Introduction to Electromagnetism
Static Electric Fields
(3-10)

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Capacitance and Capacitors

Relationship between total charge (Q) and potential:

$$\mathbf{E} = -\nabla V \quad \leftrightarrow \quad \mathbf{E} = \mathbf{a}_n \frac{\rho_s}{\varepsilon_0}$$

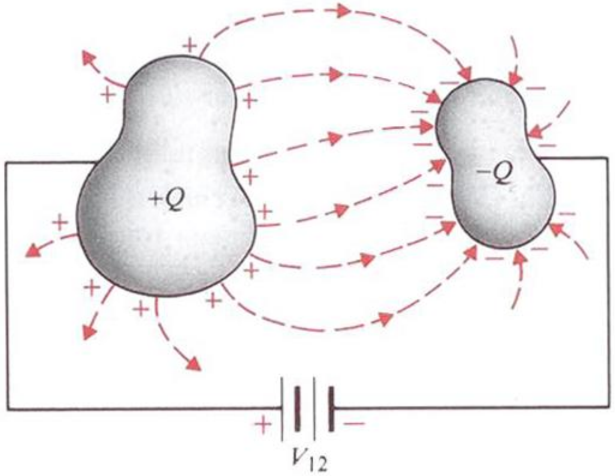
Note : The ratio Q /V remains unchanged.

Capacitance: The electric charge added to the body per unit increase in V

$$\boxed{Q = CV}$$

the body → Capacitor

e.g.

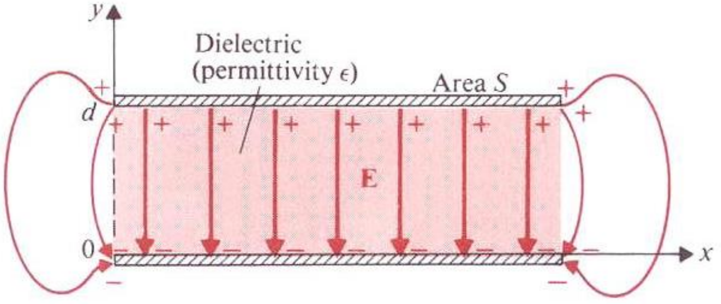


D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$\boxed{C = \frac{Q}{V_{12}} \quad \text{(F)}}$$

Example 3-17

A parallel-plate capacitor:



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Let $\rho_s = \dfrac{Q}{S}$

$$\rightarrow \mathbf{E} = -\mathbf{a}_y \frac{\rho_s}{\varepsilon} = -\mathbf{a}_y \frac{Q}{\varepsilon S}$$

$$\rightarrow V_{12} = -\int_{y=0}^{y=d} \mathbf{E} \cdot d\mathbf{l} = \frac{Q}{\varepsilon S} d$$

$$\rightarrow \boxed{C = \frac{Q}{V_{12}} = \varepsilon \frac{S}{d}}$$

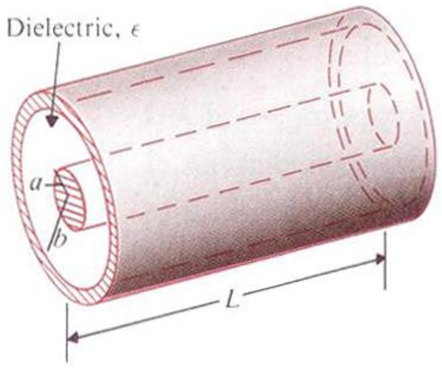
Alternatively:

$$V_{12} \rightarrow \mathbf{E} = -\mathbf{a}_y \frac{V_{12}}{d} \rightarrow \rho_s = \varepsilon E_y = \varepsilon \frac{V_{12}}{d} \rightarrow \rho_s S = Q = \varepsilon \frac{V_{12}}{d} S$$

$$\rightarrow C = \frac{Q}{V_{12}} = \varepsilon \frac{S}{d}$$

Example 3-18

A cylindrical capacitor:



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$\mathbf{E} = \mathbf{a}_r E_r = \mathbf{a}_r \frac{Q}{2\pi\varepsilon L r}$$

$$\rightarrow V_{ab} = -\int_{y=b}^{y=a} \mathbf{E} \cdot d\mathbf{l}$$

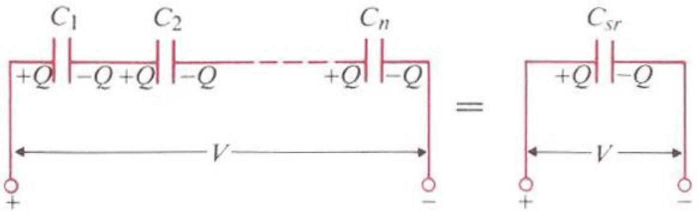
$$= -\int_b^a \left(\mathbf{a}_r \frac{Q}{2\pi\varepsilon L r} \right) \cdot \left(\mathbf{a}_r dr \right)$$

$$= \frac{Q}{2\pi\varepsilon L} \ln\left(\frac{b}{a} \right)$$

$$\rightarrow \boxed{C = \frac{Q}{V_{ab}} = \frac{2\pi\varepsilon L}{\ln\left(\frac{b}{a} \right)}}$$

Series and Parallel Connections of Capacitors

Series connections:



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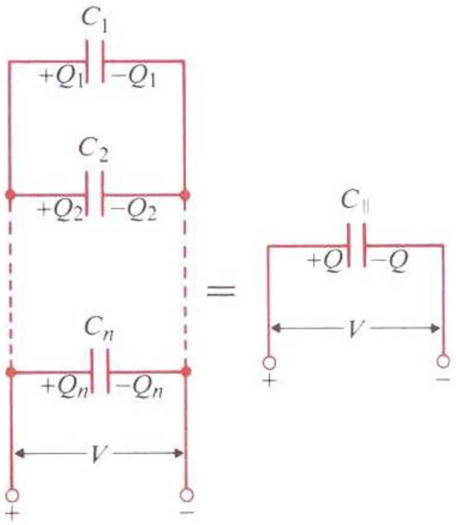
$$V = \frac{Q}{C_{sr}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \cdots + \frac{Q}{C_n} \quad \rightarrow \boxed{\frac{1}{C_{sr}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}}$$

Parallel connections:



$$\begin{aligned} Q &= Q_1 + Q_2 + \cdots + Q_n \\ &= C_1 V + C_2 V + \cdots + C_n V \\ &= C_{//} V \end{aligned}$$

$$\rightarrow \boxed{C_{//} = C_1 + C_2 + \cdots + C_n}$$

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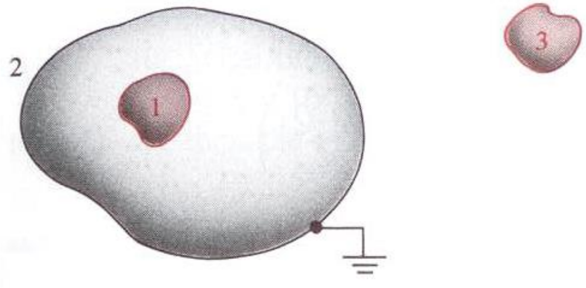
Electrostatic Shielding



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Enclosed by a grounded conducting shell:

$$\rightarrow \ V_2 = 0$$

→ *No coupling between V_1 and V_3!*

What if the conducting shell is not grounded? → HW

Faraday cage: No electric field inside a closed conductor, invented by Michael Faraday