

Introduction to Electromagnetism

Static Magnetic Fields

(6-10, 6-11)

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Boundary Conditions for Magnetostatic Fields

Two basic equations for magnetostatics:

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Boundary conditions:

$$B_{1n} = B_{2n} \quad (\text{T})$$

$$\mu_1 H_{1n} = \mu_2 H_{2n}$$

← For linear media

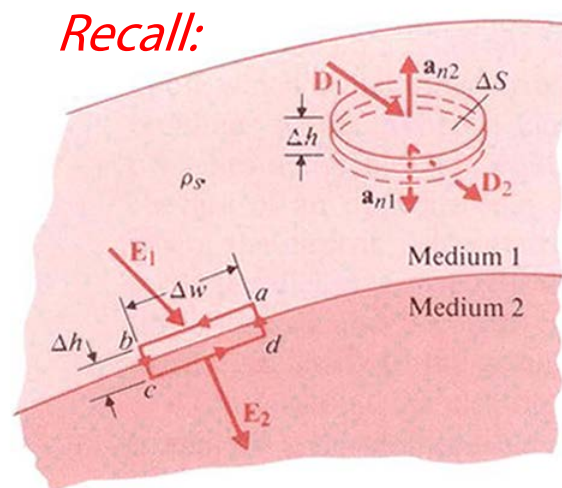
$$\rightarrow \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{s} = I$$

$$\rightarrow \oint_{abcd} \mathbf{H} \cdot d\mathbf{l} = \mathbf{H}_1 \cdot \Delta \mathbf{w} + \mathbf{H}_2 \cdot (-\Delta \mathbf{w}) = J_s \Delta w$$

$$\rightarrow H_{1t} - H_{2t} = J_s \quad (\text{A/m})$$

$$\rightarrow \mathbf{a}_{n2} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$$

← \mathbf{J}_s is non-zero only when an interface with an ideal perfect conductor is assumed.



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

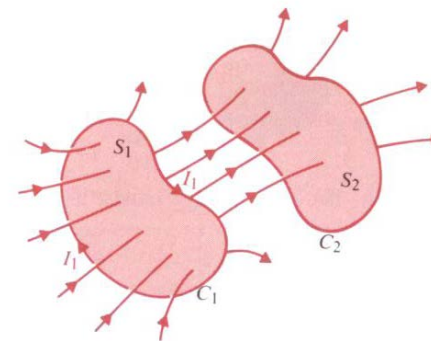
Inductance and Inductors

Magnetic flux:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} \quad (\text{Wb})$$

Any surface

Entire surface enclosed by I_1



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Inductance (Self inductance):

$$\rightarrow \Phi_{11}^L = \int_{S_1} \mathbf{B}_1 \cdot d\mathbf{s}_1 \rightarrow B_1 \propto I_1 \rightarrow \Phi_{11}^L \propto I_1$$

Total magnetic flux linkage = $L_{11} I_1$

$$\rightarrow L_{11} = \frac{\Phi_{11}^L}{I_1} \quad (\text{H}) \rightarrow L_{11} = \frac{d\Phi_{11}^L}{dI_1} \quad (\text{H})$$

Mutual inductance:

$$\rightarrow \Phi_{12}^L = \int_{S_2} \mathbf{B}_1 \cdot d\mathbf{s}_2 \leftarrow \text{Linked to } S_2$$

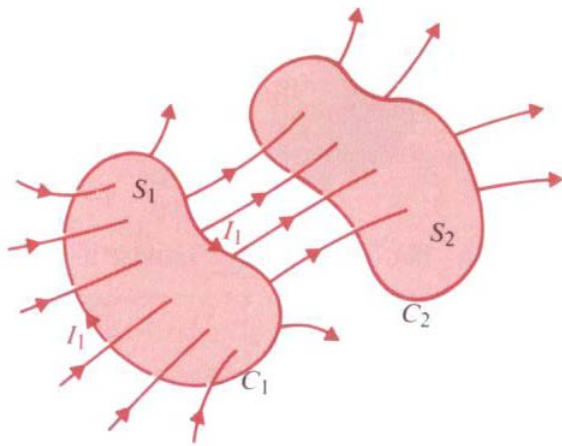
$$= L_{12} I_1 \quad \leftarrow \text{By } I_1$$

$$\rightarrow L_{12} = \frac{\Phi_{12}^L}{I_1} \quad (\text{H}) \rightarrow L_{12} = \frac{d\Phi_{12}^L}{dI_1} \quad (\text{H})$$

Question: $L_{21} = \frac{\Phi_{21}^L}{I_2} = L_{12}$?

Mutual Inductance

Mutual magnetic flux linkage:



$$\begin{aligned} \rightarrow \Phi_{12}^L &= \int_{S_2} \mathbf{B}_1 \cdot d\mathbf{s}_2 = L_{12} I_1 && \text{Entire surface enclosed by } I_2 \\ \rightarrow L_{12} &= \frac{\Phi_{12}^L}{I_1} = \frac{1}{I_1} \int_{S_2} \mathbf{B}_1 \cdot d\mathbf{s}_2 = \frac{1}{I_1} \int_{S_2} (\nabla \times \mathbf{A}_1) \cdot d\mathbf{s}_2 \\ &= \frac{1}{I_1} \oint_{C_2} \mathbf{A}_1 \cdot d\mathbf{l}_2 \quad \leftarrow \mathbf{A}_1 = \frac{\mu_0 I_1}{4\pi} \oint_{C_1} \frac{d\mathbf{l}_1}{R} \\ &= \frac{\mu_0}{4\pi} \oint_{C_2} \oint_{C_1} \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{R} \end{aligned}$$

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Similarly:

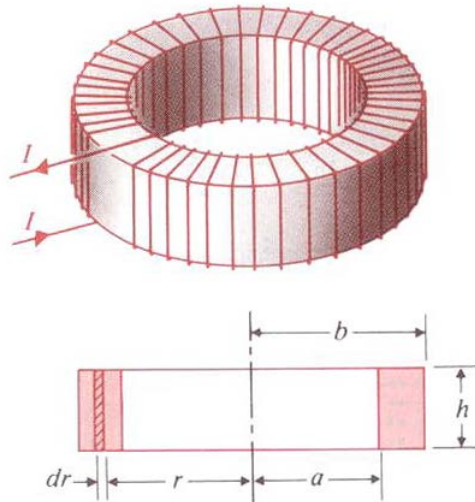
$$\rightarrow L_{21} = \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{R} \quad \leftarrow \text{Neumann formula}$$

Consequently:

$$\rightarrow L_{21} = L_{12}$$

Example 6-14

For a toroid:



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_S \mathbf{J} \cdot d\mathbf{s}$$

$$\rightarrow \mathbf{B} = \mathbf{a}_\phi B_\phi$$

$$\rightarrow \oint_C \mathbf{B} \cdot d\mathbf{l} = 2\pi r B_\phi = \mu_0 NI$$

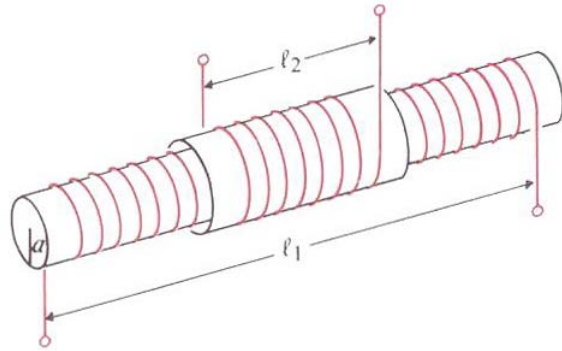
$$\rightarrow \mathbf{B} = \mathbf{a}_\phi \frac{\mu_0 NI}{2\pi r}$$

Entire surface enclosed by I

$$\begin{aligned} \rightarrow L &= \frac{\Phi^L}{I} = \frac{1}{I} \int_S \mathbf{B} \cdot d\mathbf{s} \\ &= \frac{1}{I} \int_S \left(\mathbf{a}_\phi \frac{\mu_0 NI}{2\pi r} \right) \cdot (\mathbf{a}_\phi N h dr) \\ &= \frac{\mu_0 N^2 h}{2\pi} \int_a^b \frac{1}{r} dr \\ &= \frac{\mu_0 N^2 h}{2\pi} \ln \frac{b}{a} \quad (\text{H}) \end{aligned}$$

Example 6-18

For a solenoid with two windings:



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Entire surface enclosed by I_2

$$\begin{aligned} \rightarrow \Phi_{12}^L &= \int_{S_2} \mathbf{B}_1 \cdot d\mathbf{s}_2 = L_{12} I_1 \\ &= \mu \left(\frac{N_1}{l_1} \right) I_1 \cdot N_2 \pi a^2 \quad \leftarrow B_1 l_1 = \mu N_1 I_1 \\ \rightarrow L_{12} &= \frac{\mu}{l_1} N_1 N_2 \pi a^2 \quad (\text{H}) \end{aligned}$$

Alternatively:

$$\begin{aligned} \rightarrow \Phi_{21}^L &= \int_{S_1} \mathbf{B}_2 \cdot d\mathbf{s}_1 = L_{21} I_2 \\ &= \mu \left(\frac{N_2}{l_2} \right) I_2 \cdot N_1 \left(\frac{l_2}{l_1} \right) \pi a^2 \\ \rightarrow L_{21} &= \frac{\mu}{l_1} N_1 N_2 \pi a^2 \quad (\text{H}) \end{aligned}$$

As expected:

$$\rightarrow L_{12} = L_{21}$$

