Introduction to Electromagnetism

Time-Varying Fields and Maxwell's Equations

(7-6, 7-7)

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Farewell to Electromagnetostatic Model

Electrostatic model

$$\nabla \times \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\mathbf{D} = \varepsilon \mathbf{E}$$

Magnetostatic model

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\mathbf{B} = \mu \mathbf{H}$$

Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday's law

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Ampère's law

$$\nabla \cdot \mathbf{D} = \rho$$

Gauss's law

$$\nabla \cdot \mathbf{B} = 0$$

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_o \mathbf{E} + \mathbf{P}$$

Constitutive relations

$$\mathbf{H} = \frac{1}{\mu} \mathbf{B} = \frac{1}{\mu_o} \mathbf{B} - \mathbf{M}$$

Source-Free Wave Equations

Source free, simple medium: $\rho = 0$ & $\boldsymbol{J} = 0 \; (\sigma = 0)$

$\rightarrow$ *Linear, isotropic and homogeneous*

Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_o \mathbf{E} + \mathbf{P}$$

$$\mathbf{H} = \frac{1}{\mu}\mathbf{B} = \frac{1}{\mu_o}\mathbf{B} - \mathbf{M}$$

$\rightarrow$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \mathbf{H}) = -\mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$= \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

$$\rightarrow \nabla^2 \mathbf{E} - \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$u = 1/\sqrt{\mu\varepsilon}$$

$\rightarrow$ *Phase velocity*

$$\rightarrow \nabla^2 \mathbf{E} - \frac{1}{u^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$\rightarrow \nabla^2 \mathbf{H} - \frac{1}{u^2}\frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$$

$\rightarrow$ *Homogeneous wave equations*

Electromagnetic Waves

Wave equations:

$$\nabla^2 \mathbf{E} - \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0, \quad \nabla^2 \mathbf{H} - \mu\varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$$ *(Homogeneous and no source)*

$$e.g. \quad f(x,t) = f(x - \delta x, t - \delta t)$$

Plane waves:

$$\psi = e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})}, \quad |\mathbf{k}| = \omega\sqrt{\mu\varepsilon}$$

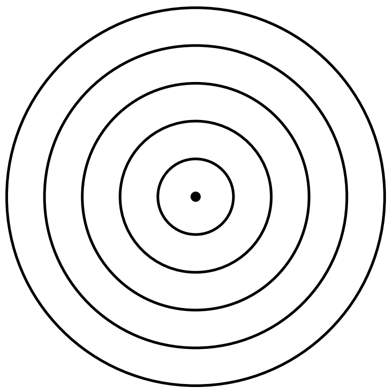
Phase velocity:

$$\omega t - \mathbf{k}\cdot\mathbf{r} = const, \quad u_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}},$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 2.997930 \times 10^8 \quad \text{(m/s)}$$

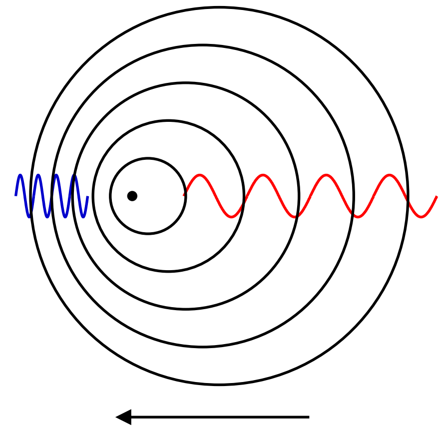
Doppler Effect in EM Waves

Stationary source:



$v = 0$

Moving source:



$v \neq 0$

Recall: Speed of light

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.997930 \times 10^8 \, \text{m/s}$$

← Is this frequency dependent?

→ *Relative to ether?*

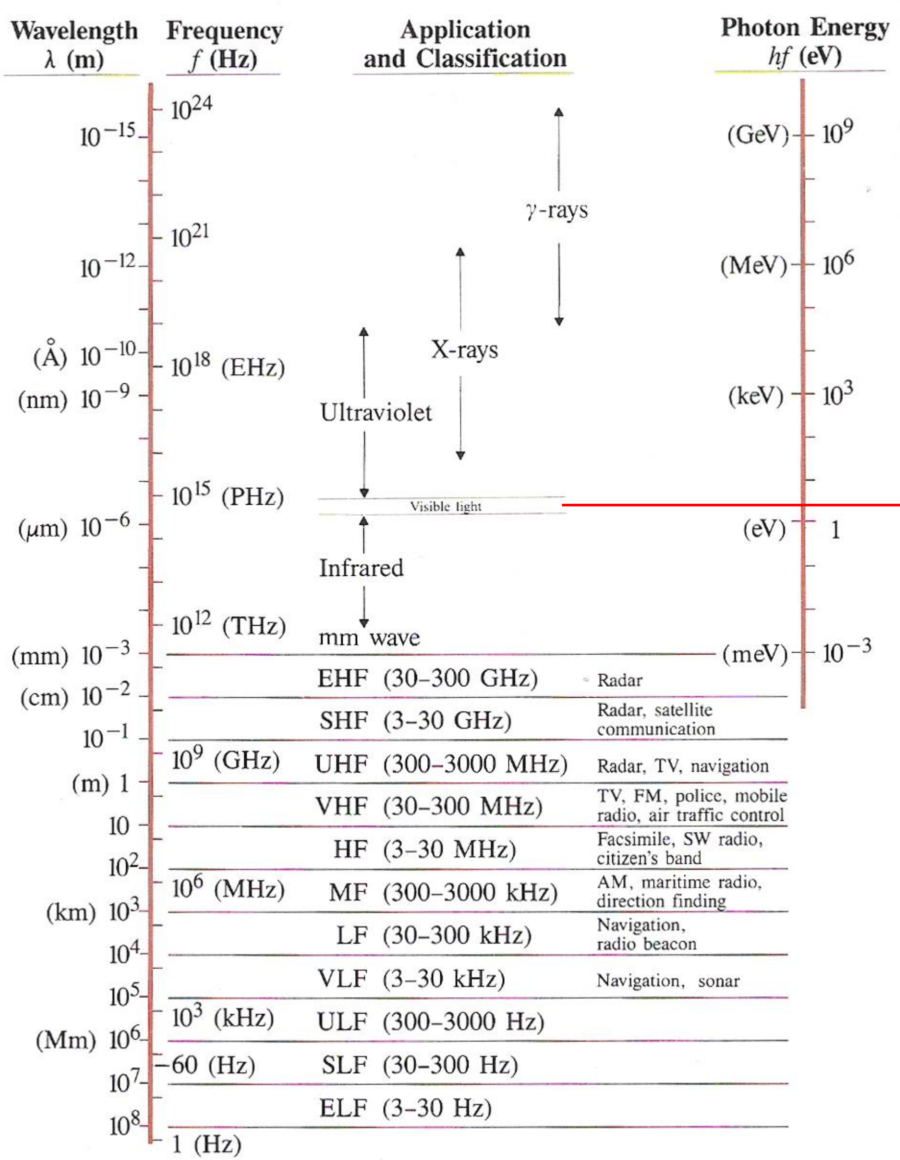
Incoming wave (blue-shifted): $c'_{in} = c + v$

Outgoing wave (red shifted): $c'_{out} = c - v$

} Is this true?

Special & general relativity presented by A. Einstein!!

Electromagnetic Spectrum



Visible light
(400 nm ~ 700 nm)

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.