3. Electrical Conduction in Solids

3A. Classical Theory

3B. Semiclassical Theory

3C. Semiconductors

3C. Semiconductors

3C.1. Band Structure and Fermi Level

3C.2. Intrinsic Semiconductor

3C.3. Extrinsic Semiconductor

3C.4. Mobility and Conductivity

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The semiconductors are the main component in optoelectronic devices. There are several semiconducting materials ranging from the single-element materials such as Si and Ge to compound semiconductors such as GaAs, InP, GaN, ZnO, ….

In the intrinsic semiconductors, the materials are highly pure, and the conductivity is very low because of the bandgap between filled and unfilled states. In extrinsic semiconductors, the charge carriers are increased by doping.

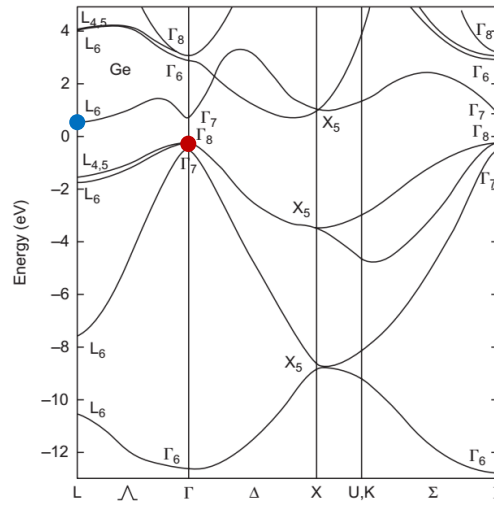**Water splitting, photo detector, light-emitting diode, solar cell, solid-state electrolyte, transistor, MOSFET, DRAM, etc.**
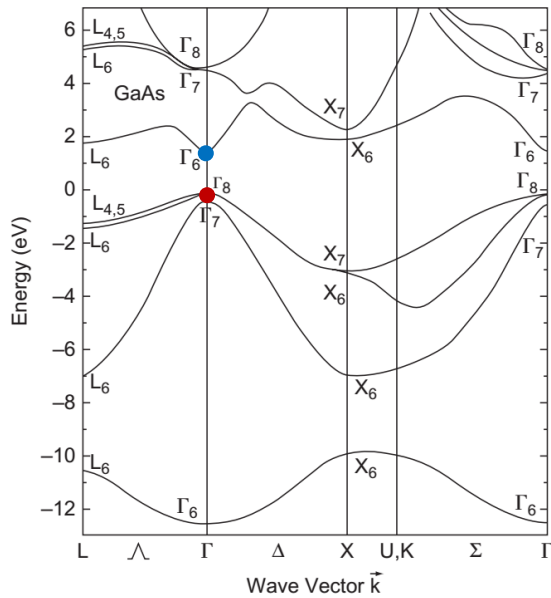
3C.1. Band Structure and Fermi Level

Si
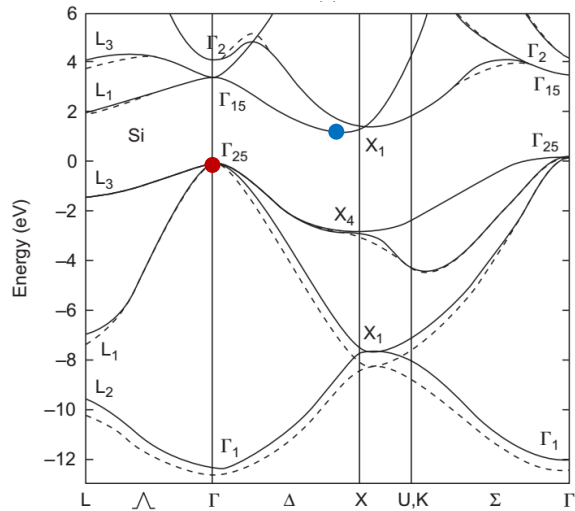
Ge

GaAs

InP
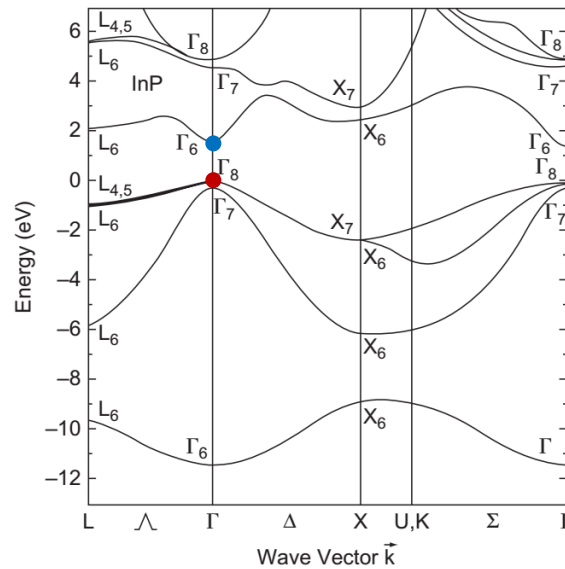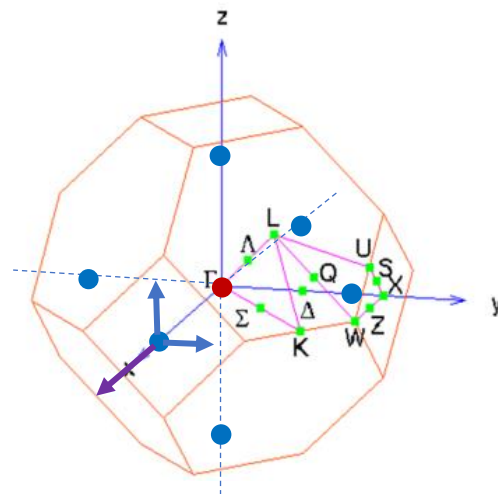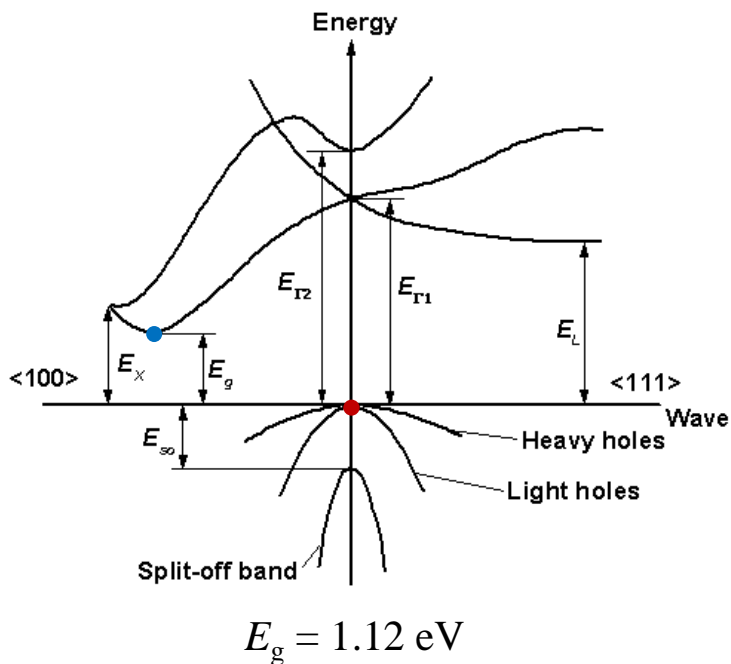
Band structure near the conduction bottom

6 "valleys"

$E_g = 1.12$ eV
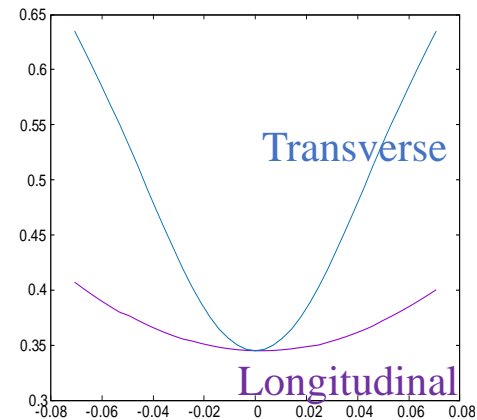
$m*$(electron) = 0.98 (longitudinal), 0.19 (transverse)
$m*$(hole) = 0.49 (heavy), 0.16 (light), 0.29 (split-off)

The split-off band is formed because of spin-orbit interactions.
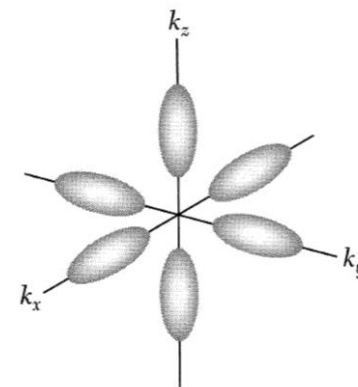
Constant energy surface for electrons

m^*(hole) = 0.49 (heavy), 0.16 (light), 0.29 (split-off)
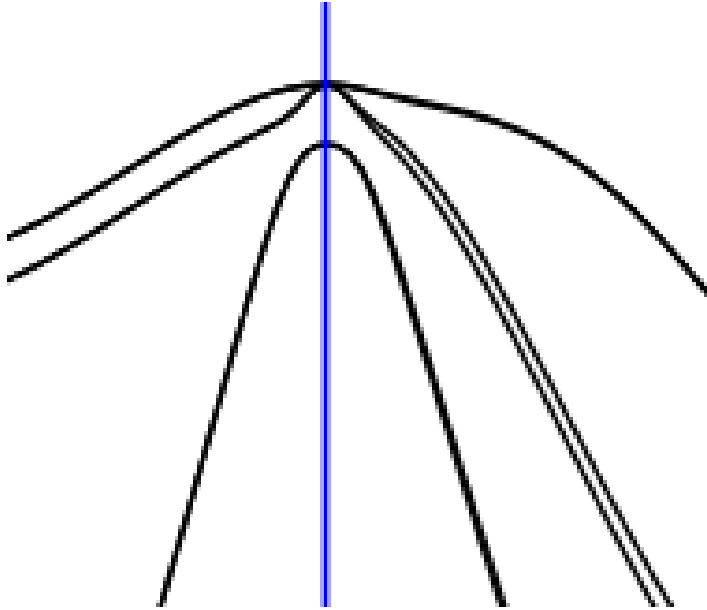
$$D(E) = \frac{1}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E}$$

ppt 2A-8
(spin-up or spin-down)
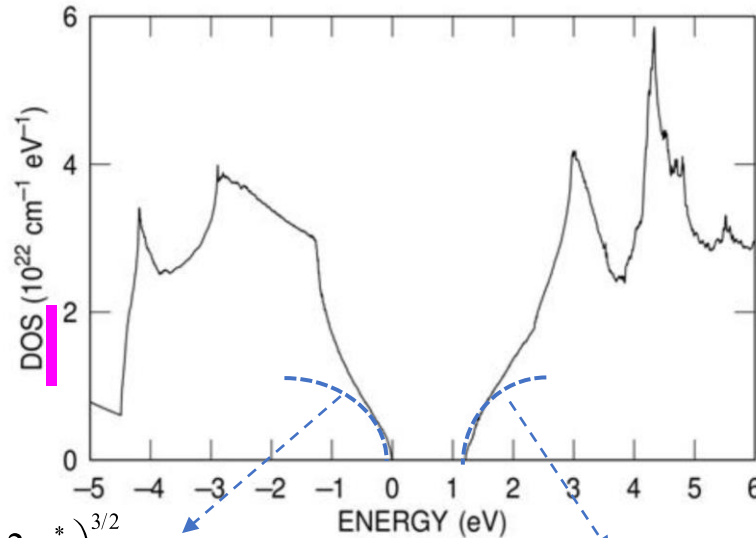
$$D_v(E) \simeq \frac{1}{2\pi^2}\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2}\sqrt{E_v - E}$$

$$D_c(E) \simeq \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}\sqrt{E - E_C}$$

The density of states near the bandedges are well approximated by the (DOS) effective mass.

In Si and Ge, effective masses are in fact anisotropic;

$$m_{\text{density}}^* = g^{\frac{2}{3}}\sqrt[3]{m_x m_y m_z}$$

g: degeneracy factor or number of valleys (6 for Si and Ge)

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Electron and Hole Concentrations

$$g(E) \propto (E - E_c)^{1/2}$$

Vacuum Level

χ: Electron Affinity

$E_c + \chi$

CB

E_c

E_F

E_v

VB

0

$D(E)$

$D(E)$

$g(E)$

$[1 - f(E)]$

For electrons

For holes

$f(E)$

Area = $\int n_E(E)dE = n$

$n_E(E)$

$p_E(E)$

Area = p

$n_E(E)$ or $p_E(E)$
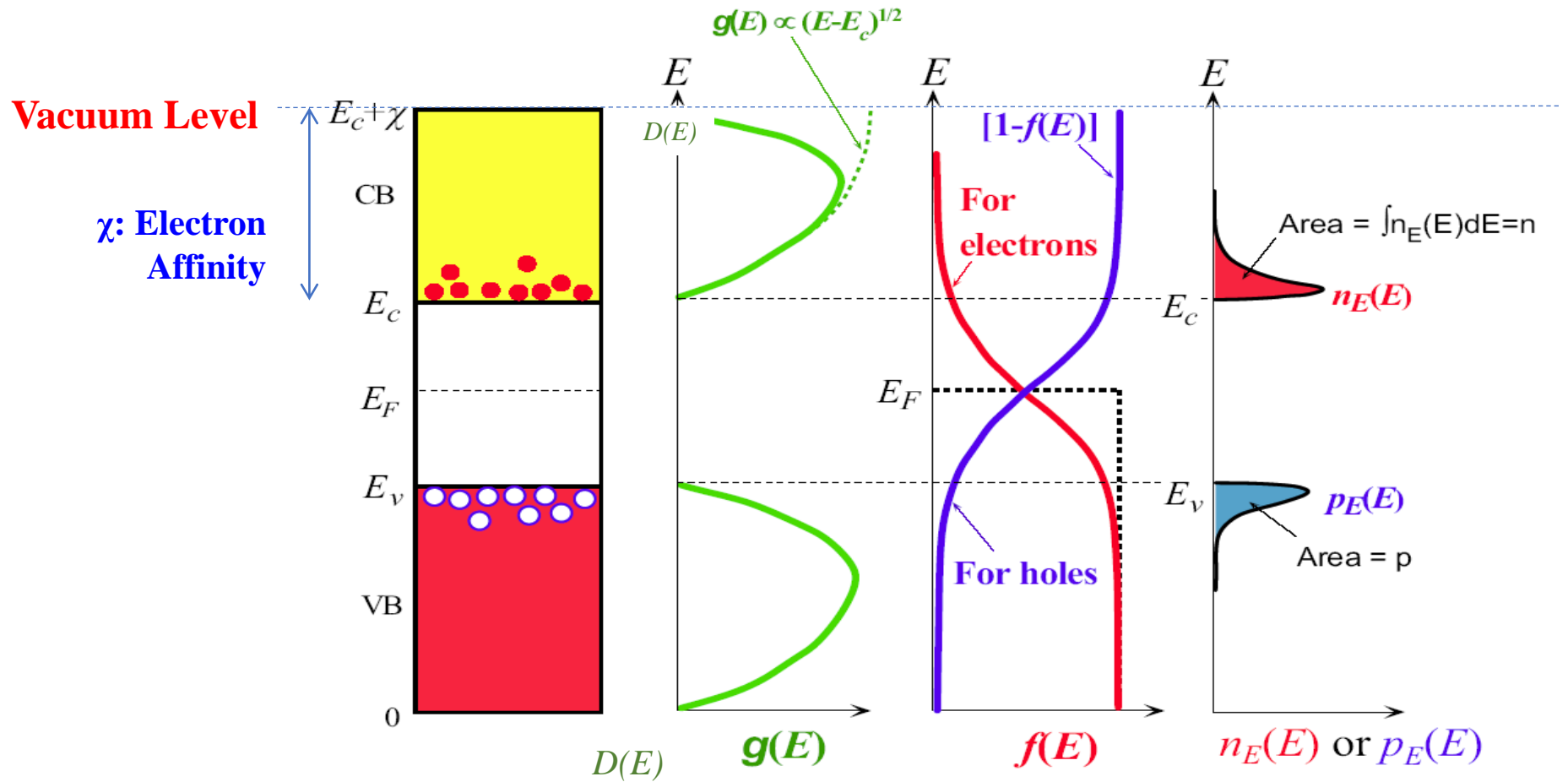
Three Independent Parameters for the Band Alignment:
- **Bandgap energy**
- **Work function**
- **Electron affinity (or Valence-band maximum)**

$$D_c(E) \simeq \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}\sqrt{E - E_C}$$

Total density of free electrons in conduction band

(spin-up + spin-down)

$$n = \int_{E_C}^{\infty} D_C(E) f(E)\, dE$$

$$= \int_{E_C}^{\infty} \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} (E - E_C)^{1/2} \frac{1}{e^{(E-E_F)/kT}+1}\, dE$$

$$\simeq \int_{E_C}^{\infty} \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} (E - E_C)^{1/2} e^{-(E-E_F)/kT}\, dE \qquad (E_c\text{-}E_F) >> kT$$

$$= \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} e^{-(E_C-E_F)/kT} \int_{E_C}^{\infty} (E - E_C)^{1/2} e^{-(E-E_C)/kT}\, dE$$

$$= 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} e^{-(E_C-E_F)/kT} = N_c e^{-(E_c-E_F)/kT} \qquad (\int_0^{\infty} E^{1/2} e^{-E/kT}\, dE = \frac{\sqrt{\pi}}{2}(kT)^{3/2})$$

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \text{ effective density of states in conduction}$$

Similarly,
$$p = N_v \exp\left[-\frac{(E_F - E_v)}{kT}\right] \qquad N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

Table 5.1 Selected typical properties of Ge, Si, InP, and GaAs at 300 K

	E_g (eV)	χ (eV)	N_c (cm^{-3})	N_v (cm^{-3})	n_i (cm^{-3})	μ_e (cm^2 V^{-1} s^{-1})	μ_h (cm^2 V^{-1} s^{-1})	m_e^*/m_e	m_h^*/m_e	ε_r
Ge	0.66	4.13	1.04×10^{19}	6.0×10^{18}	2.3×10^{13}	3900	1900	0.12a 0.56b	0.23a 0.40b	16
Si	1.10	4.01	2.8×10^{19}	1.2×10^{19}	1.0×10^{10}	1400	450	0.26a 1.08b	0.38a 0.60b	11.9
InP	1.34	4.50	5.2×10^{17}	1.1×10^{19}	1.3×10^{7}	4600	190	0.079a,b	0.46a 0.58b	12.6
GaAs	1.42	4.07	4.4×10^{17}	7.7×10^{18}	2.1×10^{6}	8800	400	0.067a,b	0.40a 0.50b	13.0

NOTE: Ge and Si are indirect whereas InP and GaAs are direct bandgap semiconductors. Effective mass related to conductivity (labeled *a*) is different than that for density of states (labeled *b*). In numerous textbooks, n_i is taken as 1.45×10^{10} cm^{-3} and is therefore the most widely used value of n_i for Si, though the correct value is actually 1.0×10^{10} cm^{-3}. (Green, M.A., *Journal of Applied Physics,* 67, 2944, 1990.) (Data combined from various sources.)

$$m_{\text{density}}^* = g^{\frac{2}{3}} \sqrt[3]{m_x m_y m_z}$$

$$m_{\text{conductivity}}^* = 3 \left[\frac{1}{m_x^*} + \frac{1}{m_y^*} + \frac{1}{m_z^*} \right]^{-1}$$

Mass Action Law

$$np = N_c N_v \exp\left(-\frac{E_g}{kT} \right)$$

Therefore, <u>the $n{\cdot}p$ product is a constant</u>, that depends on the material properties N_c, N_v, E_g, and temperature. If somehow n is increased (e.g., by doping), p must decrease to keep $n{\cdot}p$ constant. The mass action law applies in thermal equilibrium (intrinsic or extrinsic) and in the dark (no illumination).

3C.2. Intrinsic Semiconductors

$$p = N_v \exp\left[-\frac{(E_F - E_v)}{kT} \right]$$

Fermi Level

In intrinsic semiconductors, $n = p = n_i$. From the mass action law,

$$np = N_c N_v \exp\left(-\frac{E_g}{kT} \right) = n_i^2 \rightarrow n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT} \right)$$

$$p = n_i \rightarrow N_v \exp\left(-\frac{E_F - E_v}{kT} \right) = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT} \right)$$

Take log of both sides
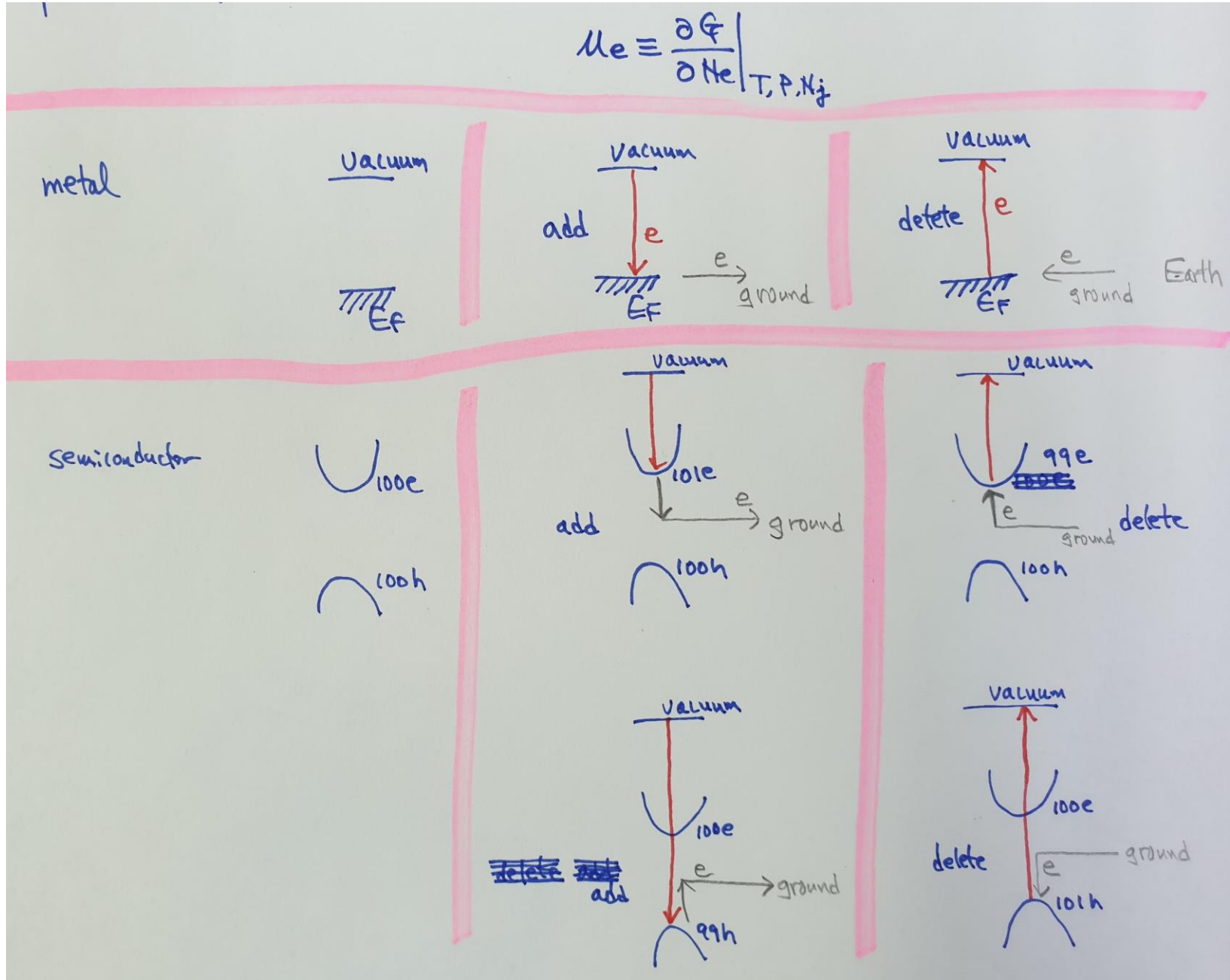
$$E_F = E_v + \frac{1}{2} E_g - \frac{1}{2} kT \ln\left(\frac{N_c}{N_v} \right) = E_v + \frac{1}{2} E_g - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*} \right)$$

If $N_c = N_v$ ($m_e^* = m_h^*$) or near 0 K, $E_F = E_v + \frac{1}{2} E_g$ That is to say, exactly the midgap.

E_F: in the middle

In general, **effective masses are not equal, and the Fermi level slightly shifts**.

Fermi Level: Metal vs. Semiconductor (*Schematics*)

$$\mu_e \equiv \left.\frac{\partial G}{\partial n_e}\right|_{T,P,N_j}$$

metal

Vacuum

add e → ground

delete e ← ground Earth

semiconductor

100e / 100h

add 101e → ground / 100h

99e / 100h delete

delete / add → 99h → ground

delete / 101h → ground

Carrier Density in Intrinsic Semiconductors

$$N_v = 2 \left(\frac{2 \rho m_h^* kT}{h^2} \right)^{3/2}$$

PPT 3C-7

$$n_i = p = N_v \exp\left(- \frac{E_F - E_v}{kT} \right) \qquad \textbf{PPT 3C-7}$$

Using $E_F = E_v + \frac{1}{2} E_g - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*} \right)$ $\qquad$ **PPT 3C-10**

$$n_i = N_v \left(\frac{m_e^*}{m_h^*} \right)^{\frac{3}{4}} \exp\left(- \frac{E_g}{2kT} \right) \qquad \textbf{PPT 3C-9}$$

Since $N_v \propto T^{\frac{3}{2}}$, plot of $\ln(n_i T^{-\frac{3}{2}})$ *vs.* $1/T$ is $\quad$ **PPT 3C-7**

a straight line with slope $- E_g / 2k$

The conductivity of the intrinsic semiconductor becomes

$$S = eN_v \left(\frac{m_e^*}{m_h^*} \right)^{\frac{3}{4}} (m_e + m_h) e^{- E_g / 2kT} \qquad \boldsymbol{n = p = n_i}$$

$$\boldsymbol{\sigma = ne\mu_n + pe\mu_p} \qquad \textbf{Bube (9.5)}$$
ppt 3A-15

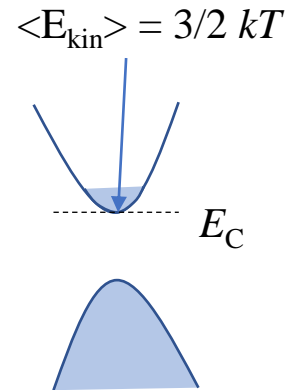$$\boldsymbol{\sigma = ne\mu = ne^2 \tau / m^*}$$

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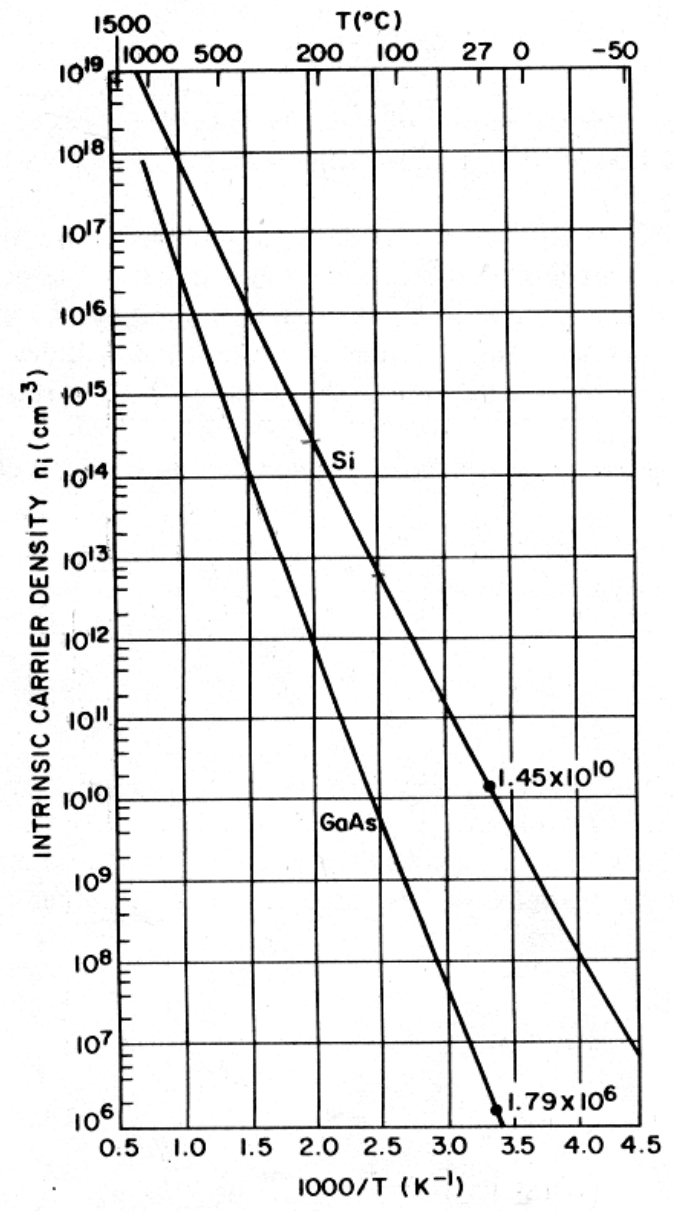
Average energy of an electron in the conduction band

$$E = \int_{E_C}^{\infty} E D_C(E) f(E) dE = \int_{E_C}^{\infty} (E - E_C) D_C(E) f(E) dE + n E_C$$ **(homework)**

$$\simeq \int_{E_C}^{\infty} \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2} \right)^{3/2} (E - E_C)^{3/2} e^{-(E-E_F)/kT} dE + n E_C$$

$$= \frac{1}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2} \right)^{3/2} e^{-(E_C - E_F)/kT} \int_{E_C}^{\infty} (E - E_C)^{3/2} e^{-(E-E_C)/kT} dE + n E_C$$

$$= \frac{3}{2} kT \, 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2} e^{-(E_C - E_F)/kT} + n E_C = \frac{3}{2} n kT + n E_C$$

$\langle E_{kin} \rangle = 3/2 \; kT$

E_C

$$\overline{\overline{E}} = E_C + \frac{3}{2} kT$$ **per one carrier electron**

This is identical to the average kinetic energy of gas molecules following classical Maxwell-Boltzmann distribution. This is because $N_c \gg n$.

PPT 3C-7

(Note that in the case of metal where electrons fully occupy states up to the Fermi level, $E_{avg} = 3/5 \; E_F$.)

ppt 2A-22

INTRINSIC CONCENTRATION AND CONDUCTIVITY OF Si Given that the density of states related <u>effective masses</u> of electrons and holes in Si are approximately $1.08m_e$ and $0.60m_e$, respectively, and the electron and hole drift <u>mobilities</u> at room temperature are 1400 and 450 cm^2 V^{-1} s^{-1}, respectively, calculate the <u>intrinsic concentration</u> and <u>intrinsic resistivity</u> of Si.

SOLUTION

We simply calculate the effective density of states N_c and N_v by

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \quad \text{and} \quad N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

Thus

$$N_c = 2\left[\frac{2\pi(1.08 \times 9.1 \times 10^{-31} \text{ kg})(1.38 \times 10^{-23} \text{ J K}^{-1})(300 \text{ K})}{(6.63 \times 10^{-34} \text{ J s})^2}\right]^{3/2}$$

$$= 2.81 \times 10^{25} \text{ m}^{-3} \quad \text{or} \quad 2.81 \times 10^{19} \text{ cm}^{-3}$$

and

$$N_v = 2\left[\frac{2\pi(0.60 \times 9.1 \times 10^{-31} \text{ kg})(1.38 \times 10^{-23} \text{ J K}^{-1})(300 \text{ K})}{(6.63 \times 10^{-34} \text{ J s})^2}\right]^{3/2}$$

$$= 1.16 \times 10^{25} \text{ m}^{-3} \quad \text{or} \quad 1.16 \times 10^{19} \text{ cm}^{-3}$$

The intrinsic concentration is

$$n_i = (N_c N_v)^{1/2} \exp\left(-\frac{E_g}{2kT}\right)$$

so that

$$n_i = [(2.81 \times 10^{19} \text{ cm}^{-3})(1.16 \times 10^{19} \text{ cm}^{-3})]^{1/2} \exp\left[-\frac{(1.10 \text{ eV})}{2(300 \text{ K})(8.62 \times 10^{-5} \text{ eV K}^{-1})}\right]$$

$$= 1.0 \times 10^{10} \text{ cm}^{-3}$$

The conductivity is

$$\sigma = en\mu_e + ep\mu_h = en_i(\mu_e + \mu_h)$$

that is,

$$\sigma = (1.6 \times 10^{-19} \text{ C})(1.0 \times 10^{10} \text{ cm}^{-3})(1400 + 450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})$$
$$= 3.0 \times 10^{-6} \; \Omega^{-1} \text{ cm}^{-1}$$

The resistivity is

$$\rho = \frac{1}{\sigma} = 3.3 \times 10^5 \; \Omega \text{ cm}$$

Au   **~10⁶ S/cm**
graphite   **~10⁴ S/cm**
graphene   **~10⁵ S/cm**

TCO   **~10⁴ S/cm**

MEAN SPEED OF ELECTRONS IN THE CB Estimate the mean speed of electrons in the conduction band of Si at 300 K. If a is the magnitude of lattice vibrations, then the kinetic theory predicts $a^2 \propto T$; or stated differently, the mean energy associated with lattice vibrations (proportional to a^2) increases with kT. Given the temperature dependence of the mean speed of electrons in the CB, what should be the temperature dependence of the drift mobility? The effective mass of an electron in the conduction band is $0.26m_e$.

SOLUTION

Suppose that v_{th} is the root mean square velocity of the electron in the CB, then the average *KE*, $\frac{1}{2}m_e^* v_{\text{th}}^2$, of this electron from Equation 5.14 is $\frac{3}{2}kT$. Thus,

PPT 3C-13

$$v_{\text{th}} = \left(\frac{3kT}{m_e^*}\right)^{1/2} = \left[\frac{(3 \times 1.38 \times 10^{-23} \times 300)}{(0.26 \times 9.1 \times 10^{-31})}\right]^{1/2} = 2.3 \times 10^5 \text{ m s}^{-1}$$

The above velocity v_{th} is called the **thermal velocity,** and it is roughly the same as the mean speed of the electron in the CB. (See Example 1.11.)

The mean free time τ of the electron between scattering events due to thermal vibrations of the atoms is inversely proportional to both the mean speed v_{th} of the electron and the scattering cross section of the thermal vibrations, that is,

$$\tau \propto \frac{1}{v_{\text{th}}(\pi a^2)} \qquad \textit{instead of Fermi velocity}$$

where a is the amplitude of the atomic thermal vibrations. But, $v_{\text{th}} \propto T^{1/2}$ and $(\pi a^2) \propto kT$, so that $\tau \propto T^{-3/2}$ and consequently $\mu_e \propto T^{-3/2}$.

in metal   ppt 3A-10
Free-Electron Model or Classical Theory

$$\rightarrow \tau \propto \frac{1}{a^2} \propto \frac{1}{T} = \frac{C}{T}$$

MEAN FREE PATH OF ELECTRONS IN THE CB Consider the motion of electrons in the CB of an undoped GaAs Si crystal. What is the mean free path of an average electron in the conduction band? How does this compare with the mean free path of a conduction electron in copper which has a drift mobility of 32 cm^2 V^{-1} s^{-1} and a mean free path of 40 nm. What is your conclusion?

SOLUTION

The drift mobility of electrons in a semiconductor is controlled by various scattering mechanisms that limit the mean scattering time or the free time of an electron in the CB. If τ is the mean scattering time for electrons in the CB, then, from Chapter 2, drift mobility $\mu_e = e\tau/m_e^*$, where m_e^* is the effective mass of the electron in the crystal. Thus,

$$\tau = \frac{\mu_e m_e^*}{e} = \frac{(1400 \times 10^{-4} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1})(0.26 \times 9.11 \times 10^{-31} \text{ kg})}{(1.602 \times 10^{-19} \text{ C})} = 2.1 \times 10^{-13} \text{ s}$$

We know from Example 5.1 that the mean velocity or the thermal velocity v_{th} of electrons in the CB is approximately 2.3×10^5 m s^{-1}. The mean free path $\ell = v_{th}\tau = (2.3 \times 10^5 \text{ m s}^{-1})(2.1 \times 10^{-13} \text{ s}) = 48 \times 10^{-9}$ m or 48 nm. The conduction electrons in copper have significantly lower drift mobility but their mean free path is almost the same as a conduction electron in Si. Recall from Chapter 4 that conduction electrons in a metal follow Fermi–Dirac statistics and their mean speed is very much larger than the thermal velocity of electrons in Si.
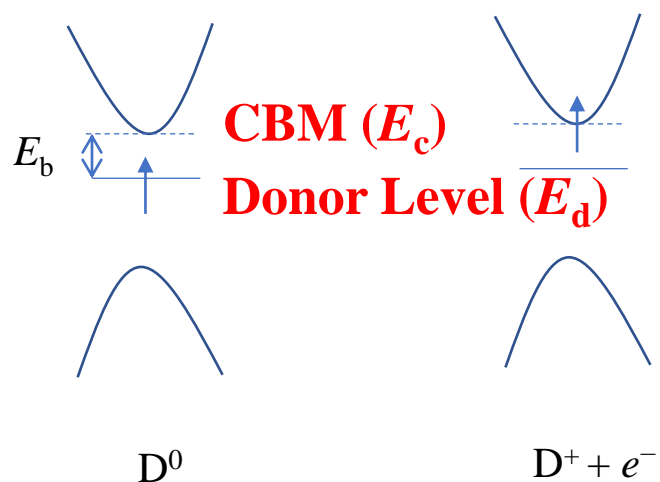
$$\sigma = ne\mu = ne^2\tau/m^*$$

3C.3. Extrinsic Semiconductors

n-type doping
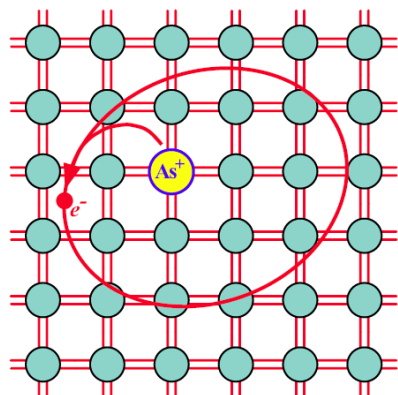
5	6	7
B	C	N
13 Al	14 Si	15 P
31 Ga	32 Ge	33 As
49 In	50 Sn	51 Sb
81 Tl	82 Pb	83 Bi

E_b

CBM (E_c)

Donor Level (E_d)

D^0 $D^+ + e^-$

Group V elements such as arsenic (As) is chemically similar to Si, and it substitutes for a Si atom when doped into Si (by experimental techniques such as ion implantation).

The As atom has five valence electrons but the diamond structure needs only four valence electrons. The redundant fifth electron may occupy the lowest state in the conduction band (conduction band minimum: CBM), but the attractive interaction with the positively charged As^+ leads to a formation of more stable states below the CBM that are called donor (impurity or dopant) states.

The energy difference between the donor level (E_d) and CBM is called the **ionization energy** or **binding energy (E_b)**. When the extra electron is **bound** at As^0, it is **spatially localized**, and so does not contribute to the current.

When E_b is small, the donor atom can be ionized by **thermal excitation** with the electrons freed into the conduction band. The electrons in the conduction band can mediate current under external bias. When E_b is small, it is called the **shallow donor** in the sense that the donor level is close to CBM. This way of increasing electron carriers is called the ***n*-type doping**.

$$E_{conduction} - E_{donor} = E_{binding\ energy}$$
$$= E_{ionization\ energy}$$

The binding energy E_b and Bohr radius can be calculated by employing hydrogen model with dielectric constant + effective mass.

Binding energy in H atom: $E_b = -E_1 = \dfrac{me^4}{8e_0^2 h^2} = 13.6$ eV

For As$^+$ in silicon, the two parameters in the formula should change. First, since the unbound electron follows the band dispersion near CBM, the mass is replaced by the **effective mass** in the conduction band minimum (considering the anisotropy, the relevant m^* is $\sim 1/3\ m$). Second, the Coulomb potential between the electron and As$^+$ is screened by polarization of electrons and ions, which is reflected by the relative **dielectric constant** ($\varepsilon_r = 11.9$ for Si).
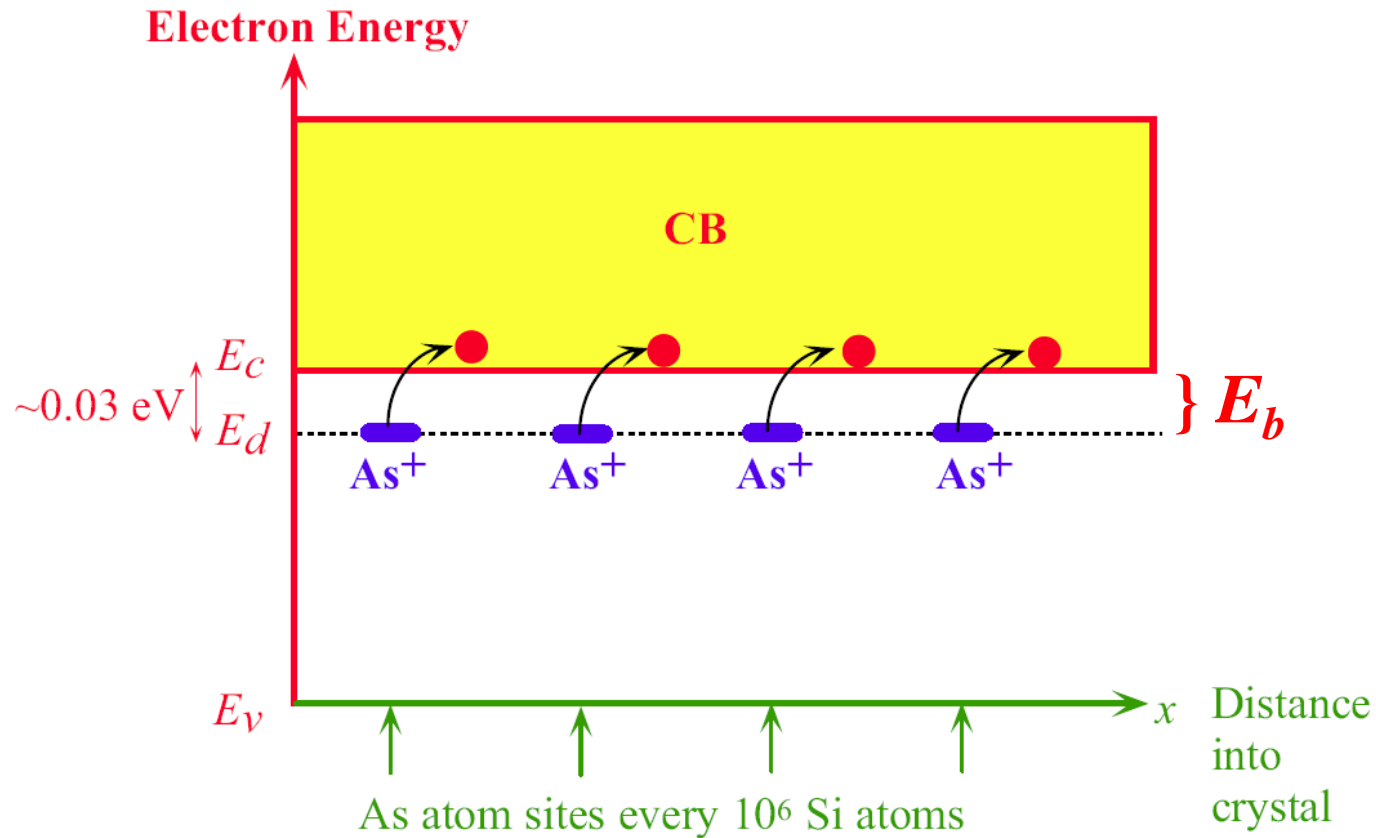
$$E_b^{Si} = \frac{m^* e^4}{8e^2 e_0^2 h^2} = (13.6\ \text{eV})\left(\frac{m^*}{m}\right)\left(\frac{1}{e^2}\right) \sim 0.032\ \text{eV}$$

Table 5.2 Examples of donor and acceptor ionization energies (eV) in Si

Donors			Acceptors		
P	**As**	**Sb**	**B**	**Al**	**Ga**
0.045	0.054	0.039	0.045	0.057	0.072

The size of donor states is the ~~Bohr radius~~ with adjusted parameters

Hydrogen atom: $a_0 = \dfrac{4\pi\varepsilon_0 \hbar^2}{me^2} \rightarrow \dfrac{4\pi\varepsilon_r \varepsilon_0 \hbar^2}{m^* e^2} = a_0 \varepsilon_r \left(\dfrac{m}{m^*}\right) = 1.9$ nm

Energy band diagram for an *n*-type Si doped with 1 ppm As. There are donor energy levels just below E_c around As$^+$ sites.

Thermal energy is ~1.5 kT = 0.04 eV (room T). Therefore, **electrons can be freed at room temperature**, and can contribute to the conductivity. In this case, the dopants are called "shallow impurity". The effective mass can be varied from ~0.1 to 10. The deep or shallow nature can depend on it.

Occupation Probability at a Donor

The probability is proportional to the Boltzmann factor. When the donor is empty (D⁺), its **electron** lies at the E_F (**chemical potential of electrons**). Therefore, the energy of D⁺ is E_F, while the <u>energy of D⁰ is E_d</u>.

$$\text{Probability} \propto \begin{cases} e^{-\frac{E_F}{kT}} & \text{empty} \\[2em] e^{-\frac{E_d}{kT}} & \text{occupied (doubly degenerate; spin up or down)} \end{cases}$$

D⁺

D⁰

The occupation probability is given by

D⁰ $$f_d(E_d) = \frac{2e^{-\frac{E_d}{kT}}}{e^{-\frac{E_F}{kT}} + 2e^{-\frac{E_d}{kT}}} = \frac{1}{1 + \frac{1}{2}e^{-(E_F - E_d)/kT}}$$

Density of ionized dopant

$$N_d^+ = N_d \times (\text{probability of not finding an electron at } E_d)$$

$$= N_d[1 - f_d(E_d)]$$

D⁺ $$= \frac{N_d}{1 + 2e^{(E_F - E_d)/kT}}$$

The Fermi level is determined by the charge neutrality condition:

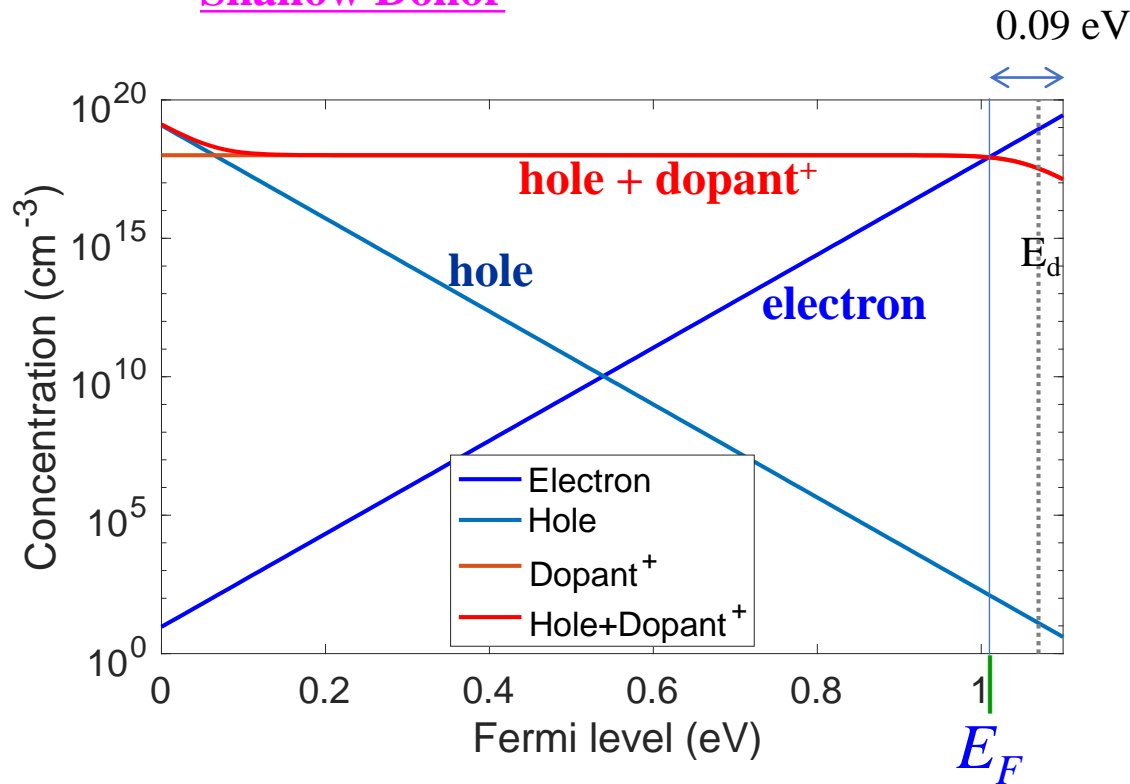Density of negative charge = density of positive charge

$$N_v = 2\left(\frac{2\rho m_h^* kT}{h^2}\right)^{3/2}$$

$n = N_i^+ + p$

ppt 3C-7

$$N_C e^{-(E_C - E_F)/kT} = \frac{N_d}{1 + 2e^{(E_F - E_d)/kT}} + N_V e^{-(E_F - E_V)/kT}$$

$\underline{E_F}$ **is determined.**

ex) Si, 300 K, $E_b = 0.03$ eV, $N_d = 10^{18}$ cm^{-3}, $E_g = 1.1$ eV

Shallow Donor

0.09 eV

At the Fermi level, **most of dopants are ionized**, and the electron density is **almost identical to the dopant density (10^{18} cm^{-3}).** This is because E_b (0.03 eV) is comparable to the **thermal energy** ($kT = 0.026$ eV).
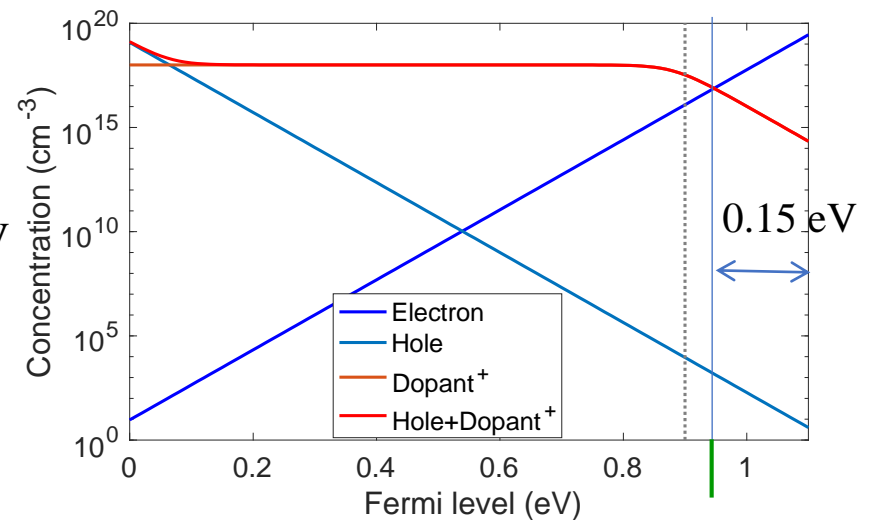
Shallow vs. Deep Donor

Ex) Si, 300 K, $E_b = 0.1$ eV, $N_d = 10^{18}$ cm^{-3}

0.11 eV

$n = 4.2 \times 10^{17}$ cm^{-3}

E_F
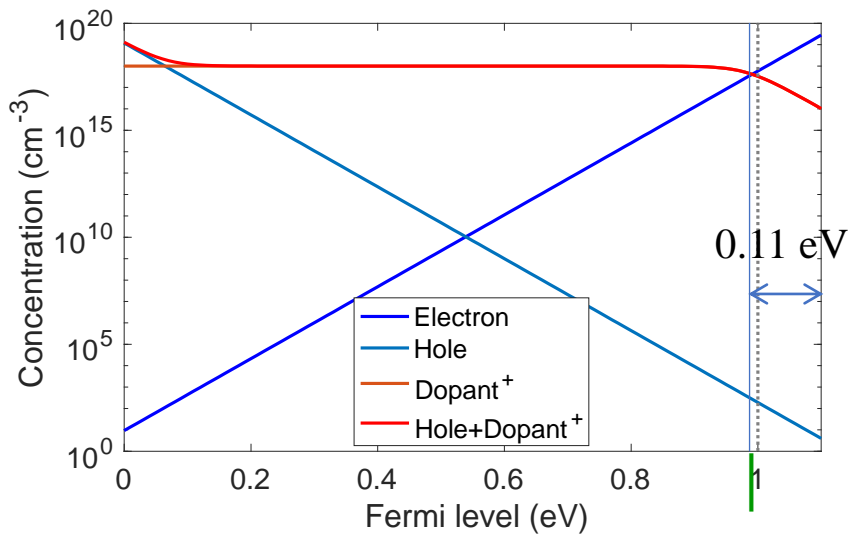
Ex) Si, 300 K, $E_b = 0.2$ eV, $N_d = 10^{18}$ cm^{-3}

0.15 eV

$n = 6.7 \times 10^{16}$ cm^{-3}

E_F

When E_b becomes bigger than ~0.2 eV, the dopants are hardly ionized and the electron carrier is significantly reduced.

For **shallow donors**, most dopants are ionized.
If $N_d \gg n_i$ (intrinsic carrier density), the
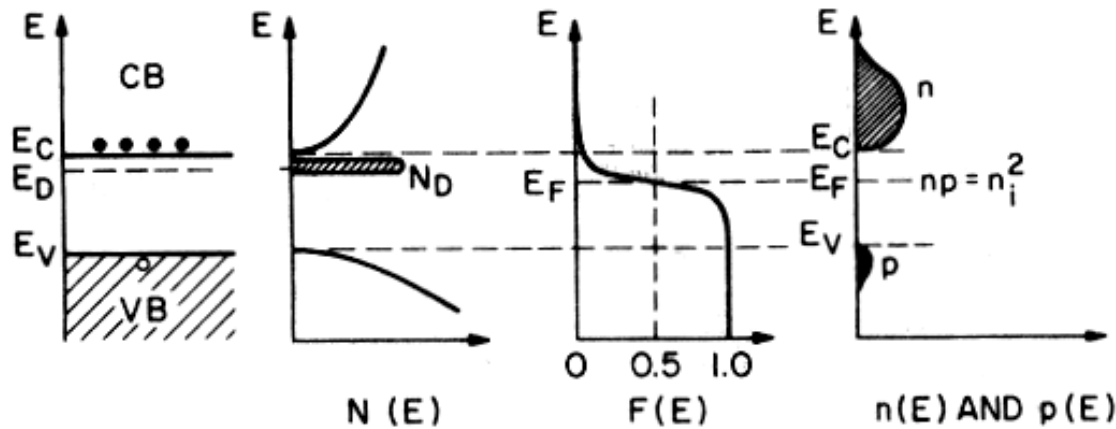**electron density is practically equal to the dopant density**.

$$n = N_d^+ + p$$

$$N_C e^{-(E_C - E_F)/kT} = \frac{N_d}{1 + 2e^{(E_F - E_d)/kT}} + N_V e^{-(E_F - E_V)/kT}$$

$$n = N_d, \quad E_C - E_F = kT \ln\left(\frac{N_C}{N_d}\right) \quad \text{at 300 K}$$

the higher donor concentration, the smaller energy difference,

i.e. Fermi level moves toward the bottom of C.B.

N (E)   F (E)   n(E) AND p(E)

$$np = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$$

The conductivity is **dominated by electron carriers**.

$$S = eN_d m_e + e\left(\frac{n_i^2}{N_d}\right) m_h \approx eN_d m_e$$

THE FERMI LEVEL IN _n_- AND _p_-TYPE Si   An _n_-type Si wafer has been doped uniformly with 10^{16} antimony (Sb) atoms cm^{-3}. Calculate the position of the Fermi energy with respect to the Fermi energy E_{Fi} in intrinsic Si. ~~The above _n_-type Si sample is further doped with 2×10^{17} boron atoms cm^{-3}. Calculate the position of the Fermi energy with respect to the Fermi energy E_{Fi} in intrinsic Si.~~ (Assume that $T = 300$ K, and $kT = 0.0259$ eV.)

SOLUTION

Sb gives _n_-type doping with $N_d = 10^{16}$ cm^{-3}, and since $N_d \gg n_i$ (=1.0×10^{10} cm^{-3}), we have

$$n = N_d = 10^{16} \text{ cm}^{-3}$$

For intrinsic Si,

$$n_i = N_c \exp\left[-\frac{(E_c - E_{Fi})}{kT}\right]$$

whereas for doped Si,

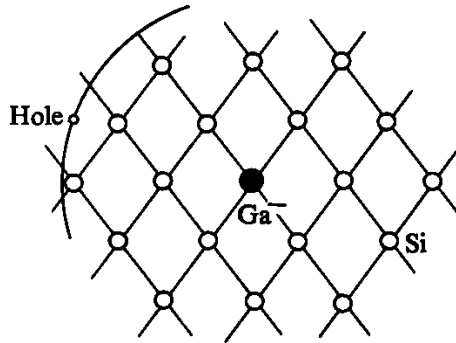$$n = N_c \exp\left[-\frac{(E_c - E_{Fn})}{kT}\right] = N_d$$

PPT 3C-7

where E_{Fi} and E_{Fn} are the Fermi energies in the intrinsic and _n_-type Si. Dividing the two expressions,

$$\frac{N_d}{n_i} = \exp\left[\frac{(E_{Fn} - E_{Fi})}{kT}\right]$$

so that

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right) = (0.0259 \text{ eV}) \ln\left(\frac{10^{16}}{1.0 \times 10^{10}}\right) = 0.36 \text{ eV}$$

p-Type Doping

Group III elements such as Ga has only three valence electrons. When it substitutes for a Si atom, one of its bonds has an electron missing, which creates a hole in the valence band top. Like the redundant electron in the *n*-type doping, this hole is bound at Ga⁻ by orbiting around it, which forms a bound empty states just above the valence band. This state is called the **acceptor state**. The energy of the acceptor level from the valence band corresponds to the ionization energy or binding energy (E_b). When E_b is small, the electrons in the valence top can be thermally promoted into the donor level, leaving hole carriers behind. This is called the *p*-type doping.

5	6	7
B	C	N
13	14	15
Al	Si	P
31	32	33
Ga	Ge	As
49	50	51
In	Sn	Sb
81	82	83
Tl	Pb	Bi
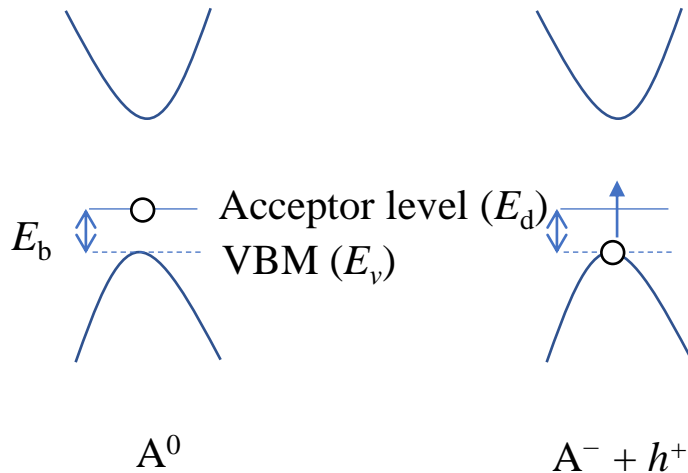
The rest of the discussions are parallel to those for the *n*-type doping.

E_b — Acceptor level (E_d) / VBM (E_v)

A^0

$A^- + h^+$

Ionization Energy ~50 meV
Bohr Radius ~5 nm
Dielectric Constant ~10
Effective Mass ~0.2 m
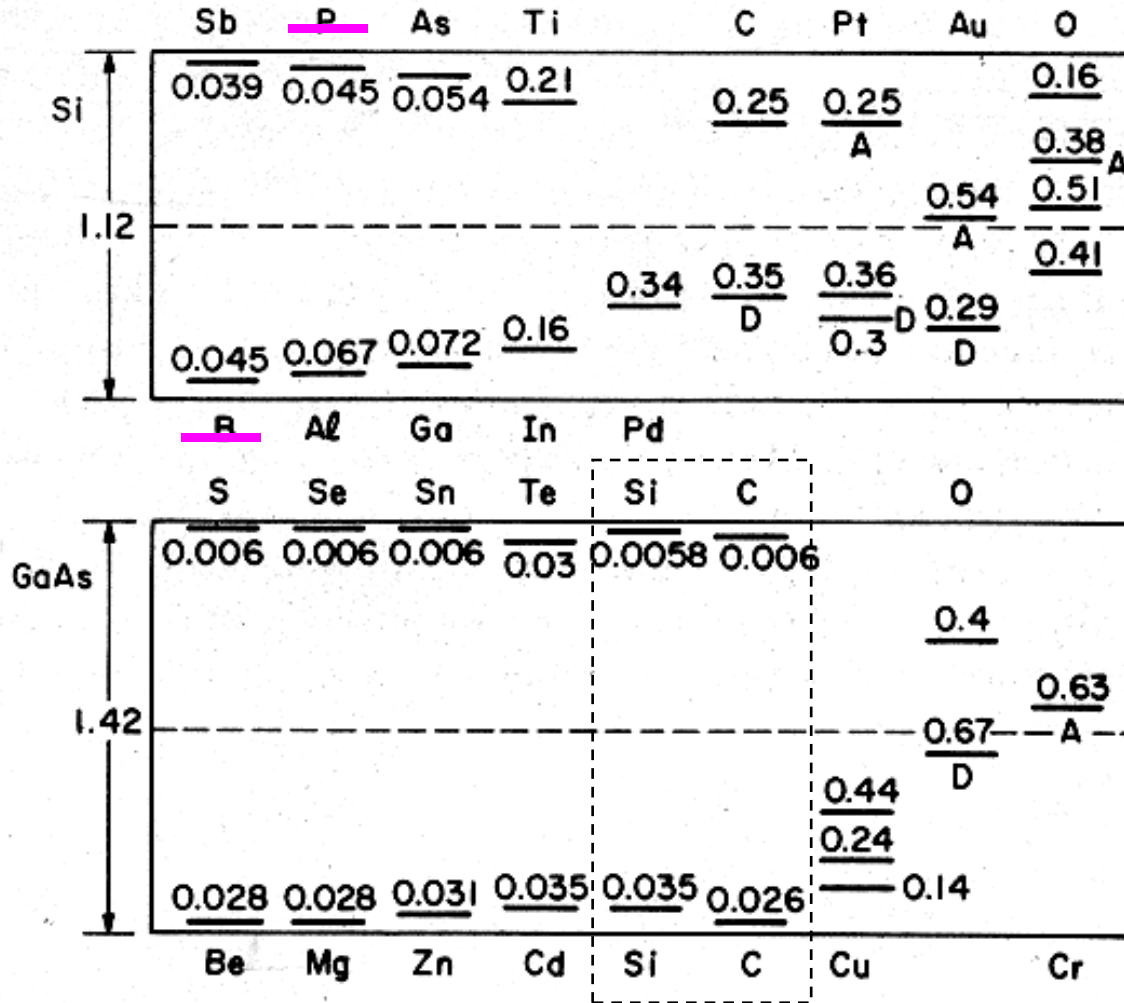
Amphoteric dopant

Intrinsic Semiconductor   *n*-Type Semiconductor   *p*-Type Semiconductor
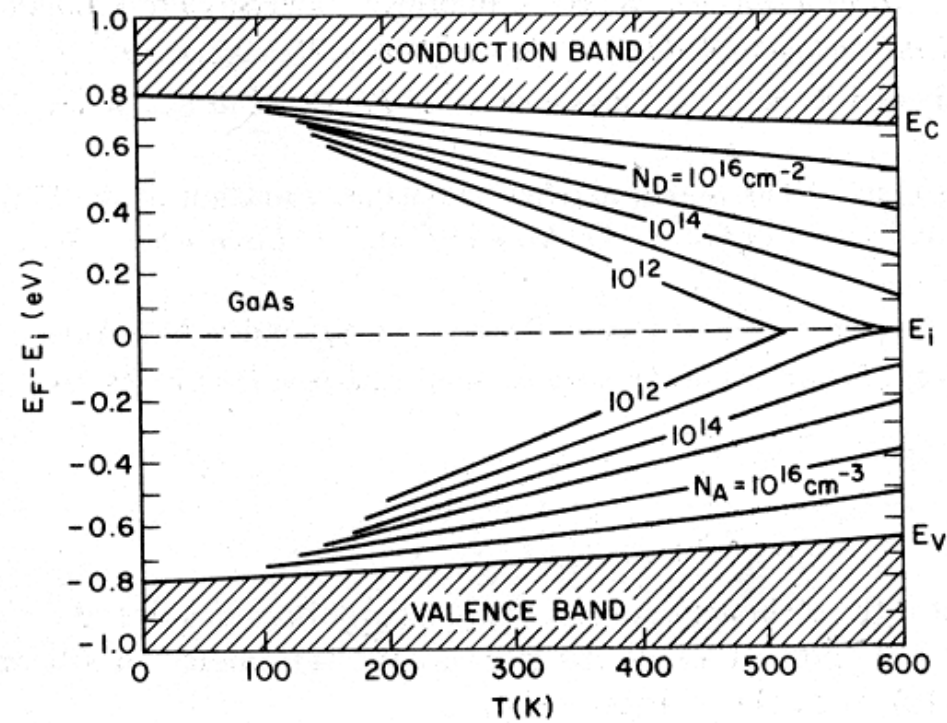
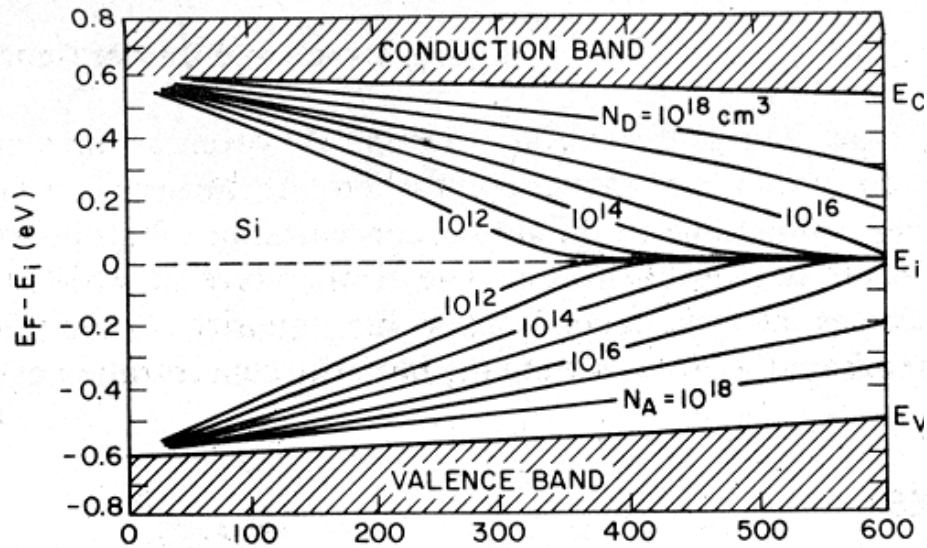(a)   (b)   (c)
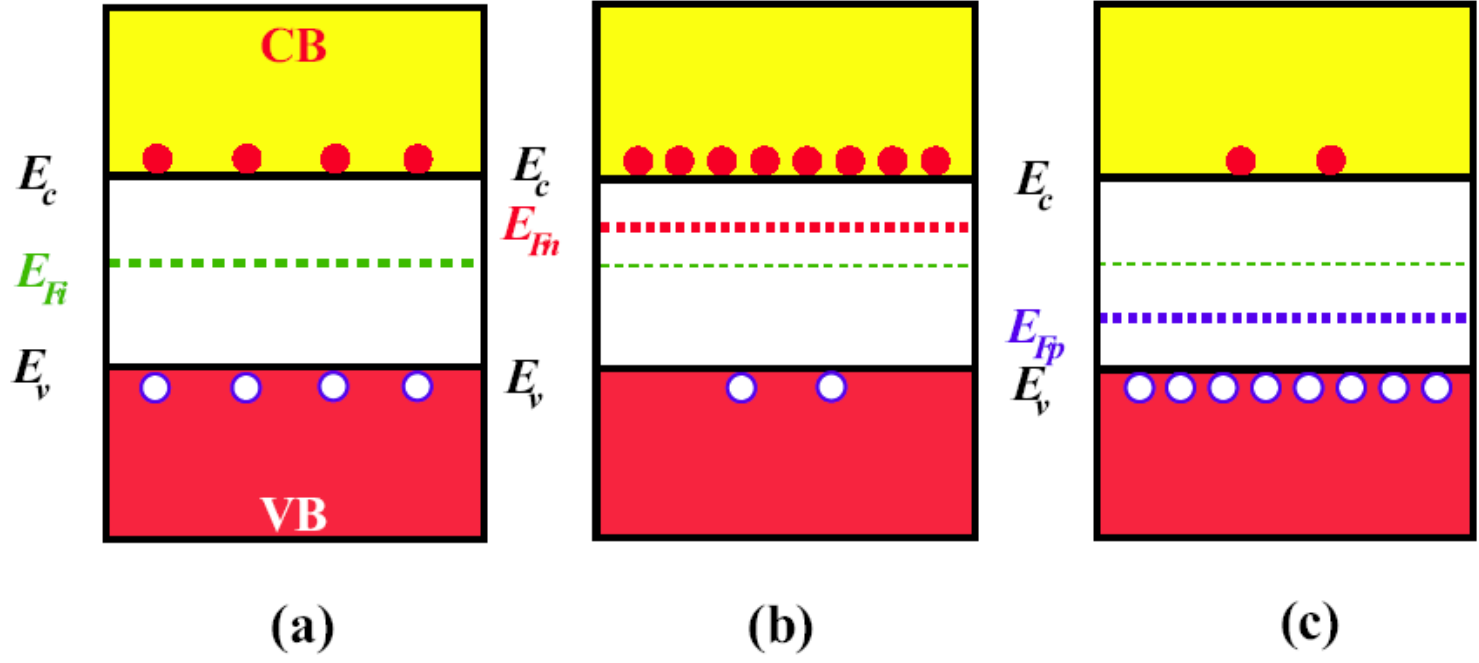
Schematic energy band diagrams

In all cases, $\quad np = N_c N_v \exp\left(-\dfrac{E_g}{kT}\right)$ **PPT 3C-9**

$$p = N_v \exp\left[-\frac{(E_F - E_v)}{kT}\right]$$ **PPT 3C-7**

Fermi levels for different doping concentrations as a function of temperature in Si and GaAs

Conductivity of a Semiconductor

$$J = env_{de} + epv_{dh}$$

$$v_{de} = \mu_e E_x \quad \text{and} \quad v_{dh} = \mu_h E_x$$

$$m_{e(h)} = \frac{et_{e(h)}}{m^*_{e(h)}}$$

v_{de} = drift velocity of the electrons

μ_e = electron drift mobility

E_x = applied electric field

v_{dh} = drift velocity of the holes

μ_h = hole drift mobility

$$v_d \equiv \mu E$$

$$J = nev_d$$

The conductivity is given by:

$$\sigma = en\mu_e + ep\mu_h$$

$$\sigma = ne\mu_n + pe\mu_p$$

$$\sigma = ne\mu = ne^2\tau / m^*$$

RESISTIVITY OF INTRINSIC AND DOPED Si   Find the resistance of a 1 cm^3 pure silicon crystal. What is the resistance when the crystal is doped with arsenic if the doping is 1 in 10^9, that is, 1 part per billion (ppb) (note that this doping corresponds to one foreigner living in China)? Note that the atomic concentration in silicon is 5×10^{22} cm^{-3}, $n_i = 1.0 \times 10^{10}$ cm^{-3}, $\mu_e = 1400$ cm^2 V^{-1} s^{-1}, and $\mu_h = 450$ cm^2 V^{-1} s^{-1}.

Assuming shallow donors.

SOLUTION

For the intrinsic case, we apply

$$\sigma = en\mu_e + ep\mu_h = en(\mu_e + \mu_h)$$

so
$$\sigma = (1.6 \times 10^{-19} \text{ C})(1.0 \times 10^{10} \text{ cm}^{-3})(1400 + 450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})$$
$$= 2.96 \times 10^{-6} \text{ } \Omega^{-1} \text{ cm}^{-1}$$

Since $L = 1$ cm and $A = 1$ cm^2, the resistance is

$$R = \frac{L}{\sigma A} = \frac{1}{\sigma} = 3.47 \times 10^5 \text{ } \Omega \qquad \text{or} \qquad 347 \text{ k}\Omega$$

When the crystal is doped with 1 in 10^9, then

$$N_d = \frac{N_{\mathrm{Si}}}{10^9} = \frac{5 \times 10^{22}}{10^9} = 5 \times 10^{13} \text{ cm}^{-3}$$

At room temperature all the donors are ionized, so

$$n = N_d = 5 \times 10^{13} \text{ cm}^{-3}$$

The hole concentration is

$$p = \frac{n_i^2}{N_d} = \frac{(1.0 \times 10^{10})^2}{(5 \times 10^{13})} = 2.0 \times 10^6 \text{ cm}^{-3} \ll n_i$$

Therefore,

$$\sigma = en\mu_e = (1.6 \times 10^{-19} \text{ C})(5 \times 10^{13} \text{ cm}^{-3})(1400 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})$$
$$= 1.12 \times 10^{-2} \ \Omega^{-1} \text{ cm}^{-1}$$

Further,
$$R = \frac{L}{\sigma A} = \frac{1}{\sigma} = 89.3 \ \Omega$$

Notice the drastic fall in the resistance when the crystal is doped with only 1 in 10^9 atoms.
Doping the silicon crystal with boron instead of arsenic, but still in amounts of 1 in 10^9, means that $N_a = 5 \times 10^{13} \text{ cm}^{-3}$, which results in a conductivity of

$$\sigma = ep\mu_h = (1.6 \times 10^{-19} \text{ C})(5 \times 10^{13} \text{ cm}^{-3})(450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1})$$
$$= 3.6 \times 10^{-3} \ \Omega^{-1} \text{ cm}^{-1}$$

Therefore,
$$R = \frac{L}{\sigma A} = \frac{1}{\sigma} = 278 \ \Omega$$

The reason for a higher resistance with p-type doping compared with the same amount of n-type doping is that $\mu_h < \mu_e$.

3C.4. Mobility and Conductivity

$$n = N_d^+ + p$$

$$N_C e^{-(E_C - E_F)/kT} = \frac{N_d}{1 + 2e^{(E_F - E_d)/kT}} + N_V e^{-(E_F - E_V)/kT}$$

Temperature Dependence of Carrier Concentration

Let's consider donor only. If $N_d \gg n_i$, most electrons are donated by ionized donors.

$$n = N_C e^{-(E_C - E_F)/kT} = \frac{N_d}{2e^{(E_F - E_d)/kT} + 1}$$

At low temperature, parts of N_d is ionized, which means that the denominator in the left-hand side should be much bigger than 1. Therefore, $2\exp[(E_F - E_d)/kT] \gg 1$ and so

$$n = N_C e^{-(E_C - E_F)/kT} \simeq \frac{N_d}{2e^{(E_F - E_d)/kT}} = \frac{N_d}{2} e^{(E_d - E_F)/kT}$$

$$n^2 = N_C e^{-(E_C - E_F)/kT} \frac{N_d}{2} e^{(E_d - E_F)/kT} = \frac{N_C N_d}{2} e^{-(E_C - E_d)/kT} = \frac{N_C N_d}{2} e^{-E_b/kT}$$
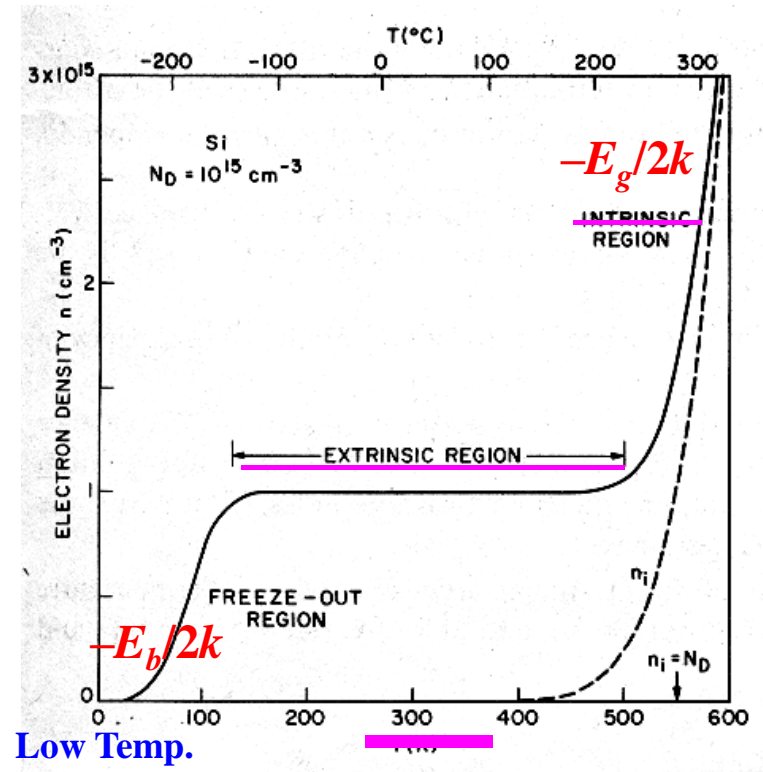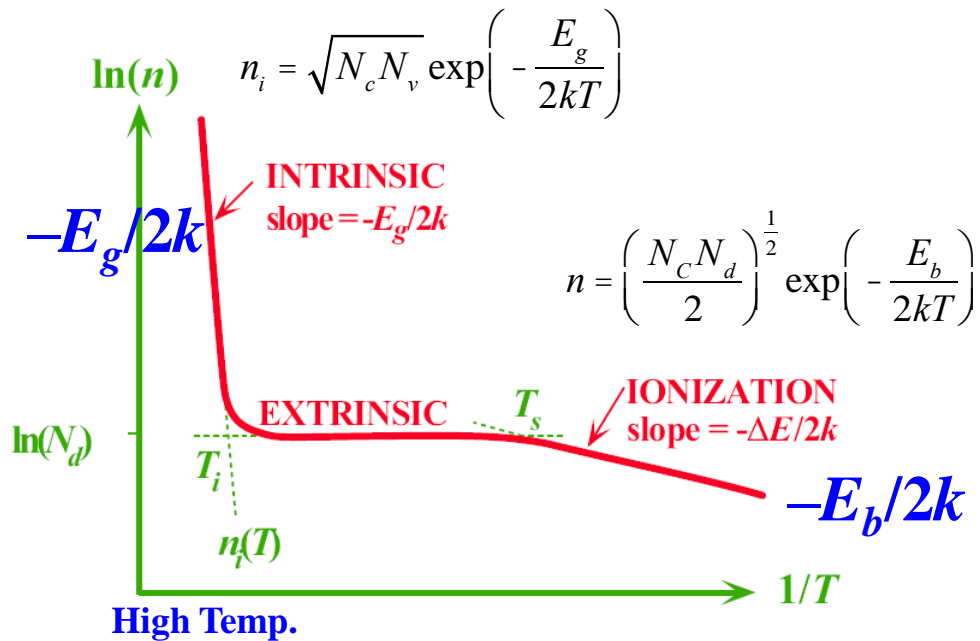
$$\rightarrow n = \left(\frac{N_C N_d}{2} \right)^{\frac{1}{2}} \exp\left(-\frac{E_b}{2kT} \right)$$

This results in a linear line in $\ln(n)$ vs $1/T$ graph with the slope of $-E_b/2k$.

Note that the formula resembles the **intrinsic carrier density**.

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT} \right)$$

ppt 3C-10
Intrinsic Semiconductor

$$n_i = \sqrt{N_c N_v}\, \exp\left(-\frac{E_g}{2kT}\right)$$

$\ln(n)$

INTRINSIC
slope $=$ -E_g/2k

$-E_g/2k$

$$n = \left(\frac{N_c N_d}{2}\right)^{\frac{1}{2}} \exp\left(-\frac{E_b}{2kT}\right)$$

$\ln(N_d)$

EXTRINSIC T_s

T_i

$n_i(T)$

IONIZATION
slope $=$ -ΔE/2k

$-E_b/2k$

$1/T$

High Temp.

$T(°C)$

-200   -100   0   100   200   300

3×10^{15}

Si
$N_D = 10^{15}\ cm^{-3}$

$-E_g/2k$

INTRINSIC
REGION

ELECTRON DENSITY n (cm^{-3})

EXTRINSIC REGION

n_i

FREEZE – OUT
REGION

$-E_b/2k$

$n_i = N_D$

0   .100   200   300   400   500   600

Low Temp.

- At low temperatures, the carrier density increases <u>by ionizing the dopant</u>.
- The dopants are **fully activated as $n = N_d$** (~300 K).
- The electron excitation is <u>directly from the valence to conduction</u> (at high temperatures).

Temperature Dependence of Carrier Density can be schematically understood as follows:
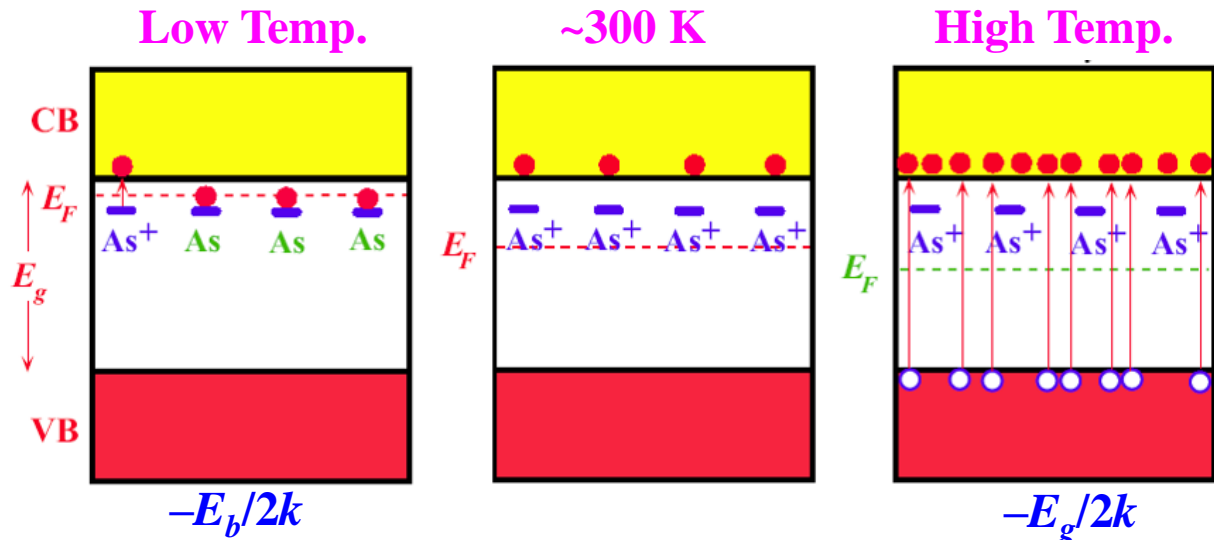
(a) At low temperatures, the electron concentration is controlled by the ionization of the donors.

(b) Between T_s and T_i (intrinsic temperature), the electron concentration is equal to the concentration of donors since they are fully ionized (**$n = N_d$**).

(c) At high temperatures, thermally generated electrons **from the valence band** exceed the number of electrons from ionized donors and the semiconductor behaves as if intrinsic.

SATURATION AND INTRINSIC ~~TEMPERATURES~~   An n-type Si sample has been doped with 10^{15} phosphorus atoms cm^{-3}. The donor energy level for P in Si is 0.045 eV below the conduction band edge energy.

a.   Estimate the temperature above which the sample behaves as if intrinsic.

b.   Estimate the lowest temperature above which most of the donors are ionized.

SOLUTION

Remember that $n_i(T)$ is highly temperature dependent, as shown in Figure 5.16 so that as T increases, eventually at $T \approx T_i$, n_i becomes comparable to N_d. Beyond T_i, $n_i(T > T_i) \gg N_d$. Thus we need to solve

$$n_i(T_i) = N_d = 10^{15} \text{ cm}^{-3}$$

From the $\log(n_i)$ versus $10^3/T$ graph for Si in Figure 5.16, when $n_i = 10^{15}$ cm^{-3}, $(10^3/T_i) \approx 1.85$, giving $T_i \approx 541$ K or 268 °C.

We will assume that most of the donors are ionized, say at $T \approx T_s$, where the extrinsic and the extrapolated ionization lines intersect in Figure 5.15:

$$n = \left(\frac{1}{2}N_c N_d\right)^{1/2} \exp\left(-\frac{\Delta E}{2kT_s}\right) \approx N_d$$

This is the temperature at which the ionization behavior intersects the extrinsic region. In the above equation, $N_d = 10^{15}$ cm^{-3}, $\Delta E = 0.045$ eV, and $N_c \propto T^{3/2}$, that is,

$$N_c(T_s) = N_c(300\text{ K})\left(\frac{T_s}{300}\right)^{3/2}$$

Clearly, then, the equation can only be solved numerically. Similar equations occur in a wide range of physical problems where one term has the strongest temperature dependence. Here, $\exp(-\Delta E/kT_s)$ has the strongest temperature dependence. First assume N_c is that at 300 K, $N_c = 2.8 \times 10^{19}$ cm^{-3}, and evaluate T_s,

$$T_s = \frac{\Delta E}{k \ln\left(\dfrac{N_c}{2N_d}\right)} = \frac{0.045 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV K}^{-1}) \ln\left[\dfrac{2.8 \times 10^{19} \text{ cm}^{-3}}{2(1.0 \times 10^{15} \text{ cm}^{-3})}\right]} = 54.7 \text{ K}$$

At $T = 54.7$ K,

$$N_c(54.7\text{ K}) = N_c(300\text{ K})\left(\frac{54.7}{300}\right)^{3/2} = 2.18 \times 10^{18} \text{ cm}^{-3}$$

With this new N_c at a lower temperature, the improved T_s is 74.6 K. Since we only need an estimate of T_s, the extrinsic range of this semiconductor is therefore from about 75 K to 541 K or −198 °C to about 268 °C.

Drift Mobility: Temperature and Impurity Dependence

- **Lattice-Scattering-Limited** Mobility

$$\mu_L = \frac{e\tau_L}{m^*}, \quad \tau_L = \frac{1}{Sv_{th}N_s}$$

The same as ppt 3C-16

$$\frac{1}{2}m^*v_{th}^2 = \frac{3}{2}kT \rightarrow v_{th} \propto T^{1/2} \text{ and } S = \pi a^2, \ a^2 \propto T$$

$$\searrow m_L \propto T^{-3/2}$$

in metal   ppt 3A-10

Free-Electron Model or Classical Theory

$$\rightarrow \tau \propto \frac{1}{a^2} \propto \frac{1}{T} = \frac{C}{T}$$

- ## Charged Impurity Scattering

In doped semiconductors, the ionized impurities play as scattering centers due to the long-range Coulomb interaction between electron and charged dopants (D^+ or A^-). **At low temperatures, this impurity scattering dominates the carrier mobility.** As shown below, electrons incident close to the impurity (kinetic energy < |potential energy|) are scattered more severely than those approaching away from the impurity (kinetic energy > |potential energy|). The approximate cross section can be obtained by assuming that kinetic energy ~ |potential energy| at the radius of cross section (r_c).
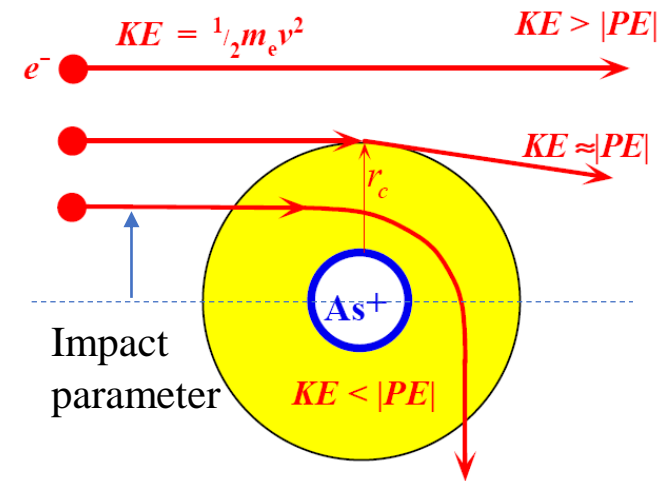
$$\frac{3}{2}kT = |PE(r_c)| = \frac{e^2}{4\pi\varepsilon_o\varepsilon_r r_c} \longrightarrow r_c = e^2/(6\pi\varepsilon_o\varepsilon_r kT)$$

$$S = \pi r_c^2 = \frac{\pi e^4}{(6\pi\varepsilon_o\varepsilon_r kT)^2} \propto T^{-2}$$

$$\tau_I = \frac{1}{Sv_{th}N_I} \propto \frac{1}{(T^{-2})(T^{1/2})N_I} \propto \frac{T^{3/2}}{N_I} \qquad \diagdown m_I \sqcup T^{3/2}$$

N_I = concentration of the ionized impurities (all ionized impurities including donors and acceptors) Below the saturation temperature, N_I itself is temperature dependent.

$$\frac{1}{m} = \frac{1}{m_I} + \frac{1}{m_L}$$

$\mu_L \propto T^{-3/2}$

$\mu_I \propto T^{3/2}$

Lattice

Impurity

$\log \mu$

$\log T$

$T^{-1.5}$

$N \le 10^{12}$

4×10^{13}

1.3×10^{17}

$Si - \mu_e(T)$

Low $\mathscr{E}$

Electron mobility (cm²/V s)
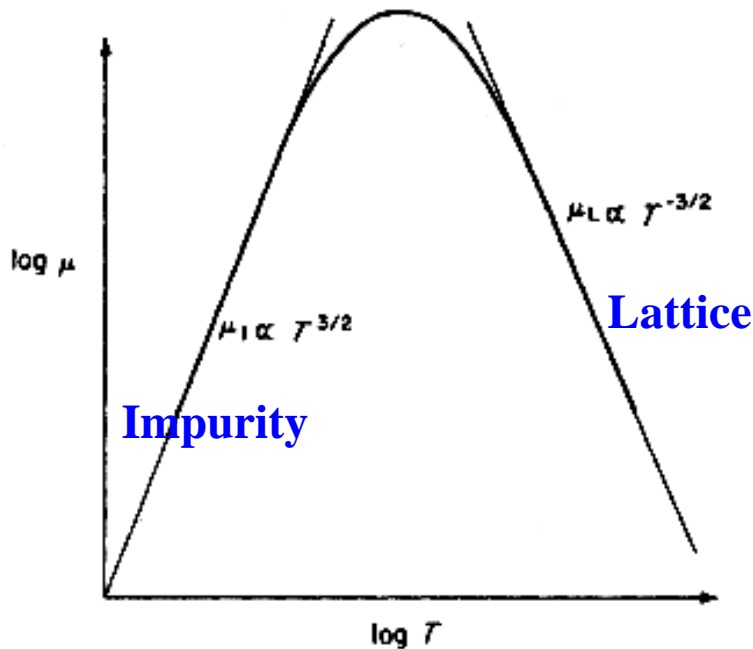
Temperature, T (K)

Temperature dependence of mobility in a semiconductor with scattering by both acoustic lattice waves and by charged imperfections, resulting in a maximum mobility at a particular temperature. As N_I decreases, μ_I increases so that maximum mobility moves to lower temperature with increasing purity of the material.
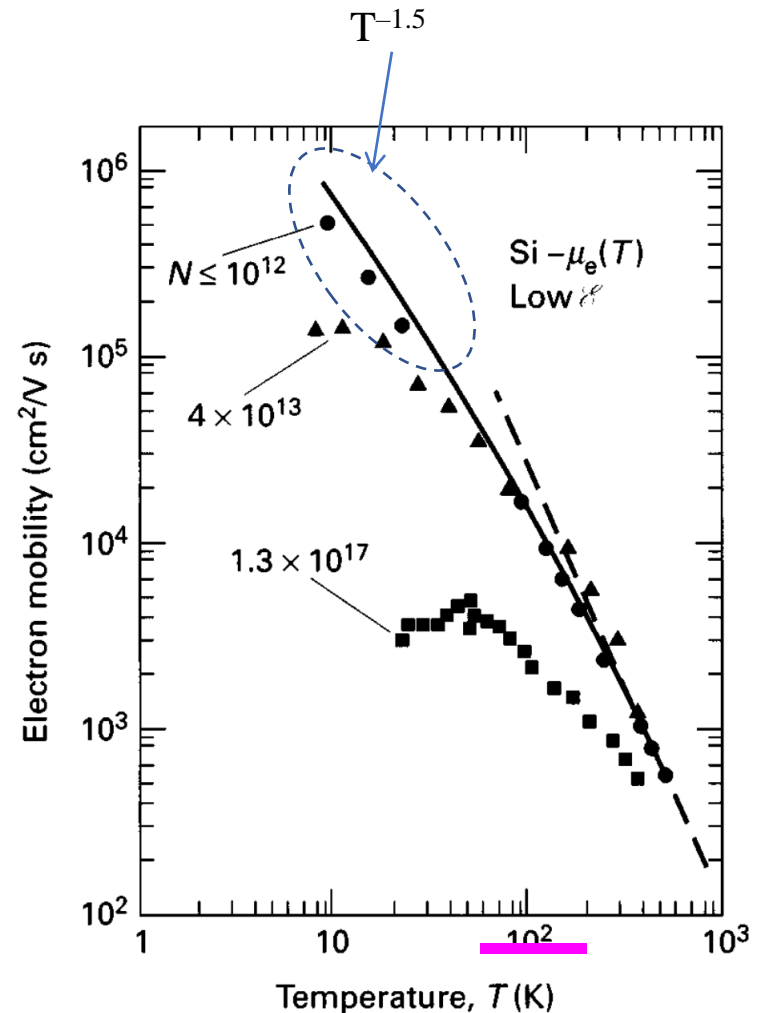
At high temperatures, μ drops more rapidly because of intervalley scattering.

- Effective or overall mobility
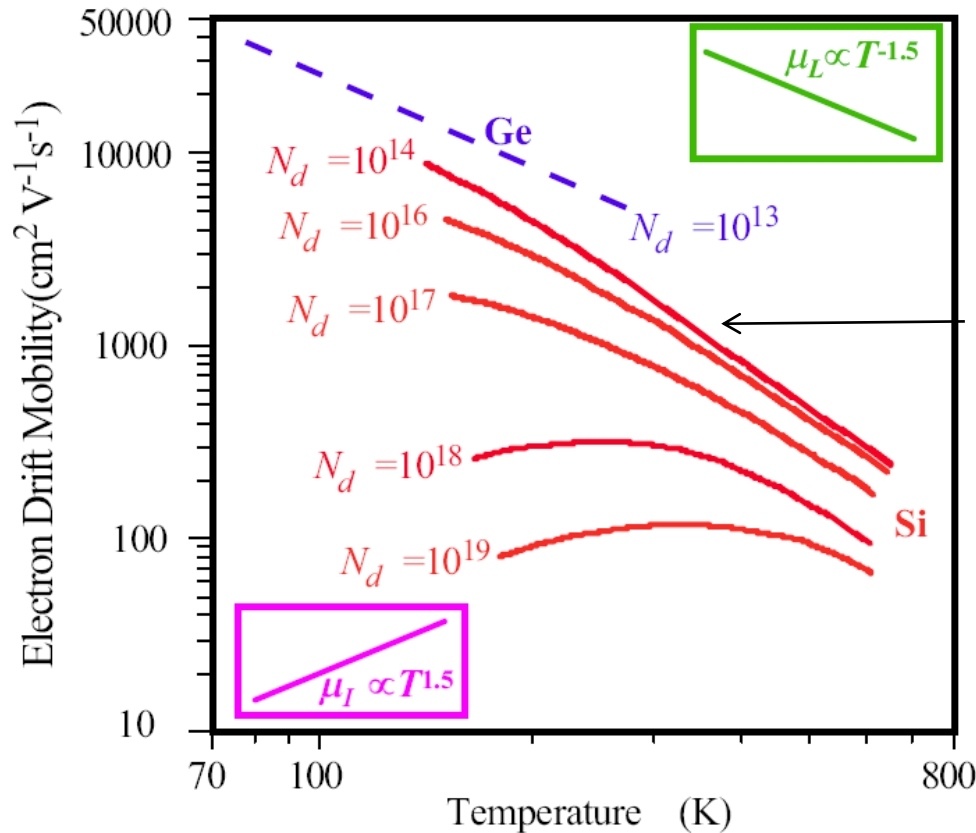
$$\frac{1}{m_e} = \frac{1}{m_I} + \frac{1}{m_L}$$

$$\mu_L \propto T^{-2.5}$$

Log-log plot of drift mobility versus temperature for n-type Ge and n-type Si samples. Various donor concentrations for Si are shown. N_d are in cm^{-3}. The upper right inset is the simple theory for lattice limited mobility, whereas the lower left inset is the simple theory for impurity scattering limited mobility.
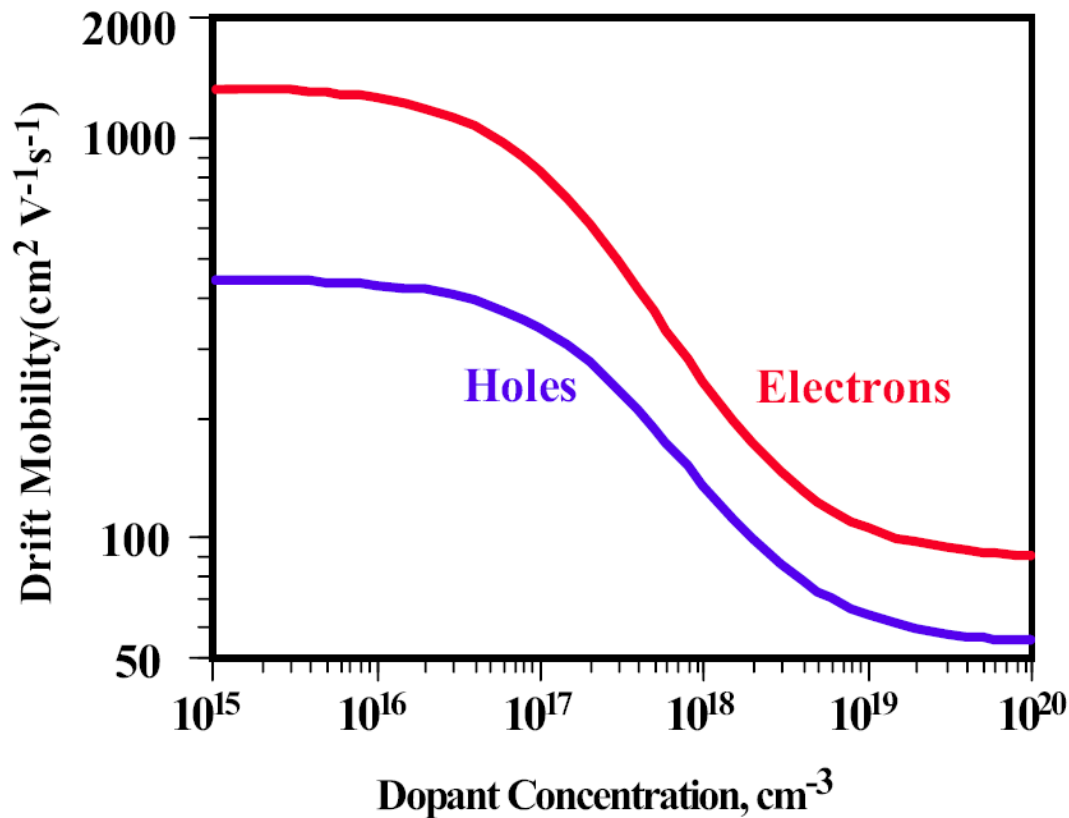
The variation of the drift mobility with dopant concentration in Si for electrons and holes at 300 K. For typical doping concentrations, the drift mobility is largely controlled by the impurity scattering.

$$\mu_e \approx 88 + \frac{1252}{1+6.984\times10^{-18}\,N_{\text{dopant}}}\,\text{cm}^2\text{V}^{-1}\text{s}^{-1} \qquad \mu_h \approx 54.3 + \frac{407}{1+3.745\times10^{-18}\,N_{\text{dopant}}}\,\text{cm}^2\text{V}^{-1}\text{s}^{-1}$$
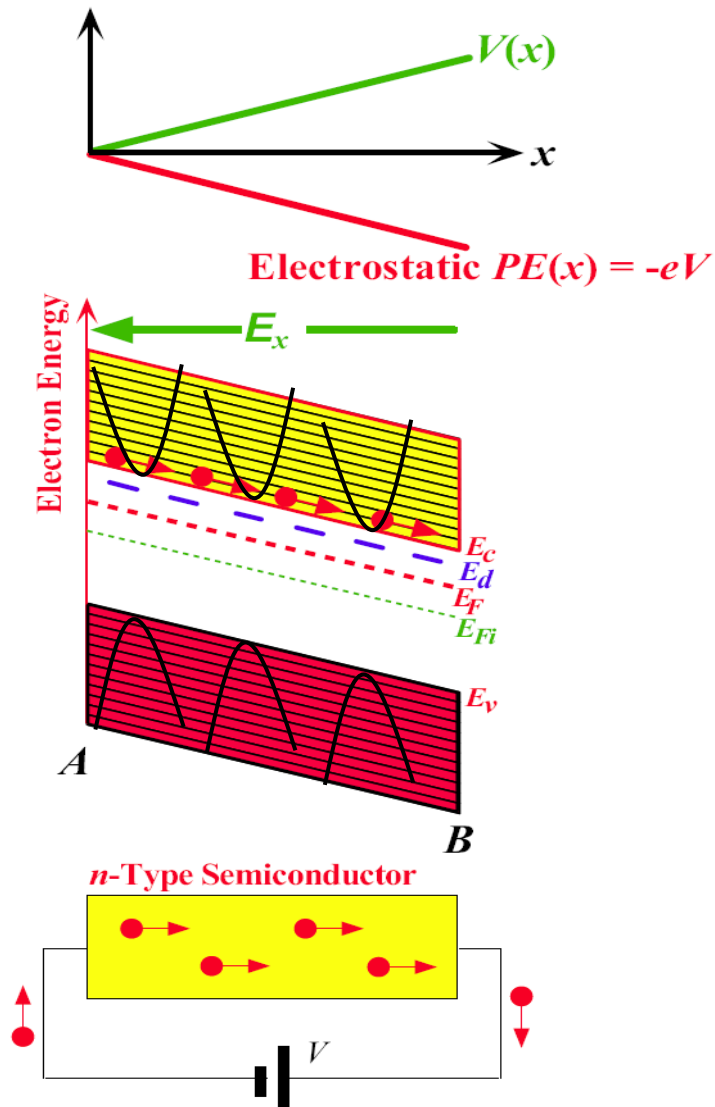
Temperature dependence of conductivity
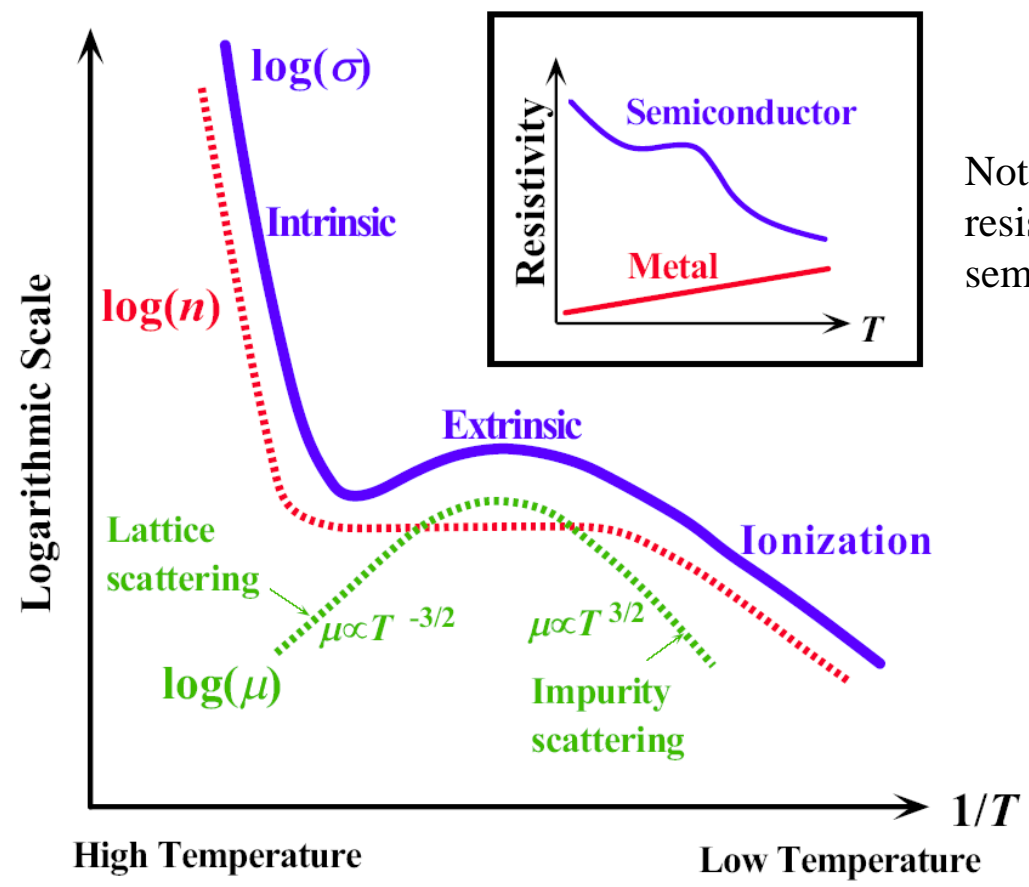
Note the opposite T-dependence of resistivity or conductivity between semiconductors and metals.

Schematic illustration of the temperature dependence of electrical conductivity for a doped (*n*-type) semiconductor.
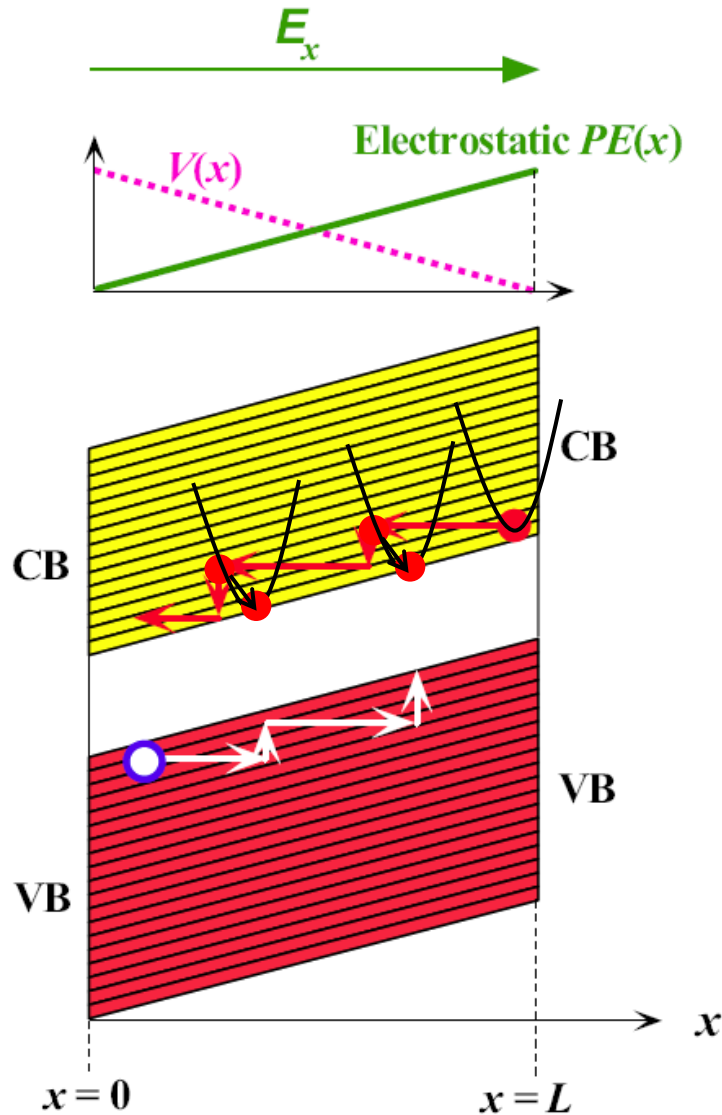
Energy band diagram in devices

Energy band diagram of an *n*-type semiconductor connected to a voltage supply of *V* volts.

The whole energy diagram tilts because the electron now also has an electrostatic potential energy.

The band structure can be defined locally with its relative energy position varies with the external electrostatic potential.

In more detail, the electrons accelerate starting from the conduction bottom (in fact slightly above by 3/2 kT) at a certain spatial point. The acceleration continue during the scattering time, which moves the electron to the higher Bloch states in the left region separated by (drift velocity) x (scattering time). As the electron is scattered, the corresponding Bloch state drops to the conduction bottom. Overall, the process can be depicted as electrons sliding through the bottom line of the conduction band as in the previous slide.

Degenerate semiconductors

For heavily doped SC, typically 10^{19}-10^{20} cm^{-3}, $n > N_c$ or $p > N_v$ and they are called degenerate semiconductors. In this case, the Fermi-Dirac distribution should be applied like free-electron model for metals. The degenerate semiconductors behave more like metals and $\rho \propto T$. In another viewpoint, the defect states with a typical radius of a few nm, begin to overlap each other and form their own bands (defect bands)
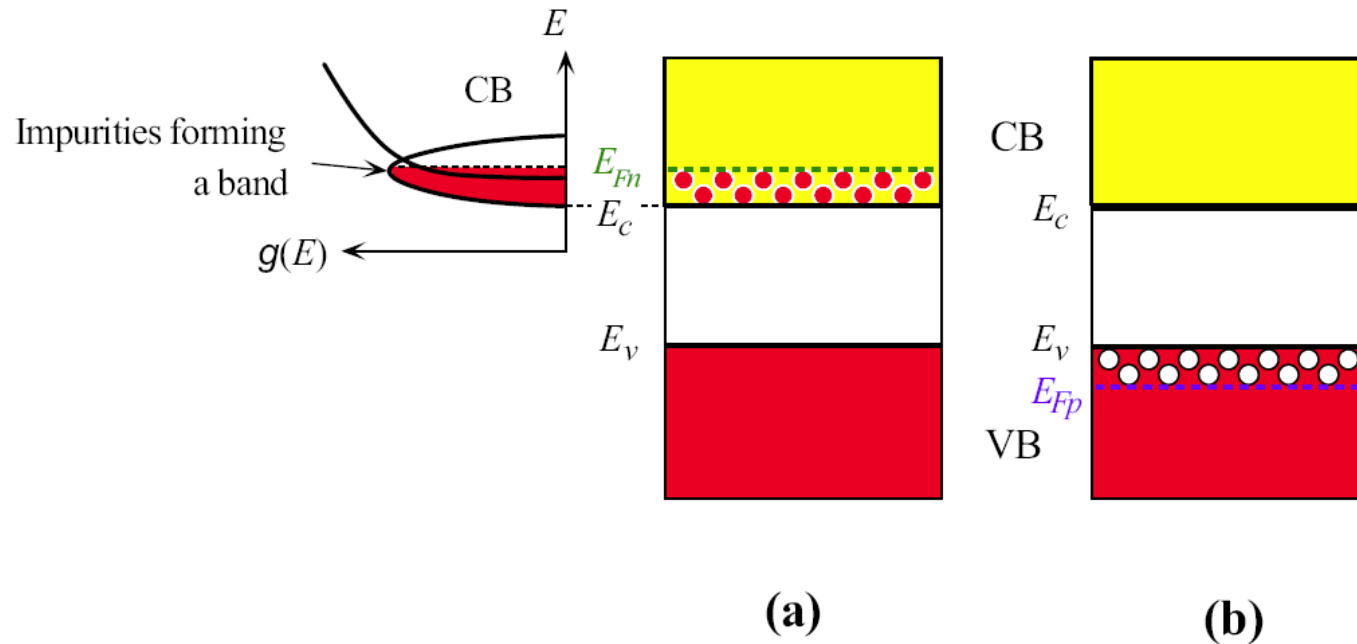

Figure 5.21 (a) Degenerate *n*-type semiconductor. Large number of donors form a band that overlaps the CB. (b) Degenerate *p*-type semiconductor.
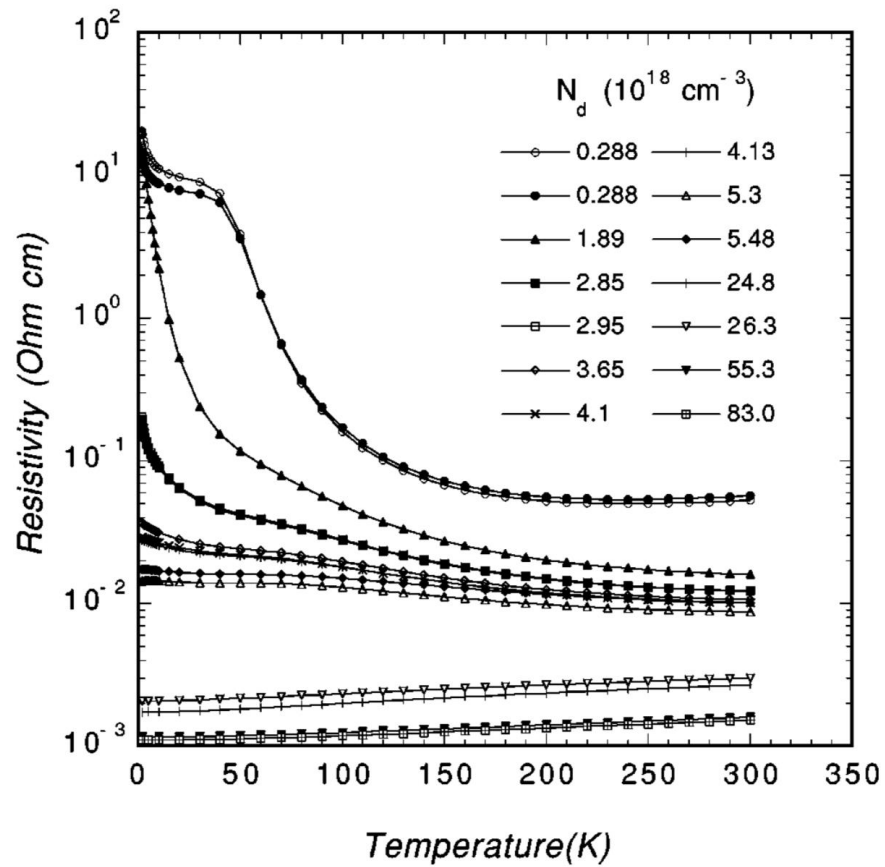
FIG. 3. Temperature dependence of the resistivity of the double-doped system Si:P,Bi with different donor impurity concentrations N_d.

PRB 60, 15824

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Band conduction
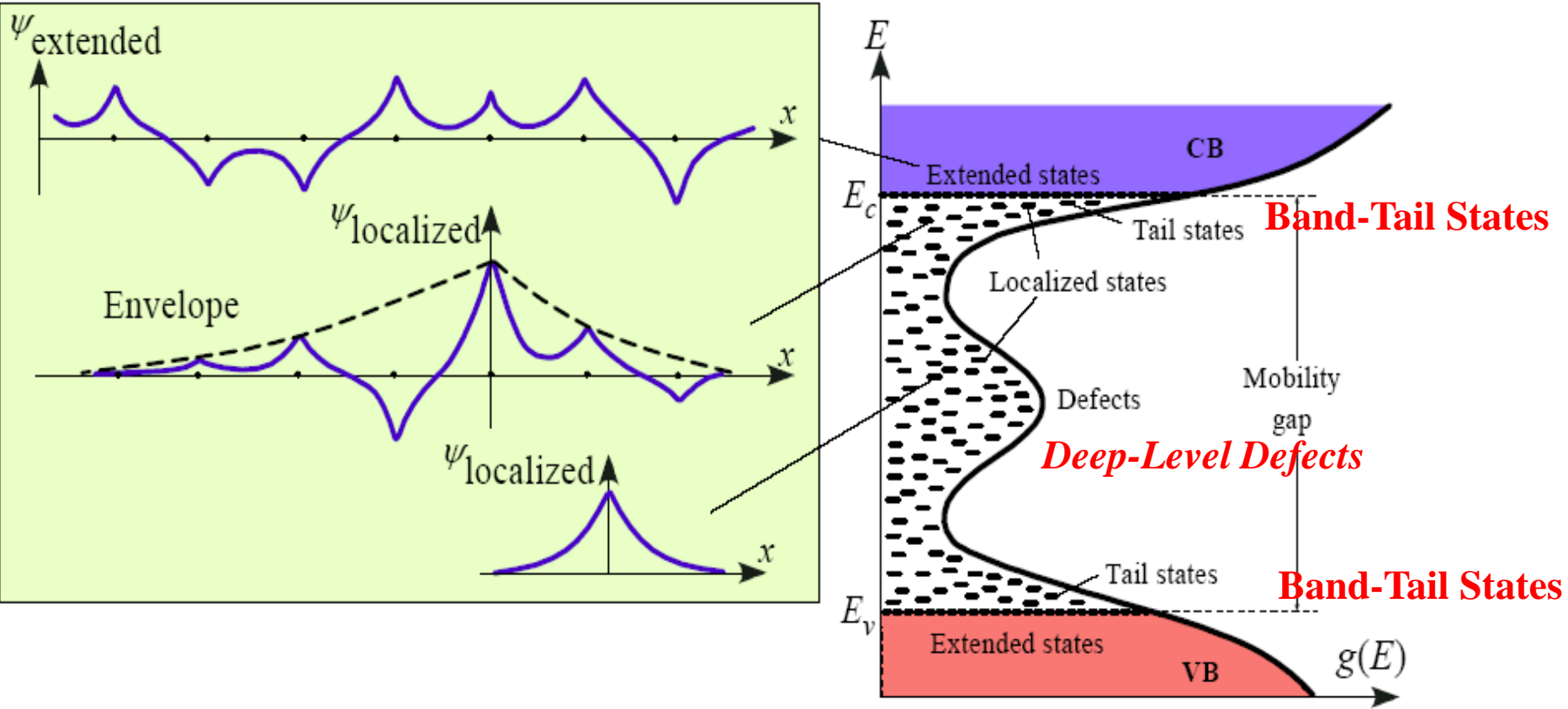Ballistic conduction
Drift conduction

Hopping conduction
Variable range hopping
Markus theory

Superconductivity

Amorphous Semiconductors

Schematic representation of the density of states $g(E)$ vs. energy E for an **amorphous semiconductor,** and the associated electron wavefunctions for an electron in the extended and localized states.

3C.5. Junctions

Properties of an Equilibrium p-n Junction

For the same bandgap energy between *p*-type and *n*-type.
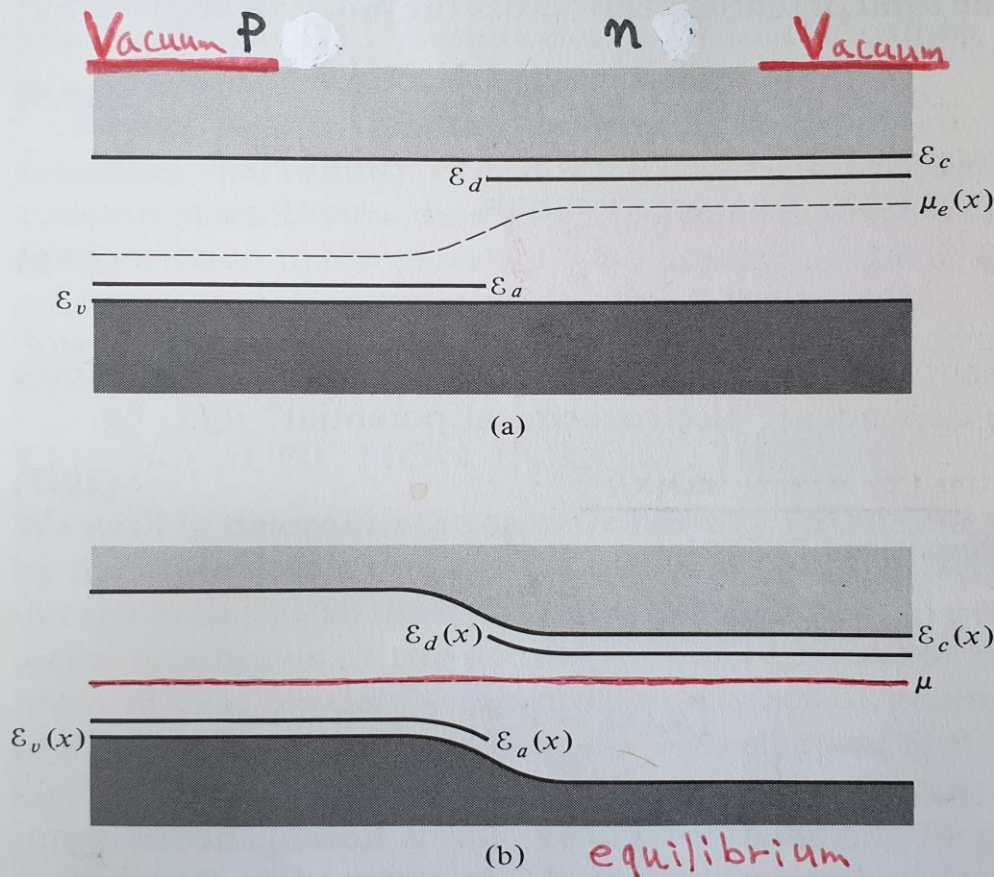

(a)

(b) equilibrium

Figure 29.2
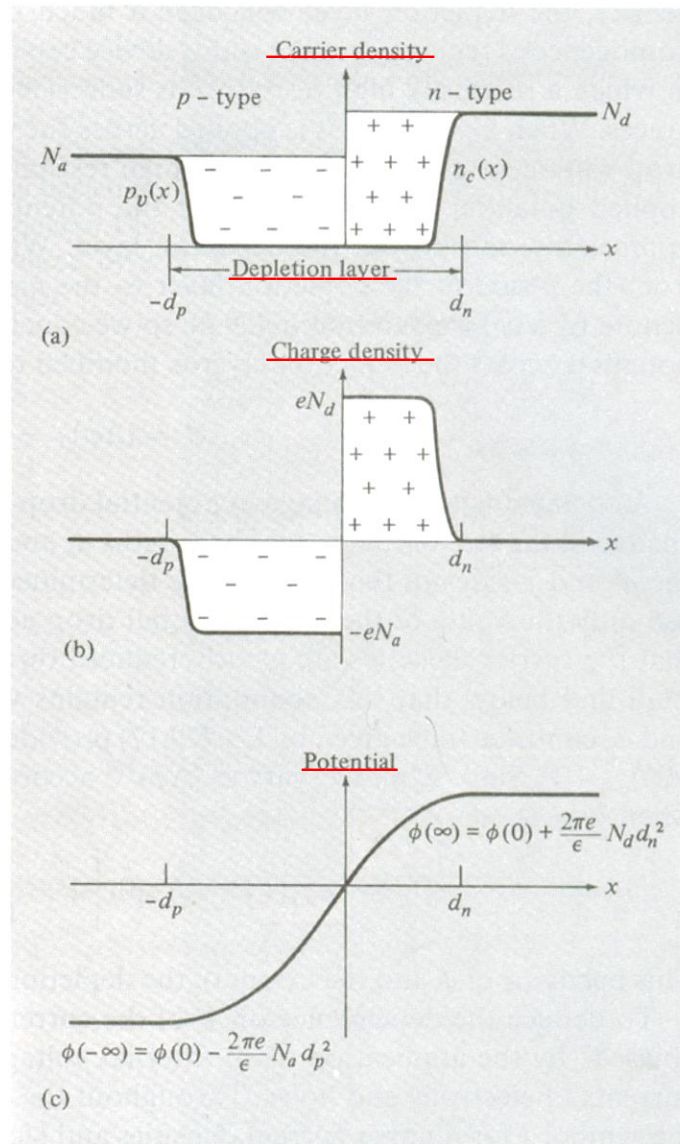Two equivalent ways of representing the effect of the internal potential $\phi(x)$ on the electron and hole densities of a *p-n* junction. (a) The electrochemical potential $\mu_e(x) = \mu + e\phi(x)$ is plotted along the *p-n* junction. The carrier densities at any point x are those that would be found in a uniform semiconductor characterized by the fixed band and impurity energies $\mathcal{E}_c$, $\mathcal{E}_v$, $\mathcal{E}_d$, and $\mathcal{E}_a$, at a chemical potential equal to $\mu_e(x)$. (b) Here $\mathcal{E}_c(x) = \mathcal{E}_c - e\phi(x)$ is the energy of an electron wave packet localized about x formed from levels very near the conduction band minimum, and similarly for $\mathcal{E}_v(x)$. The energies of the local impurity levels are $\mathcal{E}_d(x) = \mathcal{E}_d - e\phi(x)$ and $\mathcal{E}_a(x) = \mathcal{E}_a - e\phi(x)$. The (constant) chemical potential is also shown. The carrier densities at any point x are those that would be found in a uniform semiconductor characterized by band and impurity energies equal to $\mathcal{E}_c(x)$, $\mathcal{E}_d(x)$, $\mathcal{E}_a(x)$, and $\mathcal{E}_v(x)$ at the fixed chemical potential μ.

Ashcroft, Solid State Physics

Depletion Layer

$$\nabla \cdot D = 4\pi\rho$$
$$D = \varepsilon E$$
$$E = -\nabla\Phi$$

Figure 29.3

(a) Carrier densities, (b) charge density, and (c) potential $\phi(x)$ plotted vs. position across an abrupt p-n junction. In the analysis in the text the approximation was made that the carrier densities and charge density are constants except for discontinuous changes at $x = -d_p$ and $x = d_n$. More precisely (see Problem 1), these quantities undergo rapid change over regions just within the depletion layer whose extent is a fraction of order $(k_B T/E_g)^{1/2}$ of the total extent of the depletion layer. The extent of the depletion layer is typically from 10^2 to 10^4 Å.

Carrier density

p – type n – type

N_a

$p_v(x)$ $n_c(x)$ N_d

Depletion layer

$-d_p$ d_n

(a)

Charge density

eN_d

$-d_p$ d_n

$-eN_a$

(b)

Potential

$\phi(\infty) = \phi(0) + \dfrac{2\pi e}{\epsilon} N_d d_n^2$

$-d_p$ d_n

$\phi(-\infty) = \phi(0) - \dfrac{2\pi e}{\epsilon} N_a d_p^2$

(c)

$\rho(x)$

$\Phi(x)$

Carrier Density at a p-n Junction

Figure 29.4
The charge density ρ and potential ϕ in the depletion layer (a) for the unbiased junction, (b) for the junction with $V > 0$ (forward bias), and (c) for the junction with $V < 0$ (reverse bias). The positions $x = d_n$ and $x = -d_p$ that mark the boundaries of the depletion layer when $V = 0$ are given by the dashed lines. The depletion layer and change in ϕ are reduced by a forward bias and increased by a reverse bias.
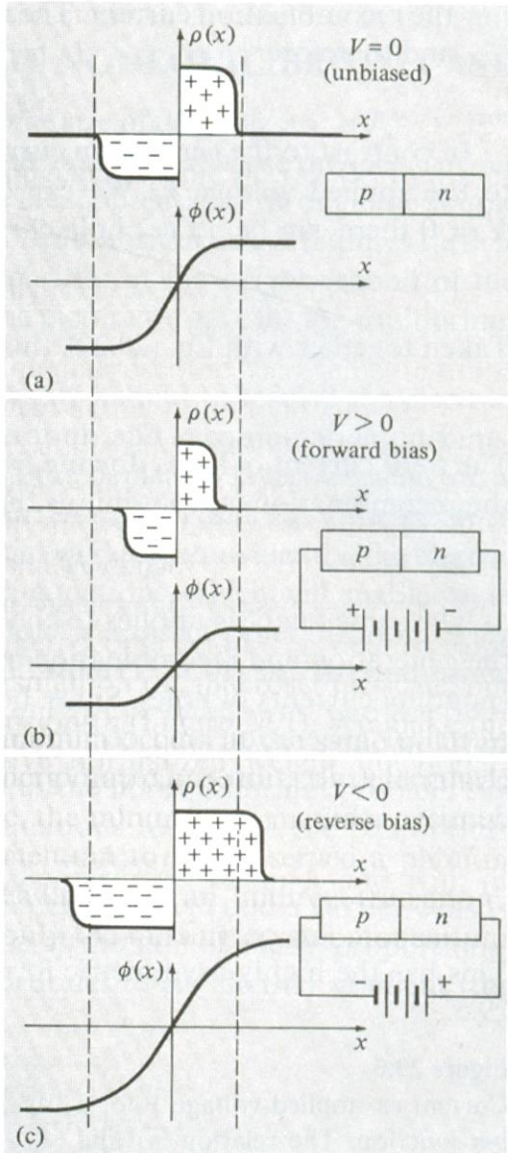
Equilibrium

Forward bias

Reverse bias

A Typical p-n Junction

Typical Si diode: ideal *I-V* characteristics.

Applied forward voltage: ~1 V.

Breakdown voltage: ~1 V – 1 kV.

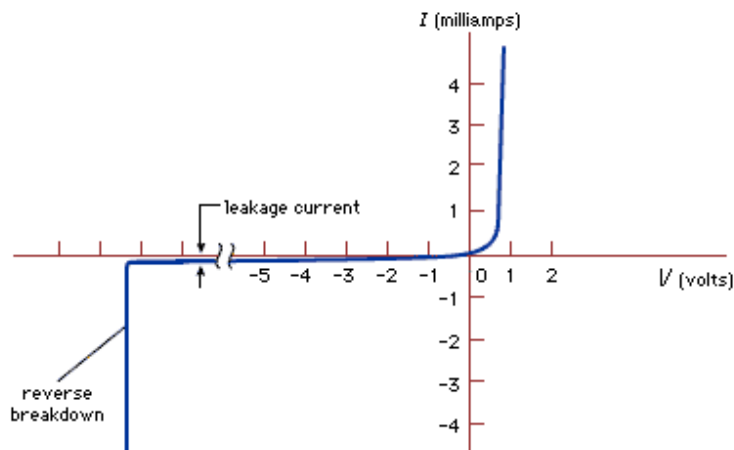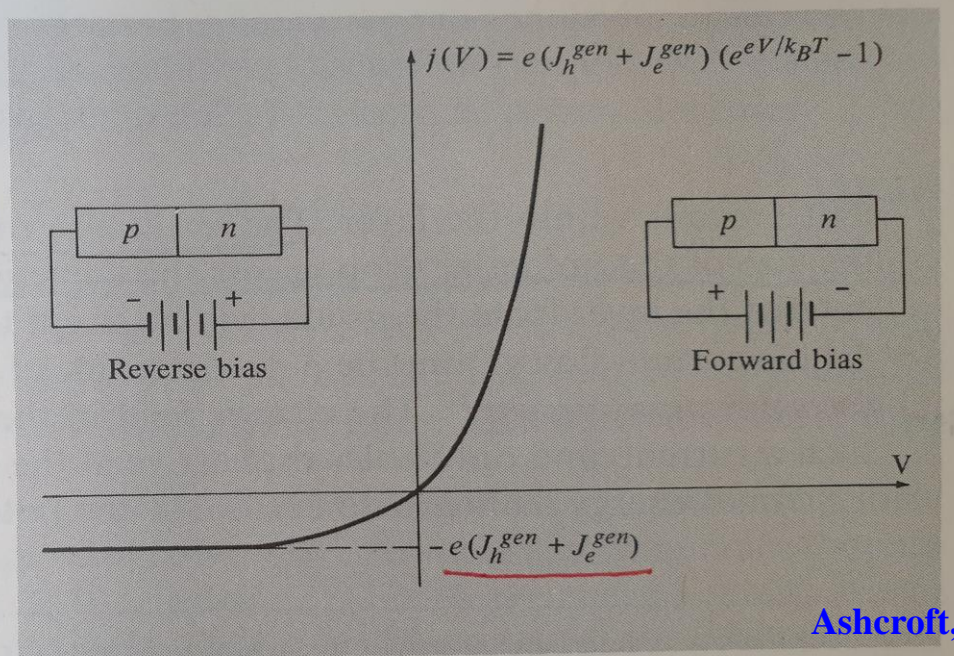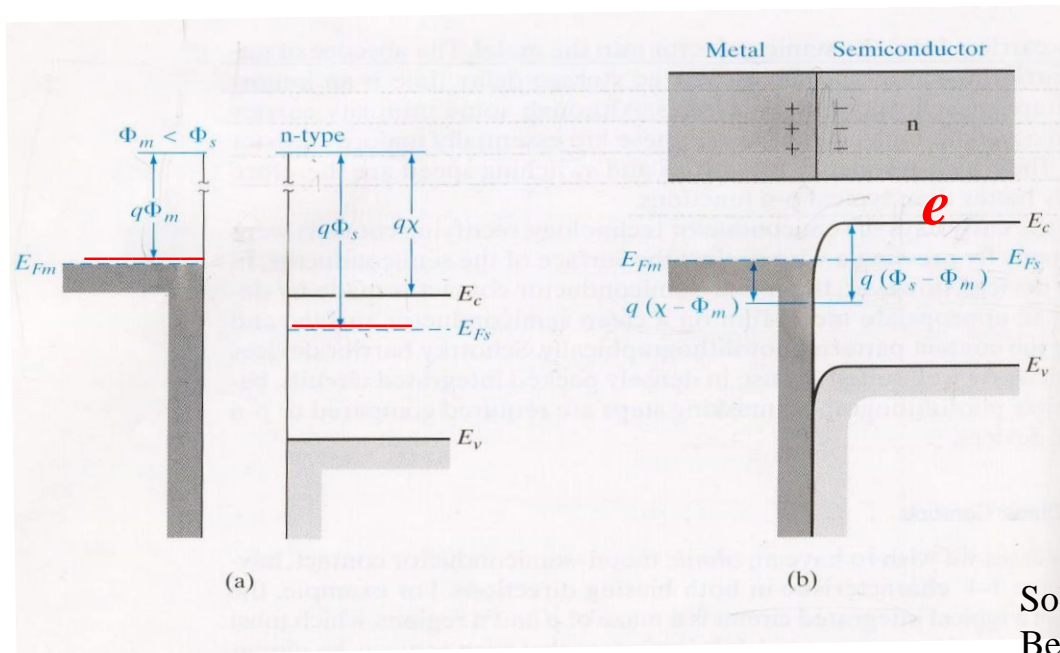*impurity concentration and/or various parameters.*

Figure 29.5
Current vs. applied voltage V for a p-n junction. The relation is valid for eV small compared with the energy gap, E_g. The saturation current $(eJ_h^{gen} + eJ_e^{gen})$ varies with temperature as $e^{-E_g/k_B T}$, as established below.

$$j(V) = e(J_h^{gen} + J_e^{gen})(e^{eV/k_B T} - 1)$$

Reverse bias

Forward bias
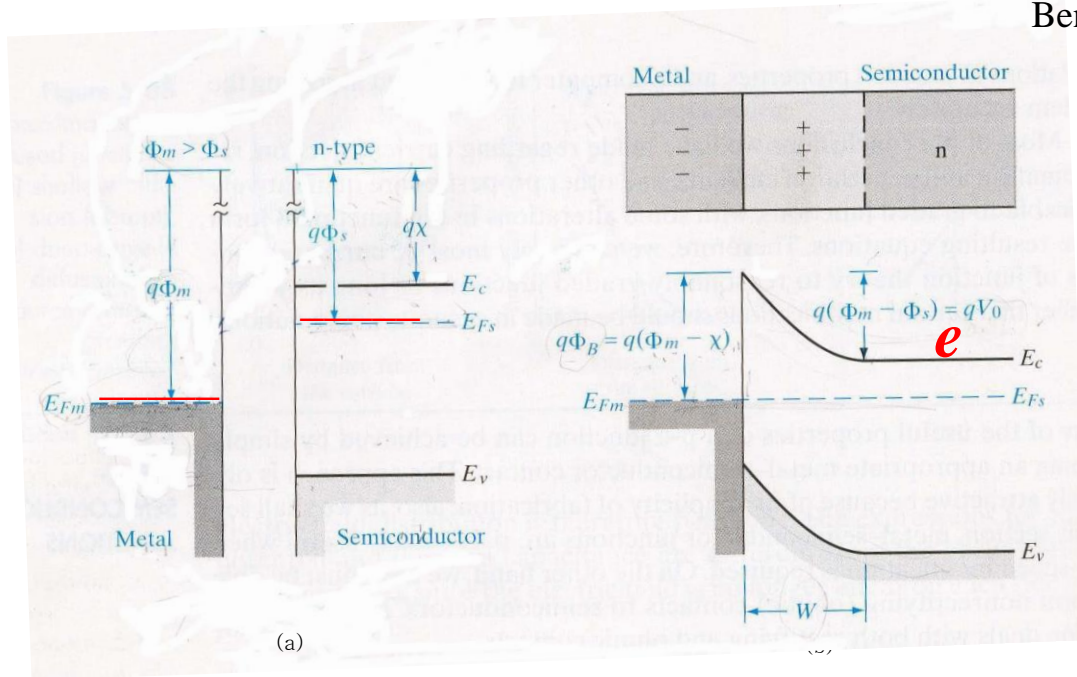
$-e(J_h^{gen} + J_e^{gen})$

$\sim 10^{-11}$ A/cm²

Ohmic Contact

Solid State Electronic Devices (6th edition, Chapter 5)
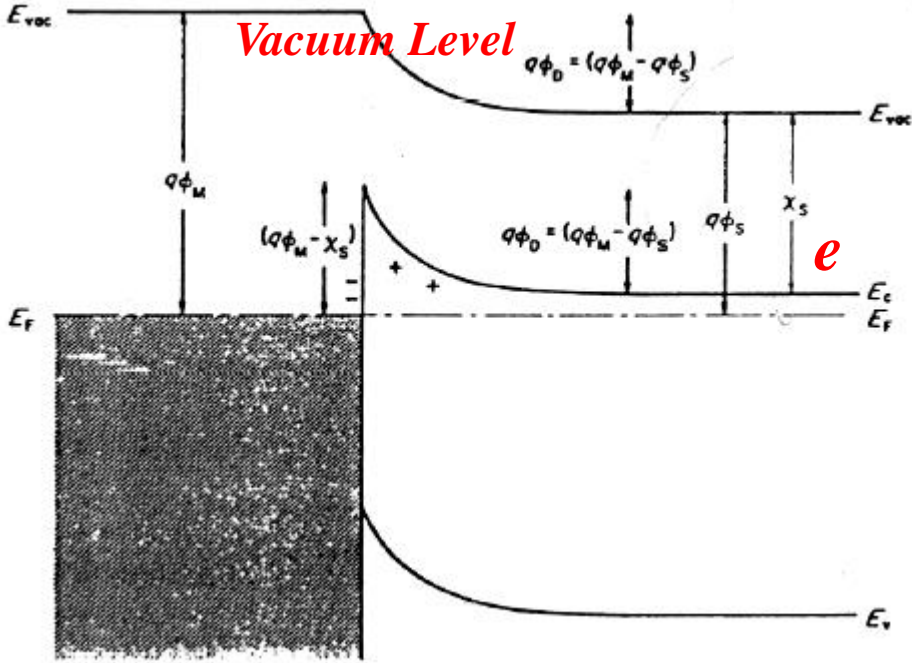Ben G. Streetman, Sanjay Kumar Banerjee

Schottky Barriers

Injection of majority carriers from the semiconductor into the metal

Barrier Height

Vacuum Level

$q\phi_D = (q\phi_M - q\phi_S)$

E_{vac}

$q\phi_M$

$(q\phi_M - x_S)$

$q\phi_0 = (q\phi_M - q\phi_S)$

$q\phi_S$

x_S

e

E_c

E_F

E_v

Vacuum Level

E_{VAC}

$(q\phi_S - q\phi_M)$

x_S

$q\phi_M$

$q\phi_S$

e

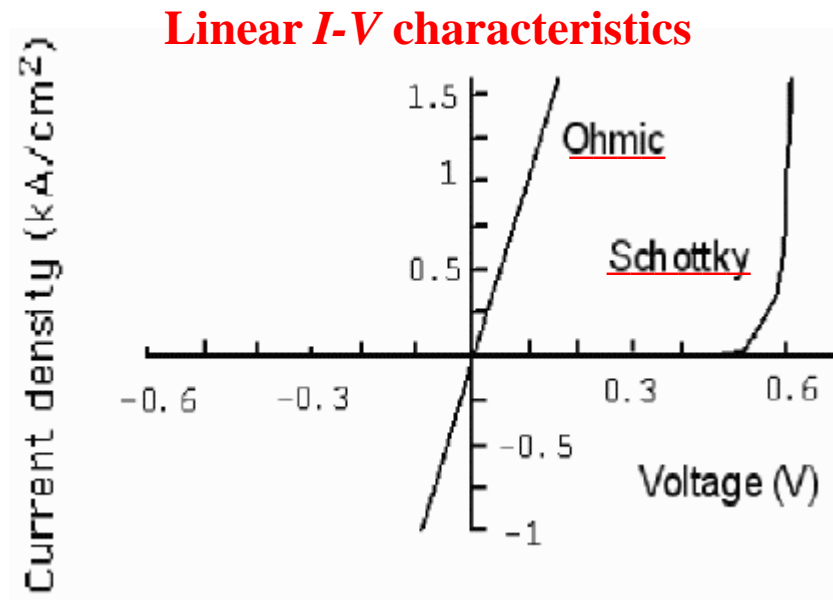E_c

E_F

$(q\phi_S - q\phi_M)$

$(x_S - q\phi_M)$

E_v

Schottky junction (*n*-type semiconductor)
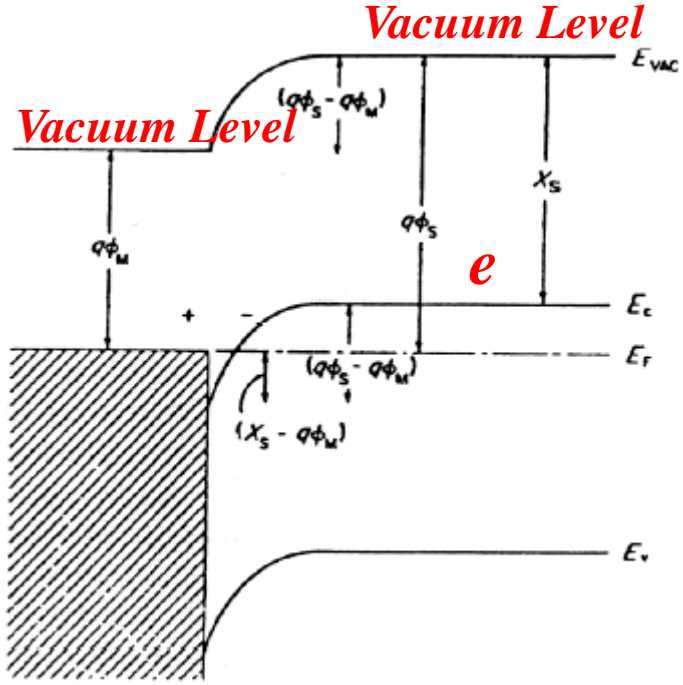
Ohmic junction (*n*-type semiconductor)

I-V Characteristics of Ohmic and Schottky Barriers

Linear *I-V* characteristics

http://www.mtmi.vu.lt/legacy/pfk/funkc_dariniai/diod/schottky.htm

Transparent Conducting Oxide (TCO)

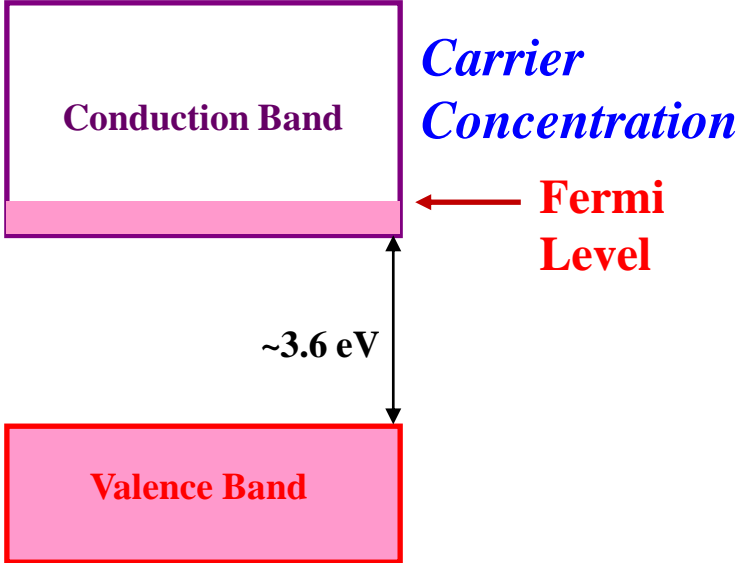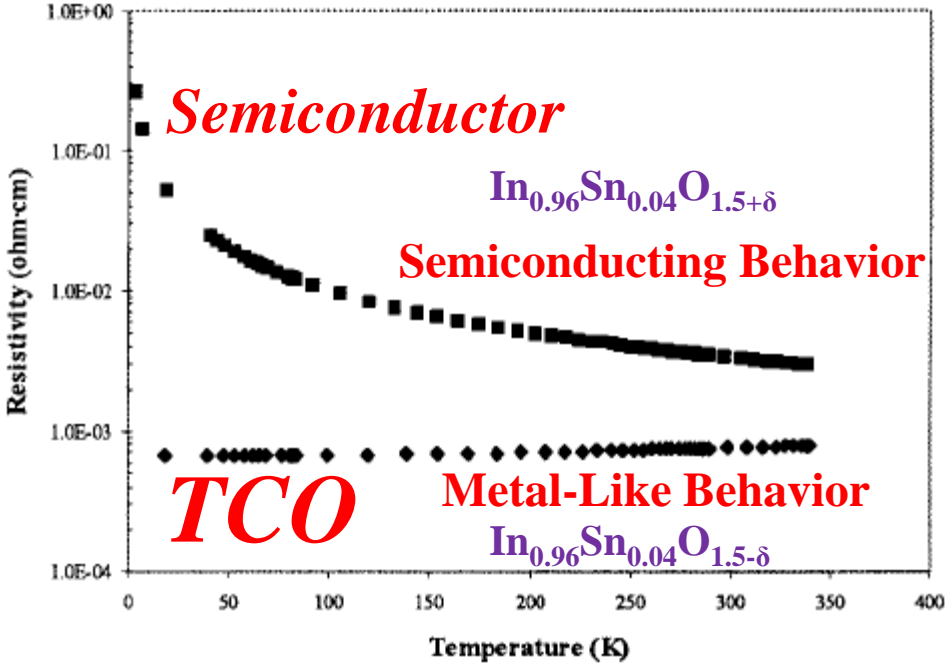Figure 3. Low-temperature resistivity of as-fired (■) and reduced (♦) ITO.

Semiconductor

$In_{0.96}Sn_{0.04}O_{1.5+\delta}$

Semiconducting Behavior

TCO **Metal-Like Behavior**

$In_{0.96}Sn_{0.04}O_{1.5-\delta}$

Conduction Band

Carrier Concentration

Fermi Level

~3.6 eV

Valence Band

Work Function

Table 2   Electron work functions[a]

(Values obtained by photoemission, except tungsten obtained by field emission.)

Element	Surface plane	Work function, in eV
Ag	(100)	4.64
	(110)	4.52
	(111)	4.74
Cs	polycrystal	2.14
Cu	(100)	4.59
	(110)	4.48
	(111)	4.98
Ge	(111)	4.80
Ni	(100)	5.22
	(110)	5.04
	(111)	5.35
W	(100)	4.63
	(110)	5.25
	(111)	4.47

[a]After H. D. Hagstrum

Kittel, Solid State Physics (Chap. 17)

-Tue/10/27/2020