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457.646 Topics in Structural Reliability

In-Class Material: Class 18

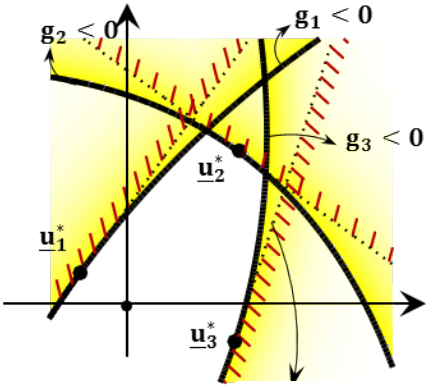
◎ FORM approximation (Hohenbichler & Rackwitz 1983)

① Series system

$$P(E_{sys}) = P(\bigcup_{i=1}^{m} E_i)$$

$$= P(\bigcup_{i=1}^{m} g_i(\mathbf{x}) \le 0)$$

$$\overset{FORM}{\cong} P(\bigcup_{i=1}^{m} \qquad \le 0)$$



$n \rightarrow$ # rv's

$m \rightarrow$ # comp's

Let $Z_i = \hat{\boldsymbol{\alpha}}_i \mathbf{u}$, $i = 1, \cdots, m$

$$G_i(\mathbf{u}) \approx G_i(\mathbf{u}_i^*) + \nabla G_i(\mathbf{u}_i^*)(\mathbf{u} - \mathbf{u}_i^*)$$

$$= \nabla G_i(\mathbf{u}_i^*)(\mathbf{u} - \mathbf{u}_i^*) \le 0$$

$$E[Z_i] =$$

$$\Leftrightarrow \qquad \le 0$$

$$Var[Z_i] = \| \quad \|^2 =$$

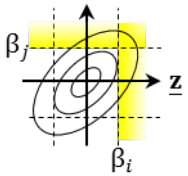
Therefore, $Z_i \sim$ (,)

$$\rho_{Z_i Z_j} = \frac{[\ ,\ ]}{} = E[\quad] - E[\ ] \cdot E[\ ]$$

$$= E[\quad \cdot \quad^{\mathrm{T}}] = E[\quad] = \quad E[\mathbf{u}\mathbf{u}^T] \quad =$$

$\therefore \mathbf{Z} \sim$ (,), $\rho_{Z_i Z_j} =$

$$\therefore P(E_{sys}) \overset{FORM}{\cong} P(\bigcup_{i=1}^{m} \qquad \le 0)$$



$$= 1 - P(\bigcap_{i=1}^{m} \quad \le \quad)$$

$$= 1 - \Phi_m(\quad, \cdots, \quad; \mathbf{R})$$

Joint normal CDF of $\mathbf{Z} \sim N(\mathbf{0}; \mathbf{R})$

Where $\Phi_m(\boldsymbol{\beta}; \mathbf{R}) = \int_{-\infty}^{\beta_1} \cdots \int_{-\infty}^{\beta_m} \varphi_m(\mathbf{Z}; \mathbf{R}) d\mathbf{z}$

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② Parallel system

$$P(E_{sys}) = P(\bigcap_{i=1}^{m} E_i)$$

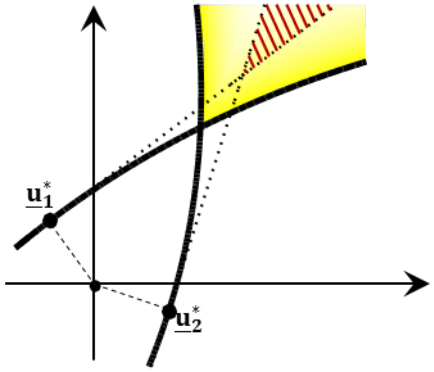
$$= P(\bigcap_{i=1}^{m} g_i(\mathbf{x}) \le 0)$$

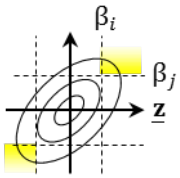
$$\overset{FORM}{\cong} P(\bigcap_{i=1}^{m} \qquad \le 0)$$

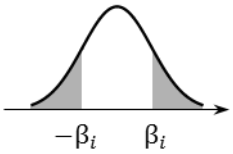
$$= P(\bigcap_{i=1}^{m} \qquad \ge \quad)$$

$$\overset{sym}{=} P(\bigcap_{i=1}^{m} \qquad \le \quad)$$

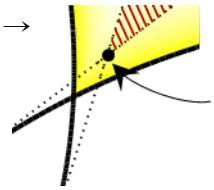
$$= \Phi_m(\quad , \cdots , \quad ; \mathbf{R})$$







→ may have huge errors due to curvatures

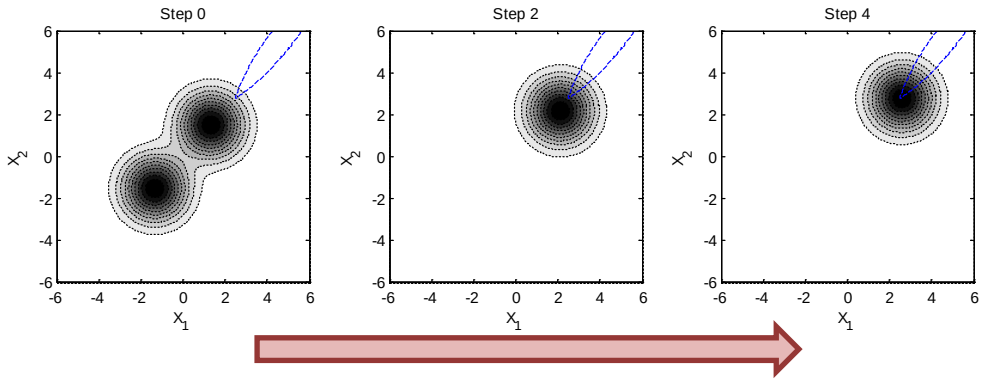
→  better linearization point?

"joint design point"

Hard to find or may not exist

Note: One could find such important domain using an adaptive sampling technique

Kurtz, N., and J. Song (2013). Cross-entropy-based adaptive importance sampling using Gaussian mixture. *Structural Safety*. Vol. 42, 35-44.



③ General system?

⇒ No direct FORM approximation