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Dept. of Civil and Environmental Engineering

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457.646 Topics in Structural Reliability

In-Class Material: Class 24

VIII-1. Probability-Based Structural Design Code

→ Cornell. C.A (1969) A probability-based structural code (J. ACI)

→ Ravindara & Galambos (1978) Load & resistance factor design for steel structures

(J. Str. Eng, Div. ASCE)

◎ Load & Resistance Factor Design (LRFD)

Replaced allowable stress design (ASD) (→safety factor)

⇒ Probability-based code

$$\phi R_n \geq \sum \gamma_k Q_{km} = \gamma_D Q_{Dm} + \gamma_L Q_{Lm} \quad \cdots\cdots\cdots\cdots\cdots (1)$$

Dead load   Live load

i. R_n : "   " resistance

→ code formula (e.g. $V_c = \frac{1}{6}\sqrt{f_c'} b_w d$)

→ nominal values used (material & dimension)

: given in "   " force, e.g. bending moment, axial force, shear force

ii. ϕ : "   " Factor ~ ϕ 1

(Dimensionless) conservatism due to the uncertainties in R

iii. Q_m : mean load effect

→ in generalized force (structural analysis)

iv. γ : "Load" factor~ γ 1

Conservatism due to

① Potential overload

② Uncertainty in load effect calculation

v. Limit-State

"U   " limit-states

e.g. frame instability, plastic mechanism formed incremental collapse

"S   " limit-states

e.g. excessive deflection, excessive vibration, premature yielding or slip

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LRFD codes suggest formulas for (   ), methods to compute (   ) from loads

provide (   ) & (   )

for each structural element (Q_m) from loads

to satisfy the (   ) reliability level

◎ Measure of (target) reliability

(or conservatism)

⇒ use

$$\beta = \frac{E\left[\ln \frac{R}{Q}\right]}{\sigma_{\ln\frac{R}{Q}}} \overset{FO}{\simeq} \quad \cdots\cdots\cdots\cdots\cdots (2)$$

$\geq$

$\mu_R \geq$

Want to split so that factors for R & Q can be determined independently

※ Lind (1971) $\sqrt{\delta_R^2 + \delta_Q^2} \simeq \bar{\alpha}(\delta_R + \delta_Q)$ where $\bar{\alpha} = 0.75$

$\therefore \qquad \geq \quad \cdots\cdots\cdots\cdots\cdots (3)$

$(\mu_R,\ \mu_Q,\ \delta_R,\ \delta_Q)?$

◎ Uncertainties in the Resistance, R

$$R = R_n \cdot M \cdot F \cdot P \quad \cdots\cdots\cdots\cdots\cdots (4)$$

R_n : nominal resistance by codes

M : "M"aterial ~

F : "F"abrication ~

P : "P"rofessional ~

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① $\mu_R \overset{FO}{\simeq}$

② δ_R ? $\ln R =$

$Var[\ln R] = \xi_R^2 =$

Note $\xi_X^2 \simeq \delta_R^2$ when $\delta \ll 1$

$\therefore \ \delta_R \cong$

◎ Uncertainties in Loads, Q

$Q = E(C_D AD + C_L BL)$ · · · · · · · · · · · · · · · (5)

① $\mu_Q \simeq$

② $$\begin{aligned}\delta_Q &\cong \delta_E^2 + \delta_{c_{DAD}+c_L BL}^2 \\ &= \delta_E^2 + \frac{c_D^2 \mu_A^2 \mu_D^2 (\delta_A^2 + \delta_D^2) + c_L^2 \mu_B^2 \mu_L^2 (\delta_B^2 + \delta_L^2)}{(c_D \mu_A \mu_D + c_L \mu_B \mu_L)^2}\end{aligned}$$

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◎ Finding target reliability index β

Initially, Eq. (3) & $\mu_R, \mu_Q, \delta_R, \delta_Q$ → existing, e.g. allowable stress code

→ can back-calculate target reliability index β embedded in the existing code

For example, 1969 AISC simply supported beams:

$\beta \cong 3.0$ (member), $\beta \cong 4.5$ (connections)

→ Provided starting points (and calibrated later)

◎ Load & Resistance Factors for given target β

Eq. (1) $\phi R_n \geq \sum_k \gamma_k Q_{km} = \gamma_E(\gamma_D C_D \mu_D + \gamma_L C_L \mu_L)$

Eq. (3) $\exp(-\bar{\alpha} \cdot \beta \cdot \delta_R) \cdot \mu_R \geq \exp(\bar{\alpha} \cdot \beta \cdot \delta_Q) \cdot \mu_Q$ ← expressions derived for $\mu_R, \mu_Q, \delta_R, \delta_Q$

From the LHS of Eq. (1) and Eq. (3): $\phi = \exp(-\alpha\beta\delta_R)\dfrac{\mu_R}{R_n}$ where $\alpha = 0.55$

From the RHS:

$$\begin{cases} \gamma_E = \exp(\alpha\beta\delta_E) \\ \gamma_D = 1 + \alpha\beta\sqrt{\delta_A^2 + \delta_D^2} \\ \gamma_L = 1 + \alpha\beta\sqrt{\delta_B^2 + \delta_L^2} \end{cases}$$

i) If $\beta \uparrow$ $\begin{cases} \phi \\ \gamma \end{cases}$

ii) $\dfrac{\mu_R}{R_n} > 1$, If $\dfrac{\mu_R}{R_n} \uparrow$, ϕ

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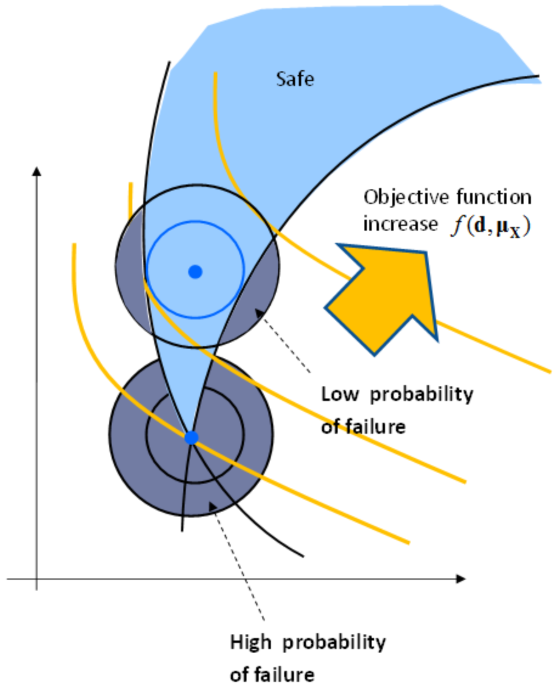
Review in Nguyen, Song & Paulino (2010)

VIII-2. Reliability-Based Design Optimization (RBDO)

◎ RBDO formulation



$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{x}} f(\mathbf{d}, \boldsymbol{\mu}_\mathbf{x})$$

s.t. $P[g(\mathbf{d}, \boldsymbol{\mu}_\mathbf{x}) \le 0] \le P_f^t$

$$\mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^u$$

$$\boldsymbol{\mu}_\mathbf{x}^L \le \boldsymbol{\mu}_\mathbf{x} \le \boldsymbol{\mu}_\mathbf{x}^u$$

Where

$f(\mathbf{d}, \boldsymbol{\mu}_\mathbf{x})$

$\mathbf{d}$

$\mathbf{x}$

$\boldsymbol{\mu}_\mathbf{x}$

P_f^t

$\mathbf{d}^L, \mathbf{d}^u$

$\boldsymbol{\mu}_\mathbf{x}^L, \boldsymbol{\mu}_\mathbf{x}^u$

◎ Reliability Index Approach (RIA; Enevaldsen & Sorensen 1994)

$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{x}} f(\mathbf{d}, \boldsymbol{\mu}_\mathbf{x})$$

s.t. β β^t

$\beta^t \leftarrow$ target reliability index $-\Phi^{-1}[P_f^t]$

$\beta \leftarrow$ generalized reliability index

$\beta = -\Phi^{-1}[$ $]$

↖ By FORM analysis (or others)

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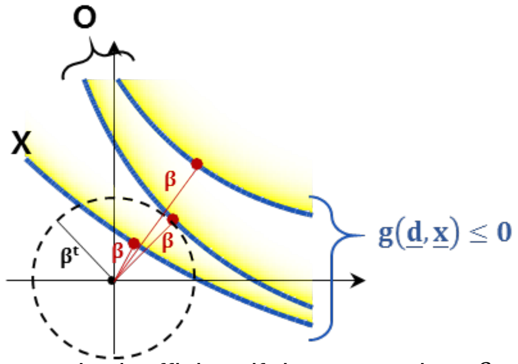
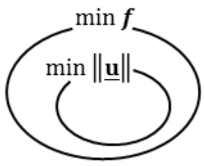


⇒ compute P_f for each interation of $\mathbf{d}$ to check if the constraint is satisfied

⇒ double loop approach



⇒ can be inefficient if the constraint $\beta \geq \beta^t$ is inactive

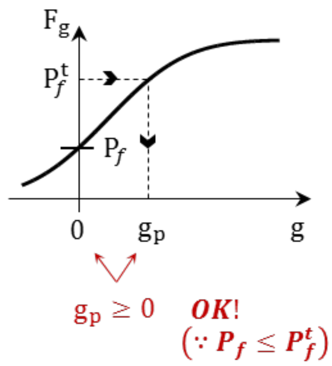
⇒ may not be able to provide an optimal solution if the failure does not occur in the feasible domain

◎ Performance Measure Approach (PMA; Tu et al., 1999) ※ double-loop

$$\min_{\mathbf{d},\boldsymbol{\mu}_\mathbf{x}} f(\mathbf{d},\boldsymbol{\mu}_\mathbf{x})$$

s.t. $g_p = F_g^{-1}\left[\Phi\left(-\beta^t\right)\right] \geq 0$ $(\Phi^{-1}[-\beta^t] = P^t)$

"Performance function" = quantile of g at P^t

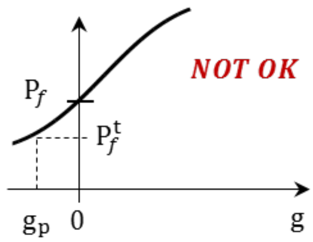


$g_p \geq 0 \iff P_f \leq P_f^t$

$\iff \beta \geq \beta^t$

Equivalent RBDO

How to find g_p?



They proposed (instead of solving FORM target β)

$$g_p = \min_{\mathbf{u}} G(\mathbf{d},\mathbf{u}) \quad \cdots\cdots\cdots\cdots\cdots (1)$$

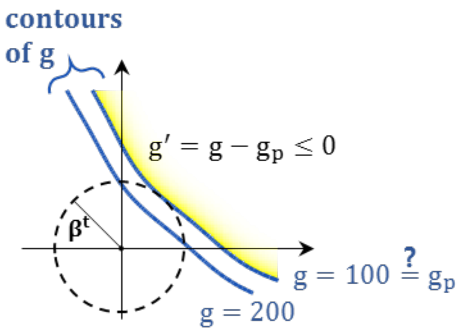
s.t. $\|\mathbf{u}\| = \beta^t$ ⇒ Minimizes g instead of $\|\mathbf{u}\|$

~ facilitates gradient-based optimization (using $\dfrac{\partial g}{\partial \mathbf{d}}$)

⇒ Overcomes the problems in RIA

(↑)

Is this g_p really $F_g^{-1}\left[P_f^t\right]$?



Set a new limit-state function

$$g^{'}(x) = g(\mathbf{x}) - g_p$$

$$P(g' \le 0) \cong \Phi(-\beta^t) = P_f^t$$

$$\parallel$$

$$P(g' \le g_p)$$

$$\parallel$$

$$F_g(g_p) \qquad g_p = F_g^{-1}[P_f^t]$$

◎ Single-Loop PMA (Liang et al., 2004)

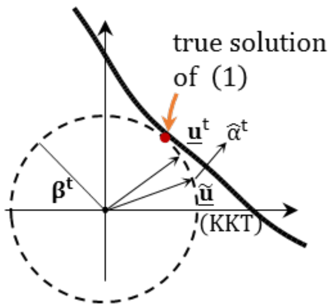
Replace the optimization in (1) with an approximation (but non-iterative)

system equation, i.e, Karush-Kuhn-Tucker (KKT)

condition

$\nabla_{\mathbf{u}} G(\mathbf{d},\mathbf{u}) + \lambda \nabla_{\mathbf{u}} (\|\mathbf{u}\| - \beta^t) = 0$ (λ →Lagrange Multiplier)

$\|\mathbf{u}\| - \beta^t = 0$



i. Solve KKT to get $\mathbf{u} = \tilde{\mathbf{u}}$

ii. Evaluate $\hat{\alpha}$ at $\mathbf{u} = \tilde{\mathbf{u}}$

iii. Approximate design point by

$\mathbf{u}^t = \beta^t \cdot \hat{\alpha}^t$

iv. Check $g(\mathbf{u}^t) \simeq g_p \ge 0$

Single loop RBDO

$$\min_{\mathbf{d},\boldsymbol{\mu}_{\mathbf{x}}} \quad f(\mathbf{d},\boldsymbol{\mu}_{\mathbf{x}})$$

s.t. $g_p \simeq g(\mathbf{d}, \mathbf{x}(\mathbf{u}^t)) \ge 0$