

Introduction to Electromagnetism

Static Electric Fields

(3-11)

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Electrostatic Potential Energy

Work done to Q_2 against the field due to Q_1 :

$$\begin{aligned}W_2 &= Q_2 V_2 = Q_2 \frac{Q_1}{4\pi\epsilon_0 R_{12}} \\ &= Q_1 \frac{Q_2}{4\pi\epsilon_0 R_{12}} = Q_1 V_1\end{aligned}$$

Stored energy for Q_1 and Q_2 as **potential energy**:

$$\rightarrow W_2 = \frac{1}{2}(Q_1 V_1 + Q_2 V_2)$$

Potential energy of a group of N discrete point charges at rest:

$$W_e = \frac{1}{2} \sum_{k=1}^N Q_k V_k \quad (\text{J}) \quad \leftarrow \quad V_k = \frac{1}{4\pi\epsilon_0} \sum_{\substack{j=1 \\ (j \neq k)}}^N \frac{Q_j}{R_{jk}}$$

$$\rightarrow 1 \text{ (eV)} = 1.60 \times 10^{-19} \text{ (J)}$$

For a continuous charge distribution:

$$W_e = \frac{1}{2} \int_{V'} \rho V dv \quad (\text{J})$$

Electrostatic Energy in Terms of Field Quantities

Electrostatic energy:

$$\begin{aligned}W_e &= \frac{1}{2} \int_{V'} \rho V dv \\ \rightarrow W_e &= \frac{1}{2} \int_{V'} (\nabla \cdot \mathbf{D}) V dv \\ &= \frac{1}{2} \int_{V'} \nabla \cdot (V \mathbf{D}) dv - \frac{1}{2} \int_{V'} \mathbf{D} \cdot \nabla V dv \\ &= \frac{1}{2} \oint_{S'} V \mathbf{D} \cdot \mathbf{a}_n ds + \frac{1}{2} \int_{V'} \mathbf{D} \cdot \mathbf{E} dv\end{aligned}$$

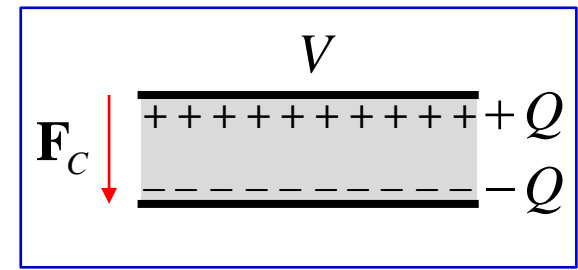
Electrostatic energy density for a simple medium:

$$\begin{aligned}W_e &= \frac{1}{2} \int_{V'} \mathbf{D} \cdot \mathbf{E} dv = \frac{1}{2} \int_{V'} \epsilon E^2 dv = \frac{1}{2} \int_{V'} \frac{D^2}{\epsilon} dv \\ &= \frac{1}{2} \int_{V'} w_e dv\end{aligned}$$

For two-conductor capacitors:

$$W_e = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C} \quad (\text{J})$$

Electrostatic Forces



System of bodies with fixed charges ($Q = \text{const.}$):

$$dW = \mathbf{F}_Q \cdot d\mathbf{l} \quad \leftarrow \text{Work done by the virtual displacement}$$

$$\rightarrow dW = -dW_{e,Q} \quad \leftarrow \text{Expense of the stored energy}$$

$$= -(\nabla W_{e,Q}) \cdot d\mathbf{l}$$

$$\rightarrow dW = (T_Q)_z d\phi$$

$$\rightarrow \mathbf{F}_Q = -\nabla W_{e,Q}$$

$$\rightarrow T_Q = -\frac{\partial W_{e,Q}}{\partial \phi} \quad (\text{N} \cdot \text{m})$$

Systems of conducting bodies with fixed potential ($V = \text{const.}$):

$$dW = \mathbf{F}_V \cdot d\mathbf{l} \quad \leftarrow \text{Work done by the virtual displacement}$$

$$dW_{e,V} = \frac{1}{2} \sum_k V_k dQ_k \quad \leftarrow \text{Electrostatic energy change}$$

$$dW_s = \sum_k V_k dQ_k \quad \leftarrow \text{Energy supplied by the source}$$

$$\rightarrow dW + dW_{e,V} = dW_s \quad \rightarrow dW = \mathbf{F}_V \cdot d\mathbf{l} = dW_{e,V} = (\nabla W_{e,V}) \cdot d\mathbf{l}$$

$$\rightarrow \mathbf{F}_V = \nabla W_{e,V}$$

$$\rightarrow T_V = \frac{\partial W_{e,V}}{\partial \phi} \quad (\text{N} \cdot \text{m})$$

$$\mathbf{F}_C = \mathbf{F}_Q = \mathbf{F}_V$$