

Introduction to Electromagnetism

Static Electric Fields

(3-4, 3-5, 3-6)

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Gauss's Law and Applications

Gauss's law:

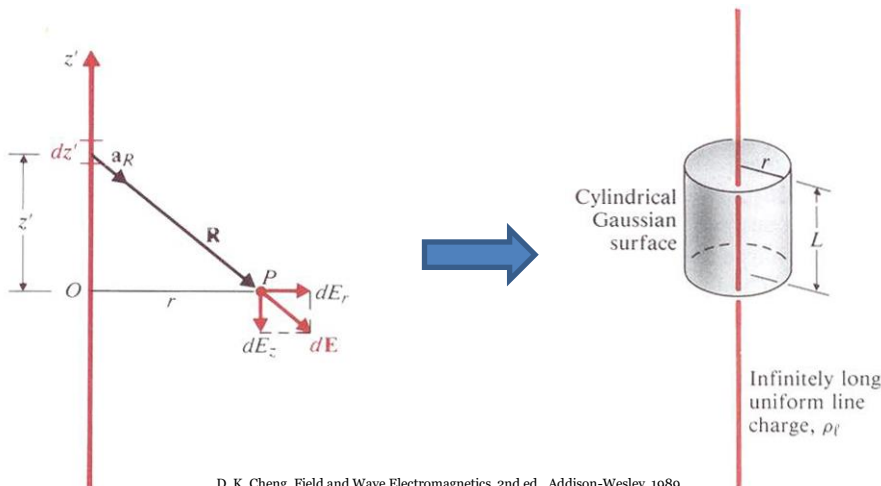
The total outward flux of the E-field over any closed surface in free space is equal to the total charge enclosed in the surface divided by ϵ_0 .

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{Q}{\epsilon_0}$$

Gaussian surface:

With some symmetry conditions, the normal component of the electric field intensity is constant over an enclosed surface.

e.g.



$$\rightarrow 2\pi r L E_r = \frac{\rho_l L}{\epsilon_0}$$

$$\rightarrow \mathbf{E} = \mathbf{a}_r \frac{\rho_l}{2\pi\epsilon_0 r} \quad (\text{V/m})$$

Example 3-7

A spherical cloud of electrons with a volume charge density:

$$\rho = \begin{cases} -\rho_0 & \text{for } 0 \leq R \leq b \\ 0 & \text{for } R > b \end{cases}$$

$$\mathbf{E} = \mathbf{a}_R E_R, \quad d\mathbf{s} = \mathbf{a}_R ds$$

a) $0 \leq R \leq b$

$$\rightarrow \oint_{S_i} \mathbf{E} \cdot d\mathbf{s} = E_R \int_{S_i} ds = E_R 4\pi R^2$$

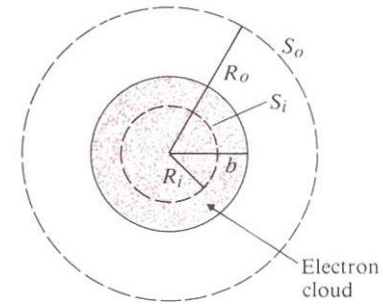
$$\rightarrow Q = \int_V \rho dv = -\rho_0 \int_V dv = -\rho_0 \frac{4\pi}{3} R^3$$

$$\rightarrow \mathbf{E} = -\mathbf{a}_R \frac{\rho_0}{3\epsilon_0} R$$

a) $R > b$

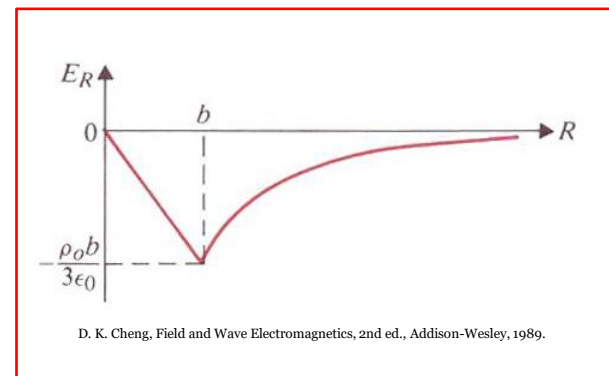
$$\rightarrow Q = -\rho_0 \frac{4\pi}{3} b^3$$

$$\rightarrow \mathbf{E} = -\mathbf{a}_R \frac{\rho_0 b^3}{3\epsilon_0 R^2}$$



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Divergence theorem valid??



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Electric Potential

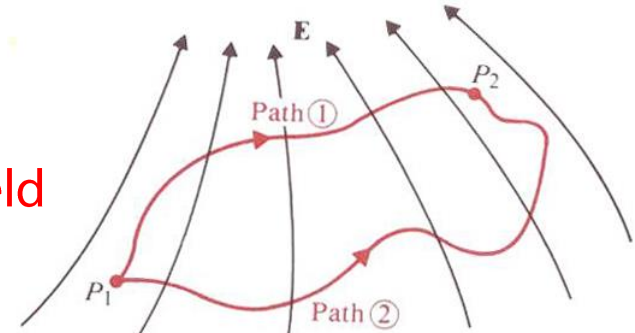
Electric potential:

A curl-free vector field can always be expressed as the gradient of a scalar field.

$$\mathbf{E} = -\nabla V$$

Physical significance: Work done “against” the field

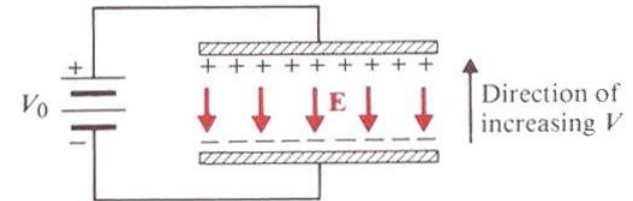
$$\frac{W}{q} = -\int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{l} \quad (\text{J/C or V})$$



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Potential difference:

$$\begin{aligned} \rightarrow -\int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{l} &= \int_{P_1}^{P_2} (\nabla V) \cdot (\mathbf{a}_l dl) \\ &= \int_{P_1}^{P_2} dV = V_2 - V_1 \end{aligned}$$



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→ In most cases the zero-potential point is taken at infinity unless otherwise stated.

Electric Potential Due to a Charge Distribution

For a point charge:

$$V = -\int_{\infty}^R \left(\mathbf{a}_R \frac{q}{4\pi\epsilon_0 R^2} \right) \cdot (\mathbf{a}_R dR)$$

$$\rightarrow V = \frac{q}{4\pi\epsilon_0 R} \quad (\text{V})$$

Potential difference:

$$V_{21} = V_{P_2} - V_{P_1} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{R_2} - \frac{1}{R_1} \right)$$

Electric potential due to a system of n discrete charges:

$$V = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^n \frac{q_k}{|\mathbf{R} - \mathbf{R}_k|} \quad (\text{V})$$

Electric Potential: An Electric Dipole

Electric potential at a distance R from an electric dipole \mathbf{p} :

$$V = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{R_+} - \frac{1}{R_-} \right)$$

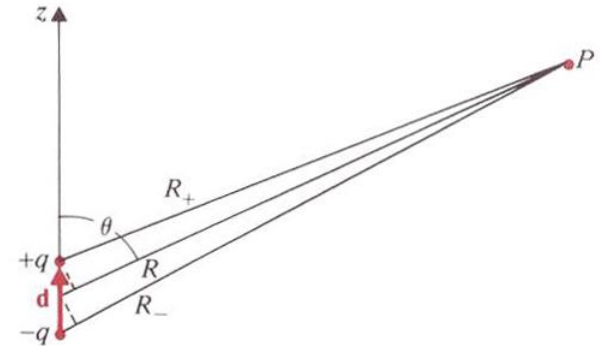
If $d \ll R$:

$$\frac{1}{R_+} = \left(R^2 + \left(\frac{d}{2}\right)^2 - Rd \cos \theta \right)^{-1/2}$$

$$\cong \frac{1}{R} \left(1 + \frac{d}{2R} \cos \theta \right)$$

$$\frac{1}{R_-} = \left(R^2 + \left(\frac{d}{2}\right)^2 + Rd \cos \theta \right)^{-1/2}$$

$$\cong \frac{1}{R} \left(1 - \frac{d}{2R} \cos \theta \right) \rightarrow V = \frac{qd \cos \theta}{4\pi\epsilon_0 R^2}$$



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$$\rightarrow V = \frac{\mathbf{p} \cdot \mathbf{a}_R}{4\pi\epsilon_0 R^2} \quad (\text{V})$$

Electric field:

$$\begin{aligned} \mathbf{E} &= -\nabla V = -\mathbf{a}_R \frac{\partial V}{\partial R} - \mathbf{a}_\theta \frac{1}{R} \frac{\partial V}{\partial \theta} \\ &= \frac{p}{4\pi\epsilon_0 R^3} (\mathbf{a}_R 2 \cos \theta + \mathbf{a}_\theta \sin \theta) \end{aligned}$$

Q:

Equipotential lines
 \perp Electric field lines

Electric Potential: Continuous Charge Distribution

For a volume charge density:

Point charge: $V = \frac{q}{4\pi\epsilon_0 R}$ *Intuitively:* $\rightarrow V = \frac{1}{4\pi\epsilon_0} \int_{v'} \frac{\rho}{R} dv'$

For a surface charge density:

$$\rightarrow V = \frac{1}{4\pi\epsilon_0} \int_{s'} \frac{\rho_s}{R} ds'$$

For a line charge density:

$$\rightarrow V = \frac{1}{4\pi\epsilon_0} \int_{L'} \frac{\rho_l}{R} dl'$$

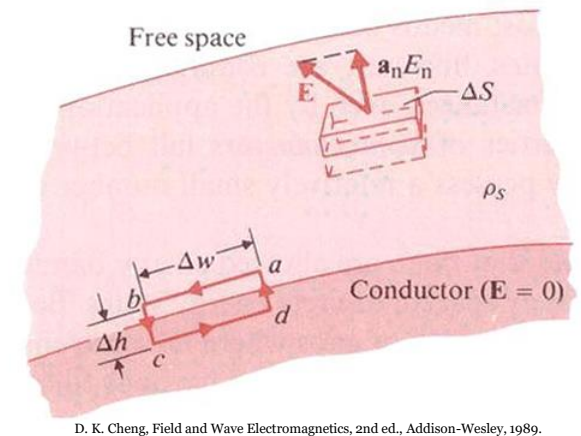
Conductors in Static Electric Field

Classification of Materials:

- Conductors
- Insulators (or Dielectrics)
- Semiconductors

Inside a conductor (under “static” conditions):

$$\rightarrow \rho = 0$$
$$\rightarrow \mathbf{E} = 0$$



Charge distribution on the boundary surface:

$$\rightarrow \oint_{abcd} \mathbf{E} \cdot d\mathbf{l} = E_t \Delta w = 0 \rightarrow E_t = 0$$

Normal component of the E-field at the boundary surface:

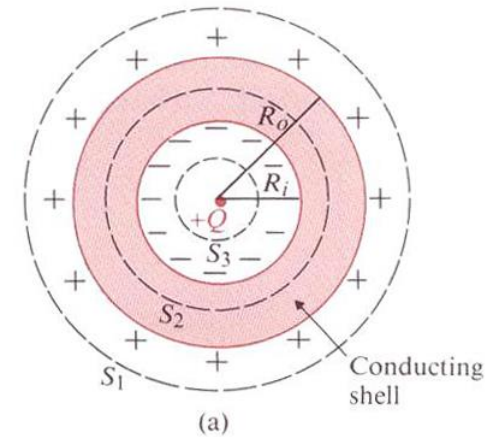
$$\rightarrow \oint_S \mathbf{E} \cdot d\mathbf{s} = E_n \Delta S = \frac{Q_{tot}}{\epsilon_0} = \frac{\rho_s \Delta S}{\epsilon_0}$$

Boundary conditions at a conductor/free space interface:

$$\rightarrow E_t = 0$$
$$\rightarrow E_n = \frac{\rho_s}{\epsilon_0}$$

Example 3-11

For a charge Q at the center of a spherical conducting shell:



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a) $R > R_o$

$$\rightarrow \oint_S \mathbf{E} \cdot d\mathbf{s} = E_{R_1} 4\pi R^2 = \frac{Q}{\epsilon_0}$$

$$\rightarrow E_{R_1} = \frac{Q}{4\pi\epsilon_0 R^2} \quad \rightarrow V_1 = -\int_{\infty}^R E_{R_1} dR = \frac{Q}{4\pi\epsilon_0 R}$$

b) $R_i < R < R_o$

$$\rightarrow E_{R_2} = 0$$

$$\rightarrow V_2 = V_1 \Big|_{R=R_o} = \frac{Q}{4\pi\epsilon_0 R_o}$$

c) $R < R_i$

$$\rightarrow E_{R_3} = \frac{Q}{4\pi\epsilon_0 R^2} \quad \rightarrow V_3 = -\int E_{R_3} dR + C = \frac{Q}{4\pi\epsilon_0 R} + C$$

$$\rightarrow V_3 \Big|_{R=R_i} = V_2 \quad \rightarrow V_3 = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R} - \frac{1}{R_i} + \frac{1}{R_o} \right)$$

