Introduction to Electromagnetism
Time-Varying Fields and Maxwell's Equations
(7-6, 7-7)

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Farewell to Electromagnetostatic Model

Electrostatic model

$$\nabla \times \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\mathbf{D} = \varepsilon \mathbf{E}$$

Magnetostatic model

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\mathbf{B} = \mu \mathbf{H}$$

Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \textit{Faraday's law}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad \textit{Ampère's law}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \textit{Gauss's law}$$

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_o \mathbf{E} + \mathbf{P}$$

$$\textit{Constitutive relations}$$

$$\mathbf{H} = \frac{1}{\mu} \mathbf{B} = \frac{1}{\mu_o} \mathbf{B} - \mathbf{M}$$

Source-Free Wave Equations

Source free, simple medium: $\rho = 0$ & $\boldsymbol{J} = 0$ ($\sigma = 0$)

→ *Linear, isotropic and homogeneous*

Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_o \mathbf{E} + \mathbf{P}$$

$$\mathbf{H} = \frac{1}{\mu}\mathbf{B} = \frac{1}{\mu_o}\mathbf{B} - \mathbf{M}$$

$\rightarrow$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \mathbf{H}) = -\mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$= \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

$$\rightarrow \nabla^2 \mathbf{E} - \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$u = 1/\sqrt{\mu\varepsilon}$$

→ *Phase velocity*

$$\rightarrow \nabla^2 \mathbf{E} - \frac{1}{u^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$\rightarrow \nabla^2 \mathbf{H} - \frac{1}{u^2}\frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$$

→ *Homogeneous wave equations*   3

Electromagnetic Waves

Wave equations:

$$\nabla^2 \mathbf{E} - \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0, \quad \nabla^2 \mathbf{H} - \mu\varepsilon \frac{\partial^2 \mathbf{H}}{\partial t^2} = 0 \quad \textit{(Homogeneous and no source)}$$

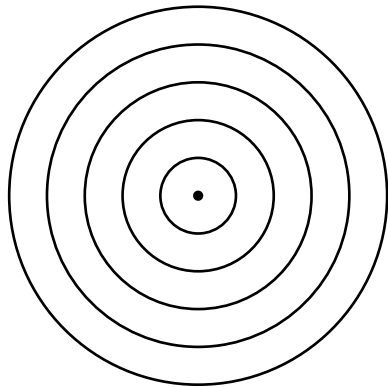$$\textit{e.g. } f(x,t) = f(x - \delta x, t - \delta t)$$

Plane waves:

$$\psi = e^{i(\omega t - \mathbf{k} \cdot \mathbf{r})}, \quad |\mathbf{k}| = \omega\sqrt{\mu\varepsilon}$$

Phase velocity:

$$\omega t - \mathbf{k} \cdot \mathbf{r} = const, \quad u_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}},$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.997930 \times 10^8 \quad (\text{m/s})$$
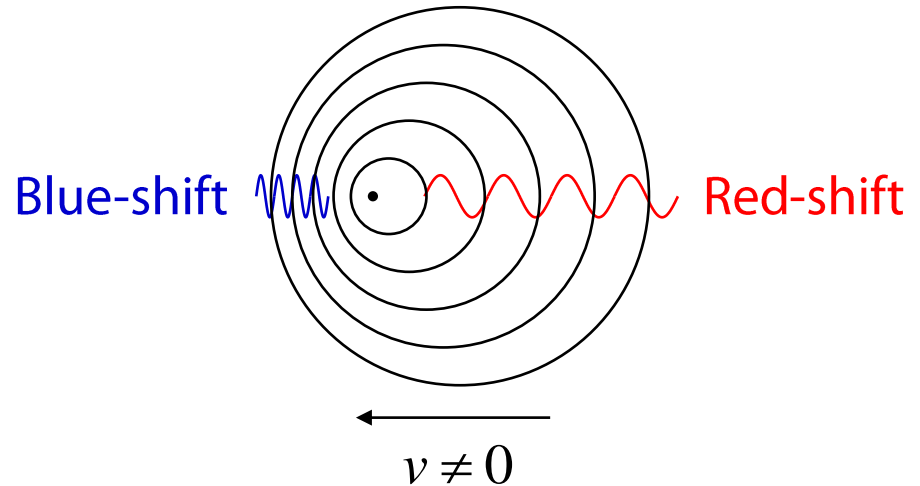
Doppler Effect in EM Waves

Stationary source: Moving source:

Blue-shift   Red-shift

$$v = 0$$ $$v \neq 0$$

Recall: Speed of light

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.997930 \times 10^8 \, \text{m/s} \quad \leftarrow \text{Is this frequency dependent?}$$
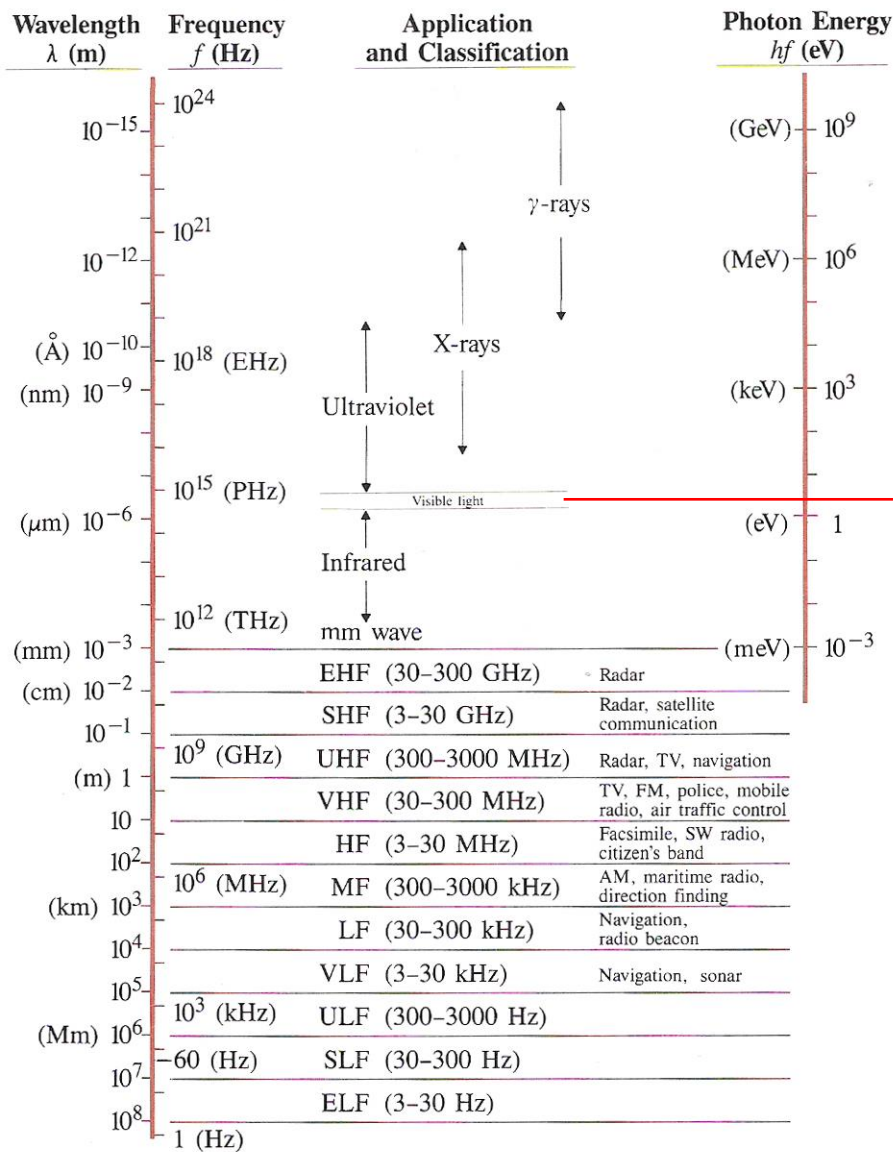
$$\rightarrow \text{Relative to ether?}$$

Incoming wave (blue-shifted): $c'_{in} = c + v$

Outgoing wave (red shifted): $c'_{out} = c - v$

Is this true?

Special & general relativity introduced by A. Einstein!!

Electromagnetic Spectrum

Wavelength λ (m)	Frequency f (Hz)	Application and Classification	Photon Energy hf (eV)

Visible light
(400 nm ~ 700 nm)

Water absorption

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.   J. D. Jackson, Classical Electrodynamics, 3rd ed. Wiley, 1999.

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