

Electromagnetics:

Oblique Incidence at a Plane Conducting Boundary

Normal Incidence at a Plane Dielectric Boundary

(8-7, 8-8)

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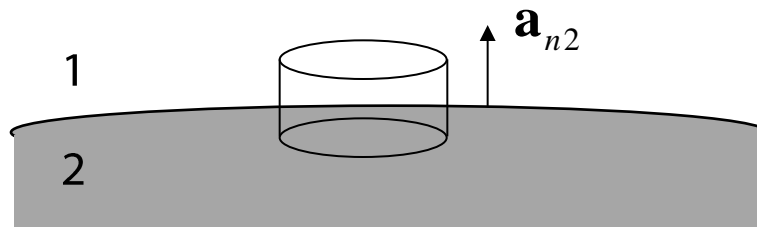
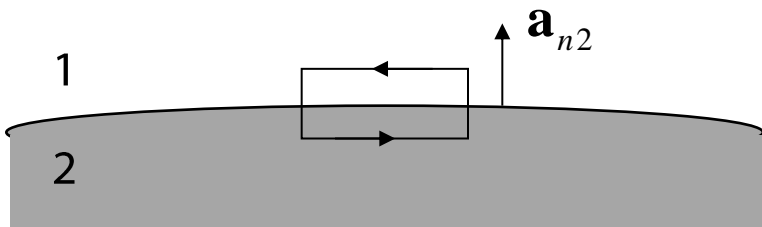
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Electromagnetic Boundary Conditions

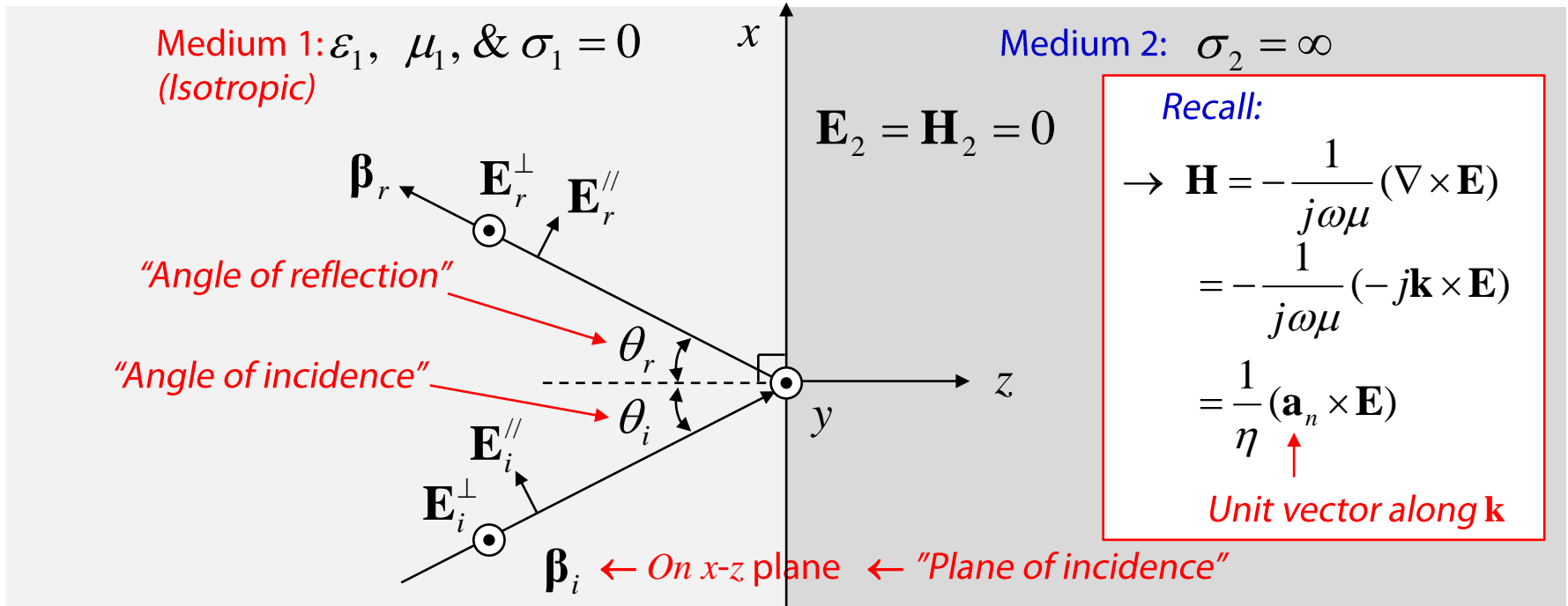
Continuity conditions:

$$\begin{aligned} \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 &\rightarrow \oint_C \mathbf{E} \cdot d\mathbf{l} = - \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} \rightarrow E_{1t} = E_{2t} \\ &\rightarrow \mathbf{a}_{n2} \times (\mathbf{E}_1 - \mathbf{E}_2) = 0 \\ \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} &\rightarrow \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{s} \\ &\rightarrow \mathbf{a}_{n2} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s \\ \nabla \cdot \mathbf{D} = \rho &\rightarrow \mathbf{a}_{n2} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s \\ \nabla \cdot \mathbf{B} = 0 &\rightarrow B_{1n} = B_{2n} \rightarrow \mathbf{a}_{n2} \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0 \end{aligned}$$



Oblique Incidence at a Plane Conducting Boundary

For plane waves: $E, H \propto e^{-j\boldsymbol{\beta} \cdot \mathbf{r}}$



$\rightarrow \boldsymbol{\beta}_i = \beta_i \mathbf{a}_{ni} = \beta_1 (\mathbf{a}_x \sin \theta_i + \mathbf{a}_z \cos \theta_i)$ \rightarrow Still orthogonal!

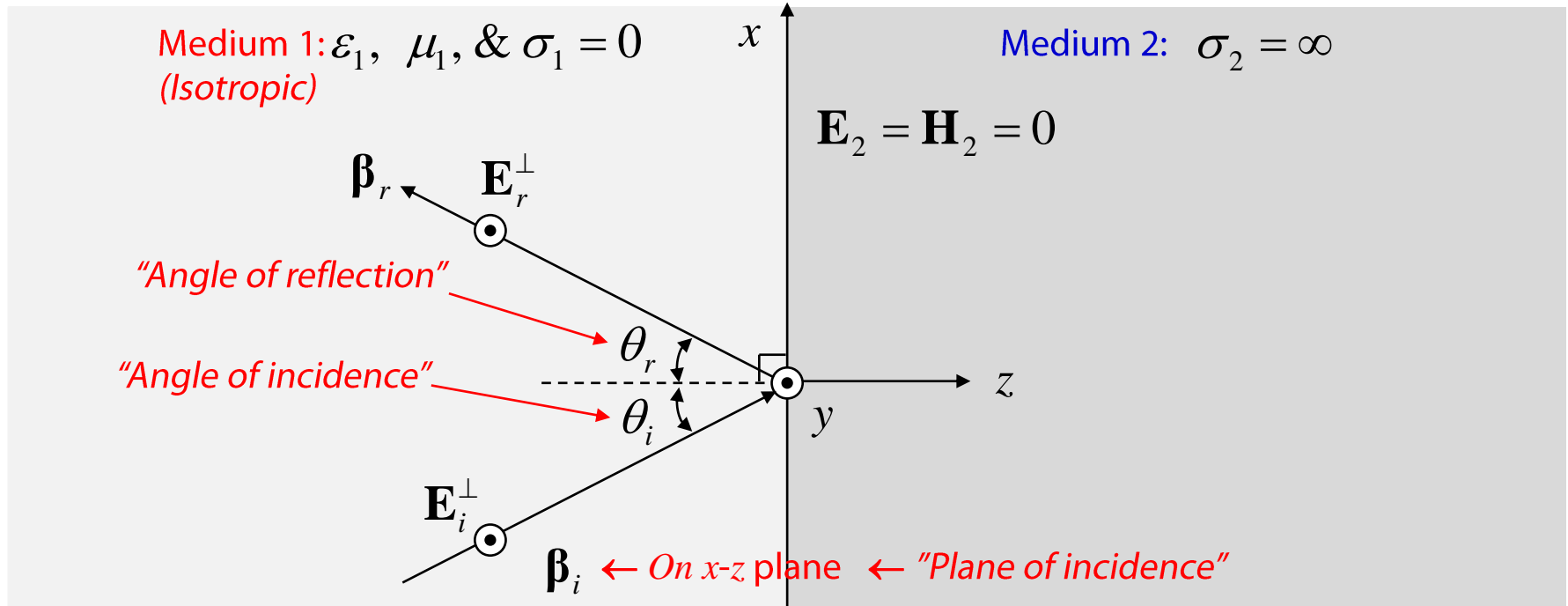
Boundary condition for E-field ($z = 0$): $\mathbf{E}_{1t} = \mathbf{E}_{2t} \rightarrow \mathbf{E}_i^\perp + \mathbf{E}_{it}^\parallel + \mathbf{E}_r^\perp + \mathbf{E}_{rt}^\parallel = 0$

Perpendicular polarization (TE)
 $\mathbf{E}_i^\perp + \mathbf{E}_r^\perp = 0 \quad (z = 0)$

Parallel polarization (TM)
 $\mathbf{E}_{it}^\parallel + \mathbf{E}_{rt}^\parallel = 0 \quad (z = 0)$

Oblique Incidence: Perpendicular Polarization (TE) (1)

For plane waves: $E, H \propto e^{-j\boldsymbol{\beta}\cdot\mathbf{r}}$



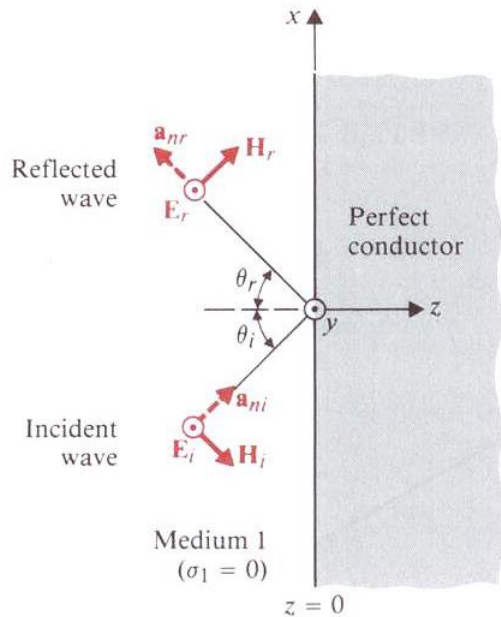
$$\rightarrow \boldsymbol{\beta}_i = \beta_i \mathbf{a}_{ni} = \beta_1 (\mathbf{a}_x \sin \theta_i + \mathbf{a}_z \cos \theta_i)$$

For the reflected field:

$$\rightarrow \boldsymbol{\beta}_r = \beta_r \mathbf{a}_{nr} = \beta_1 (\mathbf{a}_x \sin \theta_r - \mathbf{a}_z \cos \theta_r) \quad \leftarrow \text{Still on the plane of incidence}$$

Why?

Oblique Incidence: Perpendicular Polarization (TE) (2)



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

In medium 1 ($z < 0$):

Given $\mathbf{E}_i(x, z) = \mathbf{a}_y E_{i0} e^{-j\beta_i \cdot \mathbf{r}} = \mathbf{a}_y E_{i0} e^{-j\beta_1(x \sin \theta_i + z \cos \theta_i)}$

$$\mathbf{H}_i(x, z) = \frac{1}{\eta_1} [\mathbf{a}_{ni} \times \mathbf{E}_i(x, z)]$$

$$= \frac{E_{i0}}{\eta_1} (-\mathbf{a}_x \cos \theta_i + \mathbf{a}_z \sin \theta_i) e^{-j\beta_1(x \sin \theta_i + z \cos \theta_i)}$$

Unknown $\mathbf{E}_r(x, z) = \mathbf{a}_y E_{r0} e^{-j\beta_r \cdot \mathbf{r}} = \mathbf{a}_y E_{r0} e^{-j\beta_1(x \sin \theta_r - z \cos \theta_r)}$

$$\mathbf{H}_r(x, z) = \frac{1}{\eta_1} \mathbf{a}_{nr} \times \mathbf{E}_r(x, z)$$

In medium 2 ($z > 0$):

Known $\mathbf{E}_2(x, z) = 0$ for a perfect conductor

$\mathbf{H}_2(x, z) = 0$ ($\sigma = \infty$)

How many equations do we need to solve?

Oblique Incidence: Perpendicular Polarization (TE) (3)

For tangential components of E-fields:

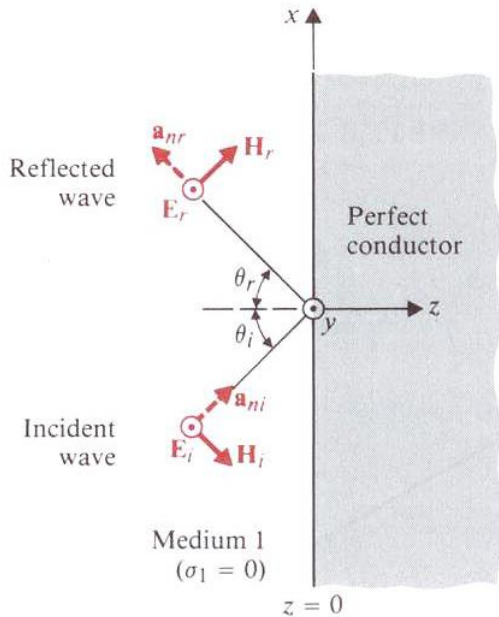
$$\mathbf{a}_{n2} \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$$

$$\rightarrow \mathbf{E}_{1t}(x,0) = \mathbf{E}_i(x,0) + \mathbf{E}_r(x,0) = \mathbf{E}_{2t}(x,0) = 0$$

$$\rightarrow \mathbf{E}_r(x,0) = -\mathbf{E}_i(x,0) = -\mathbf{a}_y E_{i0} e^{-j\beta_1 x \sin \theta_i}$$

$$\rightarrow \theta_r = \theta_i$$

$$\rightarrow \boldsymbol{\beta}_r = \beta_r \mathbf{a}_{nr} = \beta_1 (\mathbf{a}_x \sin \theta_i - \mathbf{a}_z \cos \theta_i)$$



$$\rightarrow \mathbf{E}_r(x, z) = -\mathbf{a}_y E_{i0} e^{-j\beta_1 (x \sin \theta_i - z \cos \theta_i)}$$

$$\rightarrow \mathbf{H}_r(x, z) = \frac{1}{\eta_1} \mathbf{a}_{nr} \times \mathbf{E}_r(x, z)$$

$$= -\frac{E_{i0}}{\eta_1} (\mathbf{a}_x \cos \theta_i + \mathbf{a}_z \sin \theta_i) e^{-j\beta_1 (x \sin \theta_i - z \cos \theta_i)}$$

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

What is the direction of the induced surface current density \mathbf{J}_s ?

Poynting vector: $\mathbf{E}_1 \times \mathbf{H}_1 = ?$ $\mathbf{P}_{av} = \frac{1}{2} \text{Re}[\mathbf{E}_1(x, z) \times \mathbf{H}_1^*(x, z)] = \mathbf{a}_x 2 \frac{E_{i0}^2}{\eta_1} \sin \theta_i \sin^2(\beta_1 z \cos \theta_i)$

\rightarrow Propagating wave in x-direction \rightarrow Standing wave in z-direction

Oblique Incidence: Parallel Polarization (TM)

In medium 1 ($z < 0$):

$$\mathbf{E}_i(x, z) = E_{i0} (\mathbf{a}_x \cos \theta_i - \mathbf{a}_z \sin \theta_i) e^{-j\beta_1(x \sin \theta_i + z \cos \theta_i)}$$

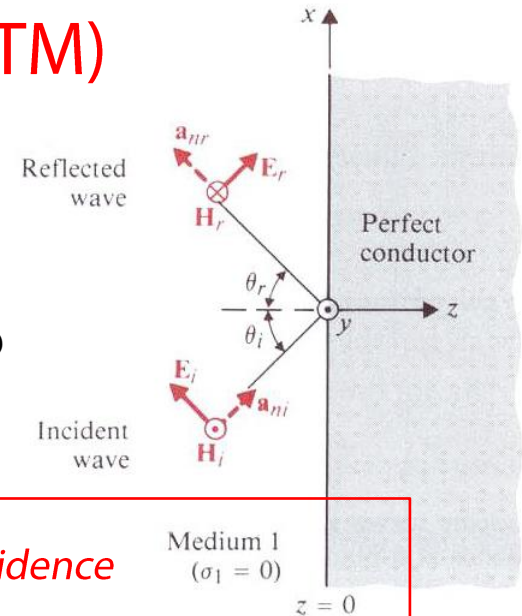
$$\mathbf{H}_i(x, z) = \frac{1}{\eta_1} [\mathbf{a}_{ni} \times \mathbf{E}_i(x, z)] = \mathbf{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1(x \sin \theta_i + z \cos \theta_i)}$$

$$\mathbf{E}_r(x, z) = \mathbf{a}_{er} E_{r0} e^{-j\beta_r \cdot \mathbf{r}} = \mathbf{a}_{er} E_{r0} e^{-j\beta_1(x \sin \theta_r - z \cos \theta_r)}$$

$$\mathbf{H}_r(x, z) = \frac{1}{\eta_1} \mathbf{a}_{nr} \times \mathbf{E}_r(x, z)$$

→ β_r : Still on the plane of incidence

Why?



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

For tangential components of E-fields:

$$\mathbf{a}_{n2} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0 \quad \rightarrow \quad \mathbf{E}_{1t}(x, 0) = \mathbf{E}_{it}(x, 0) + \mathbf{E}_{rt}(x, 0) = \mathbf{E}_{2t}(x, 0) = 0$$

$$\rightarrow \mathbf{E}_{rt}(x, 0) = -\mathbf{E}_{it}(x, 0) = -\mathbf{a}_x E_{i0} \cos \theta_i e^{-j\beta_1 x \sin \theta_i}$$

$$\rightarrow \theta_r = \theta_i \quad \rightarrow \quad \boldsymbol{\beta}_r = \beta_r \mathbf{a}_{nr} = \beta_1 (\mathbf{a}_x \sin \theta_i - \mathbf{a}_z \cos \theta_i)$$

$$\rightarrow \mathbf{a}_{er} E_{r0} = -E_{i0} (\mathbf{a}_x \cos \theta_i + \mathbf{a}_z \sin \theta_i) \quad \text{Why?}$$

$$\rightarrow \mathbf{E}_r(x, z) = -E_{i0} (\mathbf{a}_x \cos \theta_i + \mathbf{a}_z \sin \theta_i) e^{-j\beta_1(x \sin \theta_i - z \cos \theta_i)}$$

$$\rightarrow \mathbf{H}_r(x, z) = \mathbf{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1(x \sin \theta_i - z \cos \theta_i)}$$

Normal Incidence at a Plane Dielectric Boundary (1)

Lossless dielectric media: $\sigma_1 = \sigma_2 = 0$

In medium 1 ($z < 0$):

Given $\mathbf{E}_i(z) = \mathbf{a}_x E_{i0} e^{-j\beta_1 z}$

$$\mathbf{H}_i(z) = \frac{1}{\eta_1} \mathbf{a}_{ni} \times \mathbf{E}_i(z) = \mathbf{a}_y \frac{E_{i0}}{\eta_1} e^{-j\beta_1 z}$$

Unknown

$\mathbf{E}_r(z) = \mathbf{a}_x E_{r0} e^{+j\beta_1 z}$

$$\mathbf{H}_r(z) = \frac{1}{\eta_1} \mathbf{a}_{nr} \times \mathbf{E}_r(z) = -\mathbf{a}_y \frac{E_{r0}}{\eta_1} e^{+j\beta_1 z}$$

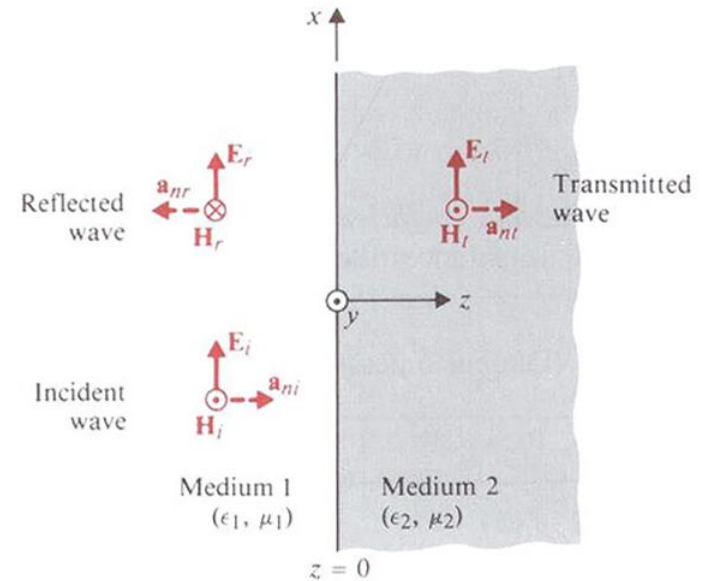
In medium 2 ($z > 0$):

Unknown

$\mathbf{E}_t(z) = \mathbf{a}_x E_{t0} e^{-j\beta_2 z}$

$$\mathbf{H}_t(z) = \mathbf{a}_y \frac{E_{t0}}{\eta_2} e^{-j\beta_2 z}$$

How many equations do we need to solve?



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$\begin{aligned} \boldsymbol{\beta}_i &= \beta_i \mathbf{a}_{ni} = \beta_1 \mathbf{a}_z \\ \boldsymbol{\beta}_r &= \beta_r \mathbf{a}_{nr} = \beta_1 (-\mathbf{a}_z) \end{aligned}$$

Normal Incidence at a Plane Dielectric Boundary (2)

For tangential components of E-fields:

$$\mathbf{a}_{n2} \times (\mathbf{E}_1 - \mathbf{E}_2) = 0 \rightarrow \mathbf{E}_i(0) + \mathbf{E}_r(0) = \mathbf{E}_t(0) \rightarrow E_{i0} + E_{r0} = E_{t0}$$

For tangential components of H-fields:

$$\mathbf{a}_{n2} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s = 0 \rightarrow \mathbf{H}_i(0) + \mathbf{H}_r(0) = \mathbf{H}_t(0)$$

$$\rightarrow \frac{1}{\eta_1} (E_{i0} - E_{r0}) = \frac{1}{\eta_2} E_{t0}$$

Reflection and transmission coefficients:

$$E_{r0} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{i0} \equiv \Gamma E_{i0} \rightarrow \text{Reflection coefficient}$$

$$E_{t0} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{i0} \equiv \tau E_{i0} \rightarrow \text{Transmission coefficient}$$

$$\rightarrow 1 + \Gamma = \tau \leftarrow \text{Continuity of E-field}$$

$$\rightarrow 1 - \Gamma^2 = \frac{\eta_1}{\eta_2} \tau^2 \leftarrow \text{Conservation of power density}$$