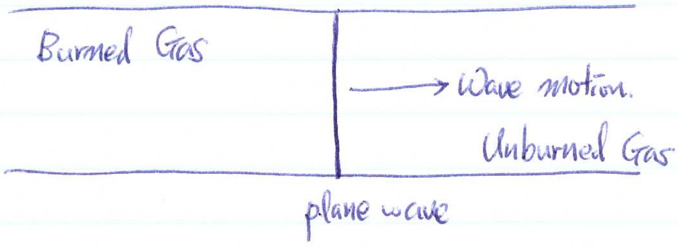
< Combustion Waves >

One-dimensional Combustion Waves.

- $-U_b \leftarrow \quad 0 \quad -U_u \leftarrow$ wave reference frame



- $\leftrightarrow W = U_u - U_b \quad \rightarrow V_w = U_u \quad 0$ laboratory reference frame

* 1-D flow without heat transfer to the surroundings.

- Conservation.

mass : $\rho_u U_u = \rho_b U_b = \dot{m}/A = \dot{m}''$ = mass flow rate per area.

momentum : $P_u + \rho_u U_u^2 = P_b + \rho_b U_b^2$

energy : $h_u + \frac{1}{2}U_u^2 = h_b + \frac{1}{2}U_b^2$

$$h = \sum_{i=1}^{N} Y_i \{ [h_i(T) - h_i(T_{ref})]_{sens} + \Delta_f H_i(T_{ref}) \}$$

Y_i : mass fraction of species i.

$$\Delta H_R = \sum (Y_i' - Y_i) \Delta_f H_i (T_{ref}) \equiv -q$$

$\therefore$ energy cons.

$$\underbrace{h_{u,sens} + \tfrac{1}{2}U_u^2}_{h_{u,0}} + q = \underbrace{h_{b,sens} + \tfrac{1}{2}U_b^2}_{h_{b,0}}$$

$h_{u,0}$ ↑ stagnation or total enthalpy.

Eqn of State : $P_b = \rho_b R_b T_b$

$$R_b \equiv \frac{\hat{R}}{\hat{M}_b}$$

$\hat{M}_b$: molecular weight of the burned gases.

mass + momentum cons.

$\Rightarrow \quad P + \dot{m}''^2 v = \text{const.}$

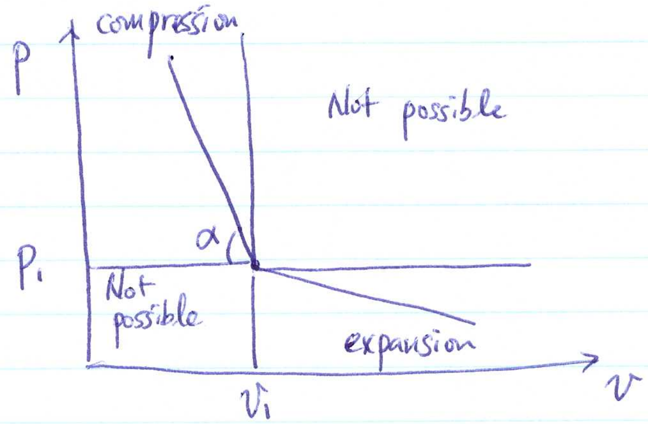
specific volume $= \frac{1}{\rho}$

$$\frac{dP}{dv} = \underbrace{-\dot{m}''^2 = \text{const.}}_{\text{mass cons.}}$$

* apply across the wave.

$$-\left(\frac{P_b - P_u}{1/\rho_b - 1/\rho_u}\right) = (\rho_u U_u)^2 = \dot{m}''^2 \Leftarrow \boxed{\text{Rayleigh line}}$$

$\Rightarrow$ indicates that $P\downarrow \Rightarrow v\uparrow$ or $P\uparrow \Rightarrow v\downarrow$ across the wave



$$U_u = \left(\frac{1}{\rho_u}\right)\Bigg(\underbrace{\frac{P_b - P_u}{1/\rho_u - 1/\rho_b}}_{= \tan\alpha}\Bigg)^{1/2} = \frac{\sqrt{\tan\alpha}}{\rho_u}$$

from mass cons. $\rho_u U_u = \rho_b U_b$

$$\therefore U_b = \frac{\sqrt{\tan\alpha}}{\rho_b}$$

$$W = U_u - U_b = \left(\left(\frac{1}{\rho_u} - \frac{1}{\rho_b}\right)(P_b - P_u)\right)^{1/2}, \quad \frac{\omega}{V_\omega} = 1 - \left(\frac{\rho_u}{\rho_b}\right)$$

i) compression wave (detonation) : $P_b > P_u$

$\therefore \frac{\omega}{V_\omega} > 0$. → the product moves in the direction of wave propagation

ii) expansion wave (deflagration) : $P_b < P_u$

$\therefore \frac{\omega}{V_\omega} < 0$ → burned gases moves away from the combustion wave

speed of sound in unburned gas.

$$a_u = \sqrt{\gamma R_u T_u} = \sqrt{\frac{\gamma P}{\rho_u}}$$

ideal eqn. of state.

$$\gamma = C_p / C_v \quad , \quad C_p = \left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{\hat{R}}{\hat{M}_u}\right)$$

$$M_u = \frac{u_u}{a_u} \quad \therefore \quad \gamma M_u^2 = \frac{\left(\frac{P_b}{P_u} - 1\right)}{\left(1 - \frac{\rho_u}{\rho_b}\right)}$$

in detonation $P_b \gg P_u$, $\rho_u < \rho_b$

$\Rightarrow M_u > 1$.

in deflagration $P_b < P_u$, $\rho_u \gg \rho_b$

$\Rightarrow M_u < 1$.

* Rankine - Hugoniot Relation

→ *assume constant C_p

$$(h_b - h_u)_{sens} = \langle C_p \rangle (T_b - T_u) = \frac{\gamma}{\gamma - 1} R (T_b - T_u)$$

Recall : $q = (h_b - h_u)_{sens} + \frac{1}{2}(u_b^2 - u_u^2)$

$$\therefore q = \frac{\gamma}{\gamma - 1}\left(\frac{P_b}{\rho_b} - \frac{P_u}{\rho_u}\right) - \frac{1}{2}(u_u^2 - u_b^2)$$

$\Rightarrow$ Recall momentum & mass cons.

$$\rho_u u_u = \rho_b u_b, \quad P_u + \rho_u u_u^2 = P_b + \rho_b u_b^2.$$

eliminate u_u & u_b

$$\Rightarrow q = \underbrace{\frac{\gamma}{\gamma-1}\left(\frac{P_b}{\rho_b} - \frac{P_u}{\rho_u}\right)}_{=(h_b - h_u)_{sens}} - \frac{1}{2}(P_b - P_u)\cdot\left(\frac{1}{\rho_u} + \frac{1}{\rho_b}\right)$$

Rankine - Hugoniot.

$$\therefore (h_b - h_u)_{sens} = \boxed{\frac{1}{2}(P_b - P_u)\left(\frac{1}{\rho_u} + \frac{1}{\rho_b}\right) + q.}$$

* in non-reacting mixture $q = 0$

Rayleigh does not depend on q) these two satisfy all of
Rankine - Hugoniot depends on q) the conservation eqns.

$\Rightarrow$ End state will be determined by intersections of these two curves.

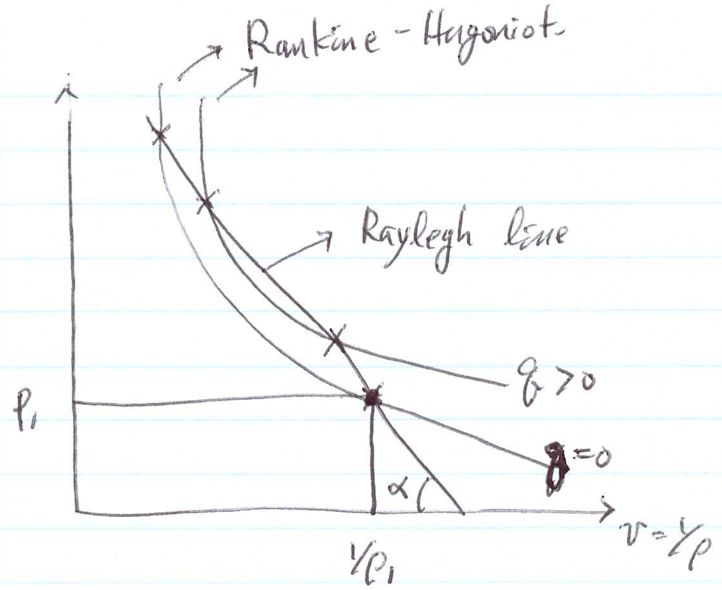
∘ Detonation.

- Shockwave thickness ~ a few mean free paths
  molecules pass through the shock in nanoseconds.
- Time scale for gas-phase combustion ~ microseconds.

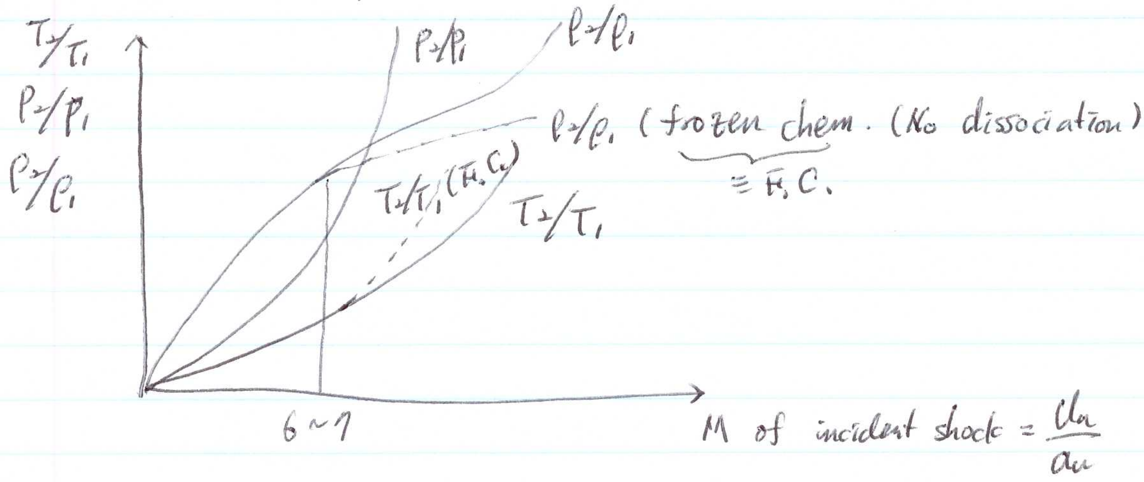
$\Rightarrow$ In detonation, ordinary shock compression raises the temp./press. of the combustible mixture before reaction occurs.

$\Rightarrow$ temp/press changes across the shock affect chemistry.

If $q=0$, the wave is an ordinary shockwave.

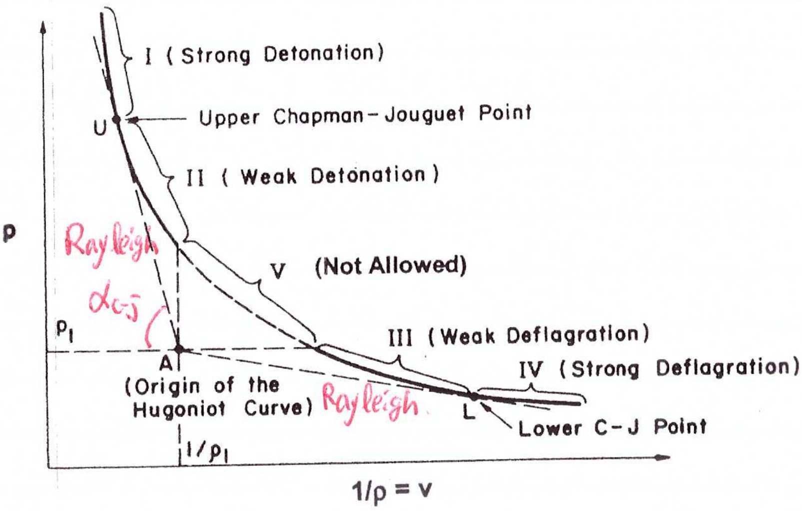


α depends on u_n (wave speed)



* Combustible mixtures will be heated by compression to temperatures sufficiently high to sustain combustion

→ from Rankine - Hugoniot relation
heat release q is proportional to pressure change.
→ heat release shifts the Hugoniot curve upward (in P-v diagram along the vertical line through the origin of the Hugoniot (@ P_1, v_1 although its form is unaltered.



- Upper C-J point, U, designates <u>the minimum</u> value of the detonation wave speed.

$$\alpha \geq \alpha_{c\text{-}j} \text{ to have solutions.}$$

$$\rho_u u_u = \sqrt{\tan\alpha}$$

$$\alpha \uparrow \Rightarrow u_u \uparrow$$

- Lower C-J point, L, designates the maximum deflagration speed

$*$ Strong detonation $\Rightarrow P_2/p_1$ is higher (more compression)

Strong deflagration $\Rightarrow P_2/p_1$ is lower (more expansion)

$\Rightarrow$ at a fixed unburned gas state (P_1, v_1) and heat release (q)
$\Rightarrow h_u, \rho_u, P_u, q$ are constant.

$$(h_b - h_u) = \frac{1}{2}(P_b - P_u)\underbrace{\left(\frac{1}{\rho_u} + \frac{1}{\rho_b}\right)}_{=(v_u + v_b)} + q$$

$$\frac{dh_b}{dP_b} - \underbrace{\frac{dh_u}{dP_b}}_{0} = \frac{1}{2}\left\{\left(\underbrace{\frac{dP_b}{dP_b}}_{1} - \underbrace{\frac{dP_u}{dP_b}}_{0}\right)(v_u + v_b) + (P_b - P_u)\left(\underbrace{\frac{dv_u}{dP_b}}_{0} + \frac{dv_b}{dP_b}\right)\right\} + \underbrace{\frac{dq}{dP_b}}_{0}$$

$$\therefore dh_b = \frac{1}{2}\Big((v_u + v_b)\,dP_b + (P_b - P_u)\,dv_b\Big)$$

Gibbs eqn: $dh = Tds + vdp$

$$dh_b = T_b dS_b + v_b dp_b = \frac{1}{2}\left((p_b - p_u) d v_b + (v_b + v_u) dp_b\right)$$

① .... $\therefore T_b dS_b = \frac{1}{2}\left((p_b - p_u) d v_b - (v_b - v_u) dp_b\right)$

* from Rankine relation

② .... $\frac{dp_b}{dv_b} = \frac{p_b - p_u}{v_b - v_u} = -(\rho_b u_b)^2 = -\rho_b^2\left(\frac{dp_b}{d\rho_b}\right)$

$\because v_b = \frac{1}{\rho_b}$ & $dv_b = -\frac{1}{\rho_b^2} d\rho_b$

at U. ① & ② both work.

from ② $p_b - p_u = (v_b - v_u)\frac{dp_b}{dv_b}$ and put this to ①

$\therefore T_b dS_b = 0 \Rightarrow dS_b = 0$ @ C-J points

and $(\rho_b u_b)^2 = \rho_b^2\left(\frac{dp_b}{d\rho_b}\right)$ @ $dS_b = 0 \Rightarrow S = \text{const.}$

$$\left(\frac{dp_b}{d\rho_b}\right)_S \equiv a_b^2$$

$\therefore u_b = a_b \Rightarrow M_b = 1$

$\Rightarrow$ Flow behind the wave is thermally-choked

① $\rightarrow T_b \frac{dS_b}{dv_b} = \frac{1}{2}\left((p_b - p_u) - (v_b - v_u)\frac{dp_b}{dv_b}\right)$

take second derivative respect to v_b

$$\frac{d^2 S_b}{dv_b^2} = \frac{1}{2T_b}(v_u - v_b)\cdot\left(\frac{d^2 p_b}{dv_b^2}\right)_{C-J}$$

@ U, $v_u - v_b > 0 \Rightarrow \frac{d^2 S_b}{dv_b^2} > 0 \Rightarrow$ minimum entropy

@ L, $v_u - v_b < 0 \Rightarrow \frac{d^2 S_b}{dv_b^2} < 0 \Rightarrow$ maximum entropy.

$$u_u = \frac{\rho_b}{\rho_u} u_b = \frac{\rho_b}{\rho_u} a_b = \frac{\rho_b}{\rho_u} a_u \left(\frac{a_b}{a_u}\right) = \frac{\rho_b}{\rho_u} a_u \sqrt{\frac{T_b}{T_u}}$$

$$\frac{\rho_b}{\rho_u} > 1, \quad \frac{T_b}{T_u} > 1.$$

$$\therefore \underline{u_u > a_u \Rightarrow \text{supersonic}} \;//$$

$\Rightarrow$ C-J detonation wave is supersonic
that is minimum possible detonation velocity.