leftrightarrow \{\qquad \le 0\}$

$$\Leftrightarrow \{U_M \le \qquad\qquad\}$$

※ Standardization

$$U_M = \frac{M}{\qquad\qquad} \qquad \begin{cases} E[U_M] = \\ Var[U_M] = \end{cases}$$

For n RVs, $\quad \mathbf{U} = \mathbf{L}^{-1}\mathbf{D}^{-1}(\mathbf{X} - \mathbf{M})$

$$\therefore P_f = P(U_M \le \qquad) = F_{U_M}(\qquad)$$

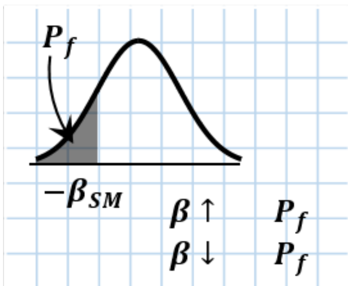
$$= F_{U_M}(\qquad)$$

β_{SM} : reliability index by safety margin

$$= \frac{\qquad}{\qquad}$$

$$= \frac{\qquad}{\qquad}$$

$$= \frac{r-1}{\sqrt{r^2\delta_R^2 + \delta_s^2 - 2r\delta_R\delta_S\rho_{RS}}}, \quad r = \mu_R / \mu_S$$



F_{U_M} : depends on distribution of R and S

e.g. special case ~ R and S are jointly normal

Then $U_M \sim$

Therefore $P_f = F_{U_M}(-\beta_{SM}) =$

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※ A. Cornell (1968. ACI codes)

Assumed R&S are jointly normal & used β_{SM} to compute P_f

④ **Reliability Index by "Safety Factor"**

$$F = \ln \frac{\qquad}{\qquad} = \ln \qquad - \ln$$

Failure $: \{ \qquad\qquad \le 0\}$ (※ used for LRFD $\phi R_n \ge \sum \gamma_k Q_k$)

$\Leftrightarrow \{ \qquad \le 0\}$

$\Leftrightarrow \{u_F \le - \frac{\qquad}{\qquad}\}$

$\therefore \beta_{SF} = \frac{\qquad}{\qquad}$

$\mu_F =$

$\sigma_F^2 =$

$P_f = F_{u_F}(- \qquad)$

⇒ special case: R & S are jointly lognormal

$U_F \sim$

$\therefore P_f = \Phi(\qquad)$

$\mu_F^{(LN)} =$

$\sigma_F^{(LN)} =$

$$\beta_{SF}^{(LN)} = \frac{\ln\left(r \cdot \sqrt{\frac{1+\delta_S^2}{1+\delta_R^2}}\right)}{\sqrt{\ln(1+\delta_R^2) - 2\ln(1+\rho_{RS}\delta_R\delta_S) + \ln(1+\delta_S^2)}}, \quad r = \frac{\mu_R}{\mu_s}$$

Safety factor-based reliability-index when R & S are jointly lognormal