Data Structure

Lecture#22: Searching 3 (Chapter 9)

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In This Lecture

- Motivation of collision resolution policy
- Open hashing for collision resolution
- Closed hashing for collision resolution

Hashing

- Given a key *k*, can we search k in a constant time?
  - Yes!
  - We can do it by hashing. It is faster than binary search , QBS, and sequential search
- Hash table HT is the array that holds the records
  - HT has M slots (slots numbered from 0 to M-1)
- Hash function *h* maps key *K* to a number (position)
  - $0 \leq h(K) \leq M - 1$
  - E.g., h(*K*) = *K* % M
- Goal of a hashing system: arrange things such that for a given key *K*, and *i* = h(*K*), the record for the key *K* is located in HT[*i*]
  - Then, the searching time would be constant

Hashing

- Goal of a hashing system: arrange things such that for a given key *K*, and i = h(*K*), the record for the key *K* is located in HT[*i*]
- Collision: two different keys k_1 and k_2 map to a slot
  - $h(k_1) = \beta = h(k_2)$

- Finding a record with key value *K* by hashing:
  - Compute the table location h(*K*)
  - Starting with slot h(*K*), locate the record containing key *K* using a <u>collision resolution policy</u>

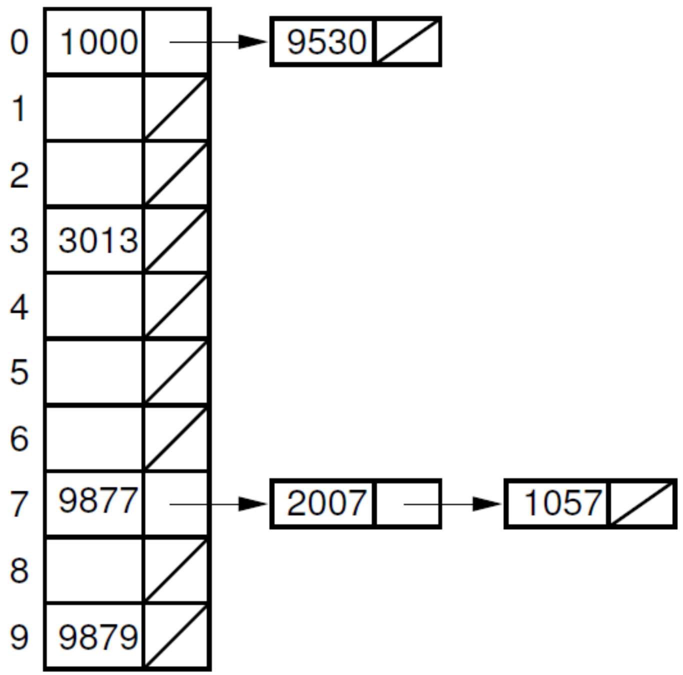
Collision Resolution

- Collision is unavoidable in many cases. How can we insert an item to hash table in case of collision?

- Collision resolution techniques
  - Open hashing (also called `separate chaining')
    - Collisions are stored outside the table
  - Closed hashing (also called `open addressing')
    - Collisions are stored at another slot in the table

Open Hashing

- Open hashing (also called `separate chaining')
  - Collisions are stored outside the table
  - Limitation: some slots in the table may not be used



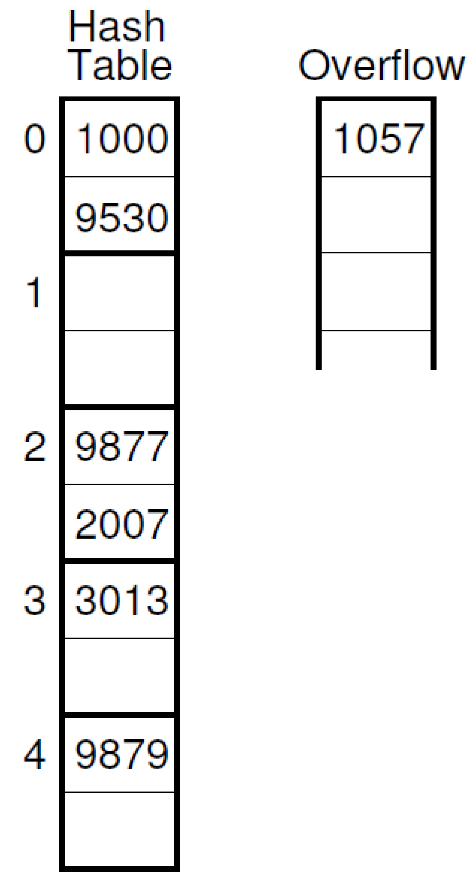
Closed Hashing

- Closed hashing (also called `open addressing')
  - Collisions are stored at another slot in the table
  - Each record R with key k_R has a home position h(k_R)
  - If another record already occupies R's home position, R will be stored at some other slot in the table

- Examples
  - Bucket Hashing
  - Linear Probing
  - …

Bucket Hashing (1)

- Group hash table slots into buckets
  - M slots are divided into B buckets (each bucket: M/B slots)
- Hash function (key->bucket number) assigns each record to the first slot in the bucket that the record is mapped to.
  - If the first slot is empty, insert
  - If the first slot is occupied, find the next empty slot in the bucket
  - If all the slots in the bucket are occupied, store in an overflow bucket

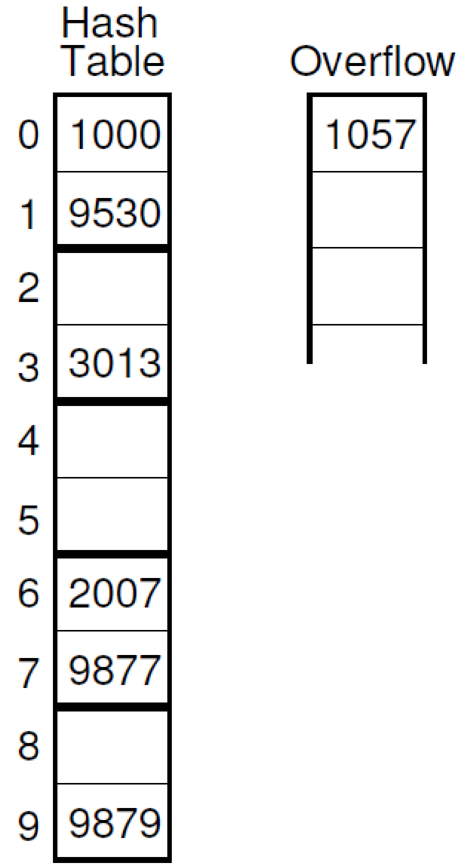


Insertion order:
9877, 2007, 1000, 9530, 3013, 9879, 1057

M = 10, B = 5
h(K) = K mod 5

Bucket Hashing (2)

- A variation on bucket hashing: hash a key to a slot in the hash table as though bucketing were not being used
  - If the slot is empty, insert
  - If the slot is occupied, find the next empty slot in the bucket
  - If all the slots in the bucket are occupied, store in an overflow bucket



Insertion order:
9877, 2007, 1000, 9530, 3013, 9879, 1057

M = 10, B = 5
h(K) = K mod 10

Bucket Hashing (3)

- Bucket hashing vs open hashing?
  - Bucket hashing has more collision => longer running time to search an item
  - Bucket hashing has less storage requirement => less space
- Limitation of Bucket Hashing
  - If a bucket is full, then all the inserts to the bucket will be stored in the overflow bucket, even when the hash table has many empty areas

Linear Probing

- Closed hashing with no bucketing, and a collision resolution policy can use any slot in the hash table
- If the home position is occupied, the new position is determined by (home + probe_function())
  - The sequence of slots is called `probe sequence'

```
/** Insert record r with key k into HT */
void hashInsert(Key k, E r) {
  int home;                          // Home position for r
  int pos = home = h(k);             // Initial position
  for (int i=1; HT[pos] != null; i++) {
    pos = (home + p(k, i)) % M;      // Next probe slot
    assert HT[pos].key().compareTo(k) != 0 :
           "Duplicates not allowed";
  }
  HT[pos] = new KVpair<Key,E>(k, r); // Insert R
}
```

Linear Probing

- Linear probing: move down *i* slots in the table
  - p(*K*, *i*) = *i*

M = 10
h(K) = K mod 10

0	9050
1	1001
2	
3	
4	
5	
6	
7	9877
8	2037
9	

(a)

Insertion order:
1001, 9050, 9877, 2037

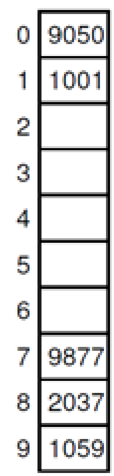
0	9050
1	1001
2	
3	
4	
5	
6	
7	9877
8	2037
9	1059

(b)

1059 is added

Linear Probing

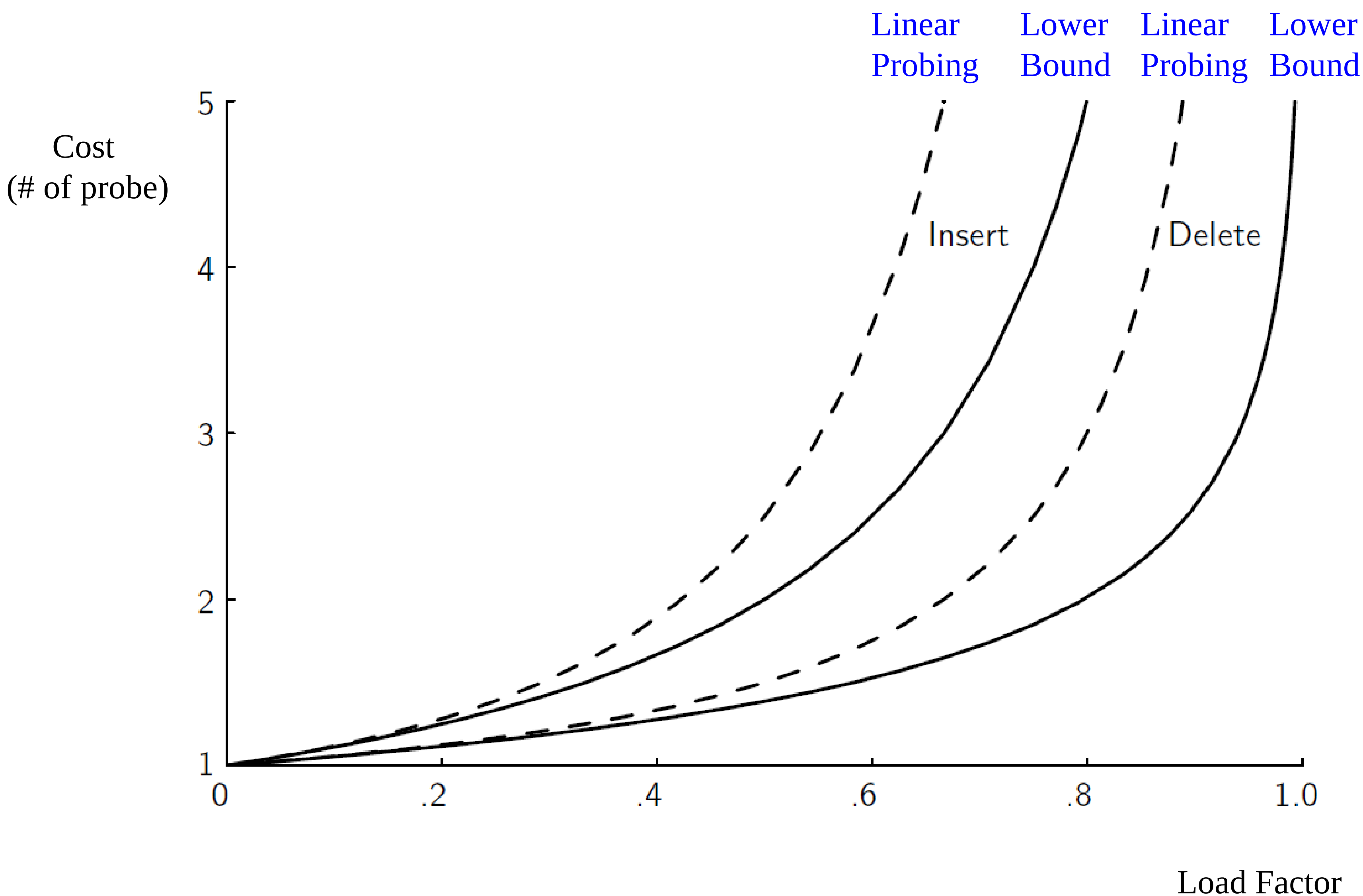
- Problem of linear probing
  - Primary clustering: nonempty slots are clustered, and thus giving unequal probability to empty slots
  - E.g., in the figure below, what is the probability that a random key k will be inserted at slot i?
    - P (slot 2) = 0.6
    - P (slot 3) = P (slot 4) = P (slot 5) = P (slot 6) = 0.1



Improved Collision Revision

- Use linear probing, but skip slots by a constant c
  - $p(K, i) = ci$
  - c should be relatively prime to M (why?)
  - Limitation: one section of slots will be used more, if inputs are skewed
  - E.g., if c = 2, and accesses are all odd numbers
- Pseudo-random probing
  - $p(K, i)$ = Perm[i – 1], where Perm is an array of length M-1 containing a random permutation of the values from 1 to M-1
- Quadratic probing
  - $p(K, i) = c_1 i^2 + c_2 i + c_3$

Performance of Closed Hashing



Discussion

- How can we make the probability of collision very small?
  - Open hashing vs. closed hashing
  - Time and space tradeoff
- Open hashing vs. bucket hashing
  - Bucket hashing uses space more efficiently, but has more collisions
- Bucket hashing vs linear probing?

What you need to know

- Collision resolution
  - Hard to avoid collision in most cases
- Open hashing
  - Simple, but some slots may not be used
- Closed hashing
  - Open hashing vs. bucket hashing
  - Bucket hashing vs. linear probing

Questions?