Introduction to Electromagnetism
Steady Electric Currents
(5-1, 5-2, 5-3, 5-4)

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Introduction to Steady Electric Currents

Charges in motion: Currents

(1) Conduction currents
(2) Electrolytic currents
(3) Convection currents

Too slow to turn on the circuit nearly instantaneously?

Conduction currents:

Electrons in the outermost shells: Loosely bounded to the nuclei

Average drift velocity: $10^{-4} \sim 10^{-3}$ m/s

Dissipation of the kinetic energy into heat

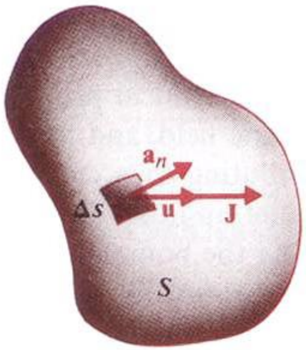
$\rightarrow$ Ohm's law: $V = IR$

Conservation of charges

Kirchhoff's voltage/current law

Current density

Charges in motion, current and current density:



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$\rightarrow \Delta Q = Nq\mathbf{u}\cdot\mathbf{a}_n \Delta s \Delta t \quad \text{(C)}$$

$$\rightarrow \Delta I = \frac{\Delta Q}{\Delta t} = Nq\mathbf{u}\cdot\mathbf{a}_n \Delta s = Nq\mathbf{u}\cdot\Delta\mathbf{s} \quad \text{(A)}$$

$$\rightarrow \mathbf{J} = Nq\mathbf{u} \quad (\text{A/m}^2)$$

$$= \rho\mathbf{u} \quad (\text{A/m}^2) \quad \leftarrow \textit{Convection current density}$$

$$\rightarrow I = \int_S \mathbf{J}\cdot d\mathbf{s} \quad \text{(A)} \quad \leftarrow \textit{Current}$$

Conduction Currents

Conduction current density:

$$\rightarrow \ \mathbf{J} = \sum_i N_i q_i \mathbf{u}_i \quad (\mathrm{A/m^2})$$ $\leftarrow$ *Electrons, holes and ions*

Average drift velocity with E-field for conductors:

Damping const.

$$\rightarrow \ \mathbf{u} = -\mu_e \mathbf{E} \quad (\mathrm{m/s}) \qquad \leftarrow \ \cancel{m\frac{d\mathbf{u}}{dt}} + \gamma \mathbf{u} = -e\mathbf{E}$$

Mobility

In a steady state, the acceleration of the electron drift is reduced to zero by collisions, i.e., damping!

$$\rightarrow \ \mathbf{J} = -\rho_e \mu_e \mathbf{E} = \sigma \mathbf{E} \quad (\mathrm{A/m^2})$$

Conductivity

Conductivity for semiconductors:

$$\rightarrow \ \mathbf{J} = \sigma \mathbf{E} \quad \leftarrow \ \sigma = -\rho_e \mu_e + \rho_h \mu_h \quad (\mathrm{S/m})$$

Resistivity:

$$\rightarrow \ \rho_{res} = \frac{1}{\sigma} \quad (\Omega \cdot \mathrm{m})$$

Ohm's Law

Point form of Ohm's law:

$$\mathbf{J} = \sigma \mathbf{E}$$

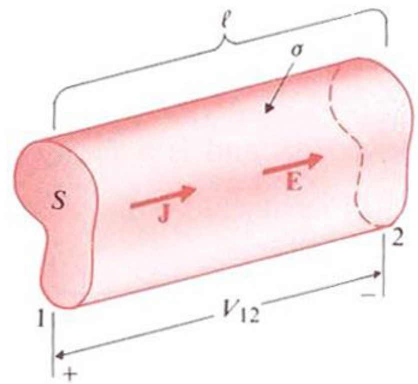
$$\rightarrow V_{12} = El \quad \rightarrow E = \frac{V_{12}}{l}$$

$$\rightarrow I = \int_S \mathbf{J} \cdot d\mathbf{s} = JS \quad \rightarrow J = \frac{I}{S}$$

$$\rightarrow \frac{I}{S} = \sigma \frac{V_{12}}{l}$$

$$\rightarrow V_{12} = \left(\frac{l}{\sigma S} \right) I \quad = RI$$

$\leftarrow$ Ohm's law



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Resistance:

$$\rightarrow \boxed{R = \frac{l}{\sigma S} \quad (\Omega)}$$

Conductance:

$$\rightarrow \boxed{G = \frac{1}{R} = \sigma \frac{S}{l} \quad (\mathrm{S})}$$

Series connection:

$$\rightarrow R_{sr} = R_1 + R_2$$

Parallel connection:

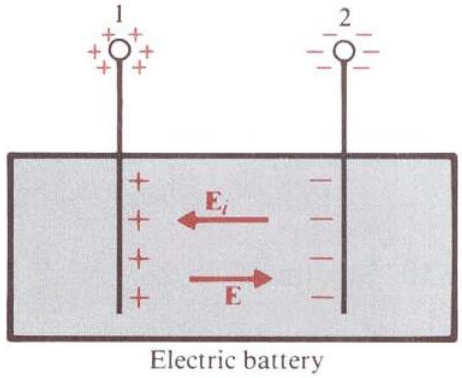
$$\rightarrow \frac{1}{R_{//}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Electromotive Force

Conservative static electric field:

$$\rightarrow \oint_C \mathbf{E} \cdot d\mathbf{l} = 0$$

$$\rightarrow \oint_C \frac{1}{\sigma} \mathbf{J} \cdot d\mathbf{l} = 0$$



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$\rightarrow$ *A steady current cannot be maintained in the same direction in a closed circuit by an electrostatic field.*

$\rightarrow$ *The ohmic loss must be compensated by a "nonconservative" field.*

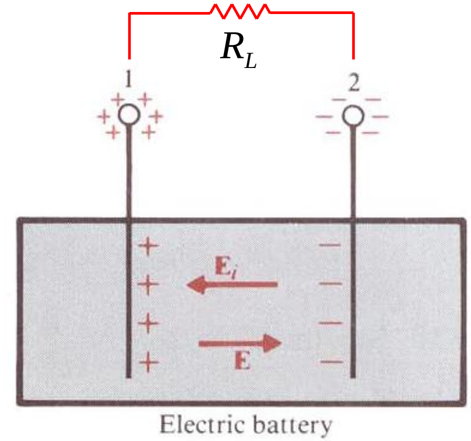
Source of nonconservative field:

- Electric batteries (chemical energy to electric energy)
- Electric generator (mechanical energy to electric energy)
- Thermocouples (thermal energy to electric energy)
- Photovoltaic cells (light energy to electric energy)

$\rightarrow$ *Impressed electric field intensity* $\mathbf{E}_i$

$\rightarrow$ *Electromotive force (emf in volt)*

Kirchhoff's Law



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Electromotive force:

$$\rightarrow \mathcal{V} = \int_2^1 \mathbf{E}_i \cdot d\mathbf{l} = -\underbrace{\int_2^1 \mathbf{E} \cdot d\mathbf{l}}_{\text{Inside the source}}$$

Conservative electrostatic field intensity:

$$\rightarrow \oint_C \mathbf{E} \cdot d\mathbf{l} = \underbrace{\int_1^2 \mathbf{E} \cdot d\mathbf{l}}_{\text{Outside the source}} + \underbrace{\int_2^1 \mathbf{E} \cdot d\mathbf{l}}_{\text{Inside the source}} = 0 \rightarrow \mathcal{V} = \underbrace{\int_1^2 \mathbf{E} \cdot d\mathbf{l}}_{\text{Outside the source}} = V_{12} = V_1 - V_2$$

For total electric field intensity with R_L connected:

$$\rightarrow \mathbf{J} = \sigma(\mathbf{r})[\mathbf{E}(\mathbf{r}) + \mathbf{E}_i(\mathbf{r})] \rightarrow \mathbf{E}(\mathbf{r}) + \mathbf{E}_i(\mathbf{r}) = \frac{\mathbf{J}}{\sigma(\mathbf{r})}$$

$$\rightarrow \oint_C (\cancel{\mathbf{E}} + \mathbf{E}_i) \cdot d\mathbf{l} = \int_2^1 \mathbf{E}_i \cdot d\mathbf{l} = \mathcal{V}$$

$$\rightarrow \oint_C \frac{1}{\sigma(\mathbf{r})} \mathbf{J} \cdot d\mathbf{l} = \frac{l_L}{\sigma_L S_L} I = R_L I$$

$$\rightarrow \mathcal{V} = R_L I$$

$$\rightarrow \sum_i \mathcal{V}_i = \sum_k R_k I_k$$

← Ideal voltage source

$$\rightarrow R_s = 0$$

← Kirchhoff's voltage law

Equation of Continuity

Net charge within V

Principle of conservation of charge:

$$\rightarrow\ I = \oint_S \mathbf{J} \cdot ds = -\frac{dQ}{dt} = -\frac{d}{dt}\int_V \rho dv$$

By divergence theorem:

$$\rightarrow\ \int_V \nabla \cdot \mathbf{J} dv = -\int_V \frac{\partial \rho}{\partial t} dv$$

Equation of continuity:

$$\rightarrow\ \boxed{\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \quad (\mathrm{A/m^3})}$$

For steady currents:

$$\rightarrow\ \nabla \cdot \mathbf{J} = 0$$ ← *Divergenceless or solenoidal*

→ *The streamlines of steady currents close upon themselves.*

$$\rightarrow\ \oint_S \mathbf{J} \cdot d\mathbf{s} = 0$$

$$\rightarrow\ \sum_j I_j = 0 \quad (\mathrm{A})$$ ← *Kirchhoff's current law*

Charges Introduced inside a Conductor

For charges introduced inside a conductor:

$$\rightarrow \rho = 0, \mathbf{E} = 0$$ ← Inside a conductor under equilibrium conditions

Continuity equation:

$$\rightarrow \nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

$$\rightarrow \sigma \nabla \cdot \mathbf{E} = -\frac{\partial \rho}{\partial t} \quad \leftarrow \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon}$$ for a simple medium

$$\rightarrow \frac{\partial \rho}{\partial t} + \frac{\sigma}{\varepsilon} \rho = 0$$

Solution:

$$\rightarrow \boxed{\rho = \rho_0 e^{-(\sigma/\varepsilon)t} \quad (\mathrm{C/m^3})}$$

Relaxation time:

$$\rightarrow \tau = \frac{\varepsilon}{\sigma} \quad (\mathrm{s})$$

e.g. copper: $$\rightarrow \tau \approx \frac{8.85 \times 10^{-12}}{5.80 \times 10^{7}} = 1.52 \times 10^{-19} \quad (\mathrm{s})$$