

# Introduction to Electromagnetism

## Steady Electric Currents

(5-1, 5-2, 5-3, 5-4)

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# Introduction to Steady Electric Currents

## Charges in motion: Currents

- (1) Conduction currents
- (2) Electrolytic currents
- (3) Convection currents

*Too slow to turn on the circuit instantaneously?*

## Conduction currents:

Electrons in the outermost shells: Loosely bounded to the nuclei

Average drift velocity:  $10^{-4} \sim 10^{-3}$  m/s

Dissipation of the kinetic energy into heat

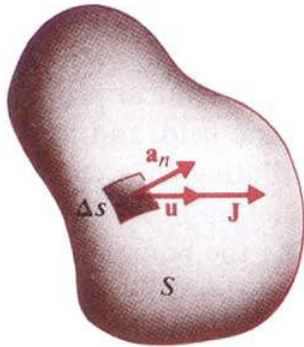
→ Ohm's law:  $V = IR$

Conservation of charges

Kirchhoff's voltage/current law

# Current density

Charges in motion, current and current density:



$$\rightarrow \Delta Q = Nq\mathbf{u} \cdot \mathbf{a}_n \Delta s \Delta t \quad (\text{C})$$

$$\rightarrow \Delta I = \frac{\Delta Q}{\Delta t} = Nq\mathbf{u} \cdot \mathbf{a}_n \Delta s = Nq\mathbf{u} \cdot \Delta \mathbf{s} \quad (\text{A})$$

$$\rightarrow \mathbf{J} = Nq\mathbf{u} \quad (\text{A/m}^2)$$

$$= \rho\mathbf{u} \quad (\text{A/m}^2) \quad \leftarrow \text{Convection current density}$$

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$\rightarrow I = \int_S \mathbf{J} \cdot d\mathbf{s} \quad (\text{A}) \quad \leftarrow \text{Current}$$

# Conduction Currents

Conduction current density:

$$\rightarrow \mathbf{J} = \sum_i N_i q_i \mathbf{u}_i \quad (\text{A/m}^2) \quad \leftarrow \text{Electrons, holes and ions}$$

Average drift velocity with E-field for conductors:

$$\rightarrow \mathbf{u} = -\mu_e \mathbf{E} \quad (\text{m/s}) \quad \leftarrow m \frac{d\mathbf{u}}{dt} + \gamma \mathbf{u} = -e\mathbf{E}$$

$\downarrow$  Mobility  $\swarrow$  Damping const.

*In a steady state, the acceleration of the electron drift is reduced to zero by collisions, i.e., damping!*

$$\rightarrow \mathbf{J} = -\rho_e \mu_e \mathbf{E} = \sigma \mathbf{E} \quad (\text{A/m}^2)$$

$\downarrow$  Conductivity

Conductivity for semiconductors:

$$\rightarrow \mathbf{J} = \sigma \mathbf{E} \quad \leftarrow \sigma = -\rho_e \mu_e + \rho_h \mu_h \quad (\text{S/m})$$

Resistivity:

$$\rightarrow \rho_{res} = \frac{1}{\sigma} \quad (\Omega \cdot \text{m})$$

# Ohm's Law

Point form of Ohm's law:

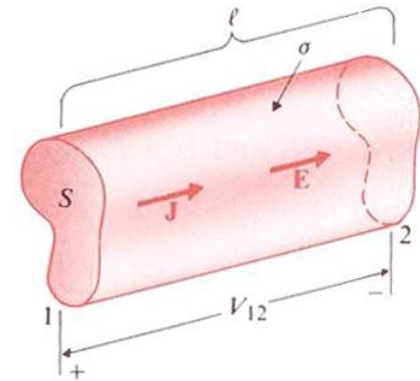
$$\mathbf{J} = \sigma \mathbf{E}$$

$$\rightarrow V_{12} = El \rightarrow E = \frac{V_{12}}{l}$$

$$\rightarrow I = \int_S \mathbf{J} \cdot d\mathbf{s} = JS \rightarrow J = \frac{I}{S}$$

$$\rightarrow \frac{I}{S} = \sigma \frac{V_{12}}{l}$$

$$\rightarrow V_{12} = \left( \frac{l}{\sigma S} \right) I = RI \quad \leftarrow \text{Ohm's law}$$



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Resistance:

$$\rightarrow R = \frac{l}{\sigma S} \quad (\Omega)$$

Conductance:

$$\rightarrow G = \frac{1}{R} = \sigma \frac{S}{l} \quad (\text{S})$$

Series connection:

$$\rightarrow R_{sr} = R_1 + R_2$$

Parallel connection:

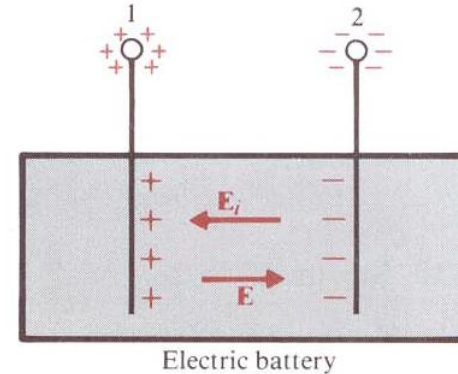
$$\rightarrow \frac{1}{R_{//}} = \frac{1}{R_1} + \frac{1}{R_2}$$

# Electromotive Force

Conservative static electric field:

$$\rightarrow \oint_C \mathbf{E} \cdot d\mathbf{l} = 0$$

$$\rightarrow \oint_C \frac{1}{\sigma} \mathbf{J} \cdot d\mathbf{l} = 0$$



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

→ *A steady current cannot be maintained in the same direction in a closed circuit by an electrostatic field.*

→ *The ohmic loss must be compensated by a “nonconservative” field.*

Source of nonconservative field:

- Electric batteries (chemical energy to electric energy)
- Electric generator (mechanical energy to electric energy)
- Thermocouples (thermal energy to electric energy)
- Photovoltaic cells (light energy to electric energy)

→ *Impressed electric field intensity  $\mathbf{E}_i$*

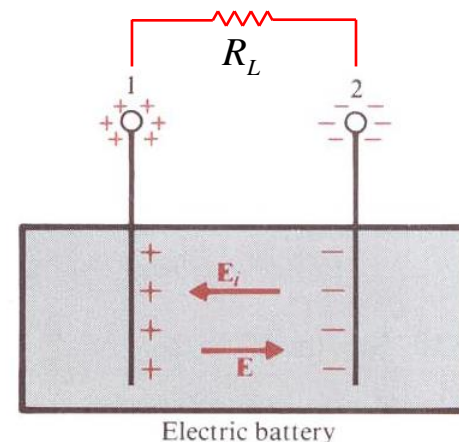
→ *Electromotive force (emf in volt)*

# Kirchhoff's Law

Electromotive force:

$$\rightarrow \mathcal{V} = \int_2^1 \mathbf{E}_i \cdot d\mathbf{l} = - \int_2^1 \mathbf{E} \cdot d\mathbf{l}$$

Inside  
the source



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Conservative electrostatic field intensity:

$$\rightarrow \oint_C \mathbf{E} \cdot d\mathbf{l} = \int_1^2 \mathbf{E} \cdot d\mathbf{l} + \int_2^1 \mathbf{E} \cdot d\mathbf{l} = 0 \rightarrow \mathcal{V} = \int_1^2 \mathbf{E} \cdot d\mathbf{l} = V_{12} = V_1 - V_2$$

Outside  
the source      Inside  
the source      Outside  
the source

For total electric field intensity with  $R_L$  connected:

$$\rightarrow \mathbf{J} = \sigma(\mathbf{r})[\mathbf{E}(\mathbf{r}) + \mathbf{E}_i(\mathbf{r})] \rightarrow \mathbf{E}(\mathbf{r}) + \mathbf{E}_i(\mathbf{r}) = \frac{\mathbf{J}}{\sigma(\mathbf{r})}$$

$$\rightarrow \oint_C (\mathbf{E} + \mathbf{E}_i) \cdot d\mathbf{l} = \int_2^1 \mathbf{E}_i \cdot d\mathbf{l} = \mathcal{V}$$

$$\rightarrow \oint_C \frac{1}{\sigma(\mathbf{r})} \mathbf{J} \cdot d\mathbf{l} = \frac{l_L}{\sigma_L S_L} I = R_L I$$

*← Ideal voltage source*

$$\rightarrow R_s = 0$$

$$\rightarrow \mathcal{V} = R_L I$$

$$\rightarrow \sum_i \mathcal{V}_i = \sum_k R_k I_k$$

*← Kirchhoff's voltage law*

# Equation of Continuity

Principle of conservation of charge:

$$\rightarrow I = \oint_S \mathbf{J} \cdot d\mathbf{s} = -\frac{dQ}{dt} = -\frac{d}{dt} \int_V \rho dv$$

*Net charge within V*

By divergence theorem:

$$\rightarrow \int_V \nabla \cdot \mathbf{J} dv = -\int_V \frac{\partial \rho}{\partial t} dv$$

Equation of continuity:

$$\rightarrow \nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \quad (\text{A/m}^3)$$

For steady currents:

$$\rightarrow \nabla \cdot \mathbf{J} = 0 \quad \leftarrow \text{Divergenceless or solenoidal}$$

*→ The streamlines of steady currents close upon themselves.*

$$\rightarrow \oint_S \mathbf{J} \cdot d\mathbf{s} = 0$$

$$\rightarrow \sum_j I_j = 0 \quad (\text{A}) \quad \leftarrow \text{Kirchhoff's current law}$$



# Charges Introduced inside a Conductor

For charges introduced inside a conductor:

$$\rightarrow \rho = 0, \mathbf{E} = 0 \quad \leftarrow \text{Inside a conductor under equilibrium conditions}$$

Continuity equation:

$$\rightarrow \nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

$$\rightarrow \sigma \nabla \cdot \mathbf{E} = -\frac{\partial \rho}{\partial t} \quad \leftarrow \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon} \quad \text{for a simple medium}$$

$$\rightarrow \frac{\partial \rho}{\partial t} + \frac{\sigma}{\epsilon} \rho = 0$$

Solution:

$$\rightarrow \rho = \rho_0 e^{-(\sigma/\epsilon)t} \quad (\text{C/m}^3)$$

Relaxation time:

$$\rightarrow \tau = \frac{\epsilon}{\sigma} \quad (\text{s})$$

e.g. copper:  $\rightarrow \tau \approx \frac{8.85 \times 10^{-12}}{5.80 \times 10^7} = 1.52 \times 10^{-19} \quad (\text{s})$