< Combustion Wave II >

- Detonation

$U_b \left(= \frac{\sqrt{\tan\alpha}}{\rho_b} \right)$ should be equal of greater than C-J detonation

Velocity to get solutions, strong and weak detonations.
(I) (II).

Specific volume of II is larger than that of C-J point.
Recall that $M_{b\,C-J} = 1$.

$\hookrightarrow U_b = \underline{v_b \tan\alpha} > U_{b\,C-J}$

$\therefore \quad M_b > 1.$

Recall that $M_u > 1$.

Normal shock produces subsonic flow right behind the shock

$\therefore$ Subsonic flow accelerates to be supersonic ($M_b > 1$)

$\Rightarrow$ violation of thermodynamics law.

$\boxed{\text{No weak detonation}}$

Otherwise, specific volume of I is smaller than $v_{b\,C-J}$.

$$M_b < 1 = M_{b\,C-J}$$

$\Rightarrow U_b$ is subsonic

Nevertheless, strong detonation have never been observed as a fully-developed, self-sustaining end state. They may occure in transient process or when the system is over-driven by external energy sources.

$\Rightarrow$ Only C-J detonation will be characterized

- Deflagration

for strong deflagration (IV), v_b of the end state is much greater than that at C-J point, L.

$\therefore$ as in weak detonation case $M_b > 1$

$\frac{v_u}{v_b}\sqrt{\frac{T_b}{T_u}} = M_u < 1$, the wave propagates at subsonic speed.

This violates thermodynamics law.

Weak deflagration satisfies all thermodynamics constraints, which is common process, termed flames.

C-J deflagrations are not observed in practice.

* Determining C-J detonation speed

$$q = \langle C_p \rangle (T_{o_b} - T_{o_u})$$

constant C_p assumed

T_o (stagnation temperature) $\equiv T + \frac{v^2}{2\langle C_p \rangle}$ — velocity

In a compressible flow.

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{q}{\langle C_p \rangle T_u} = \frac{\cancel{\langle C_p \rangle} T_{o_u} \left(\frac{T_{ob}}{T_{ou}} - 1 \right)}{\cancel{\langle C_p \rangle} T_u} = \left(1 + \frac{\gamma - 1}{2} M_u^2\right)\left(\frac{T_{ob}}{T_{ou}} - 1\right)$$

@ C-J point. $M_b = 1$, $M_u = M_{CJ}$

$$\therefore \frac{T_{ob}}{T_{ou}} = \frac{T_o^*}{T_{ou}} = \frac{(1 + \gamma M_{CJ})^2}{2(\gamma + 1) M_{CJ} \left(1 + \frac{\gamma - 1}{2} M_{CJ}^2\right)}$$

from isentropic relation

Temperature at choked flow

$$\therefore \frac{q}{\langle C_p \rangle T_u} = \frac{(1 - M_{CJ}^2)^2}{2(1 + \gamma) M_{CJ}^2}$$

$$M_{CJ} \approx \sqrt{\frac{2(1+\gamma) q}{\langle C_p \rangle T_u}} \longrightarrow q \uparrow \Rightarrow M_{CJ} \uparrow$$

2.3 Detonation Wave Structure

We have modeled detonations as a normal shock wave followed by an infinitesimal zone of exothermic reaction. A more detailed analysis (the ZND model due to Zeldovich, von Neumann and Döring) regards the shock as thin, but allows for a finite-thickness reaction zone, as illustrated below.

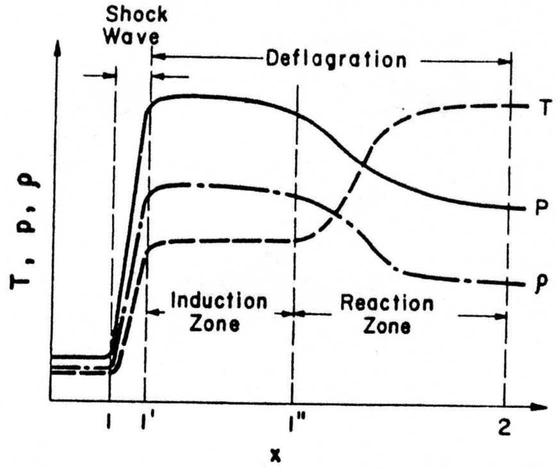

Fig. 2.9 Structure of a ZND detonation wave.

The features of this model are best seen in terms of the family of Hugoniots for different extents of reaction, shown in Fig. 2.10. The compression increases the pressure and temperature, and initiates reaction. Successive extents of reaction move the system to the ultimate end state (U), at lower pressure and higher temperature. If the reaction rate is slow, the shock weakens and the wave speed decreases, as observed near the detonation limits.

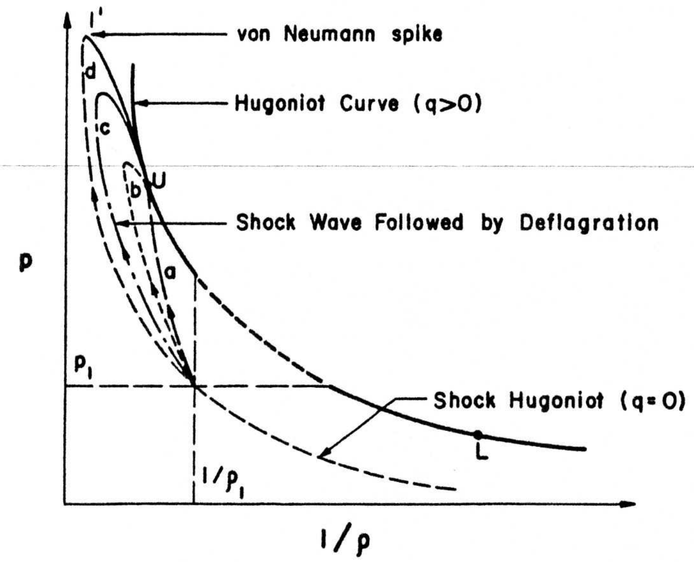

Fig. 2.10 A ZND detonation on a P-v diagram.