Introduction to Electromagnetism
Static Electric Fields
(3-11)

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Electrostatic Potential Energy

Work done to Q_2 against the field due to Q_1:

$$W_2 = Q_2 V_2 = Q_2 \frac{Q_1}{4\pi\varepsilon_0 R_{12}}$$

$$= Q_1 \frac{Q_2}{4\pi\varepsilon_0 R_{12}} = Q_1 V_1$$

Stored energy for Q_1 and Q_2 as potential energy:

$$\rightarrow W_2 = \frac{1}{2}\left(Q_1 V_1 + Q_2 V_2\right)$$

Potential energy of a group of N discrete point charges at rest:

$$W_e = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k \quad \text{(J)} \quad \leftarrow \quad V_k = \frac{1}{4\pi\varepsilon_0}\sum_{\substack{j=1 \\ (j\neq k)}}^{N} \frac{Q_k}{R_{jk}}$$

$\rightarrow$ 1 (eV) = 1.60 × 10^{-19} (J)

For a continuous charge distribution:

$$W_e = \frac{1}{2}\int_{V'} \rho V dv \quad \text{(J)}$$

Electrostatic Energy in Terms of Field Quantities

Electrostatic energy:

$$W_e = \frac{1}{2}\int_{V'} \rho V dv$$

$$\rightarrow W_e = \frac{1}{2}\int_{V'} (\nabla \cdot \mathbf{D}) V dv$$

$$= \frac{1}{2}\int_{V'} \nabla \cdot (V\mathbf{D}) dv - \frac{1}{2}\int_{V'} \mathbf{D} \cdot \nabla V dv$$

$$= \frac{1}{2}\oint_{S'} V\mathbf{D} \cdot \mathbf{a}_n ds + \frac{1}{2}\int_{V'} \mathbf{D} \cdot \mathbf{E} dv$$

Electrostatic energy density:

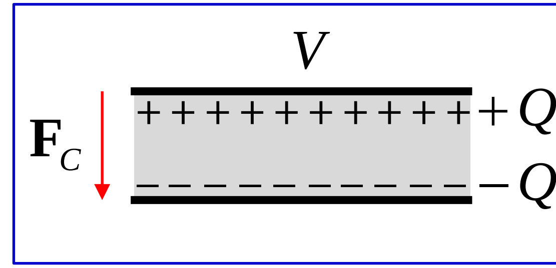
$$W_e = \frac{1}{2}\int_{V'} \mathbf{D} \cdot \mathbf{E} dv = \frac{1}{2}\int_{V'} \varepsilon E^2 dv = \frac{1}{2}\int_{V'} \frac{D^2}{\varepsilon} dv$$

$$= \frac{1}{2}\int_{V'} w_e dv$$

For two-conductor capacitors:

$$W_e = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C} \quad \text{(J)}$$

Electrostatic Forces



System of bodies with fixed charges ($Q = \text{const.}$):

$$dW = \mathbf{F}_Q \cdot d\mathbf{l} \quad \leftarrow \text{Work done by the virtual displacement}$$

$$\rightarrow \; dW = -dW_{e,Q} \quad \leftarrow \text{Expense of the stored energy}$$

$$= -(\nabla W_{e,Q}) \cdot d\mathbf{l} \quad \rightarrow \; dW = (T_Q)_z \, d\phi$$

$$\rightarrow \; \mathbf{F}_Q = -\nabla W_{e,Q} \quad \rightarrow \; T_Q = -\frac{\partial W_{e,Q}}{\partial \phi} \quad (\mathrm{N \cdot m})$$

Systems of conducting bodies with fixed potential ($V = \text{const.}$):

$$dW_s = \sum_k V_k \, dQ_k \quad \leftarrow \text{Energy supplied by the source}$$

$$dW = \mathbf{F}_V \cdot d\mathbf{l} \quad \leftarrow \text{Work done by the virtual displacement}$$

$$dW_{e,V} = \frac{1}{2} \sum_k V_k \, dQ_k = \frac{1}{2} dW_s \quad \leftarrow \text{Electrostatic energy change}$$

$$\rightarrow \; dW + dW_{e,V} = dW_s \rightarrow \; dW = \mathbf{F}_V \cdot d\mathbf{l} = (\nabla W_{e,V}) \cdot d\mathbf{l}$$

$$\rightarrow \; \mathbf{F}_V = \nabla W_{e,V} \quad \rightarrow \; T_V = \frac{\partial W_{e,V}}{\partial \phi} \quad (\mathrm{N \cdot m})$$

$$\mathbf{F}_C = \mathbf{F}_Q = \mathbf{F}_V$$