Electromagnetics:
Optical Fibers

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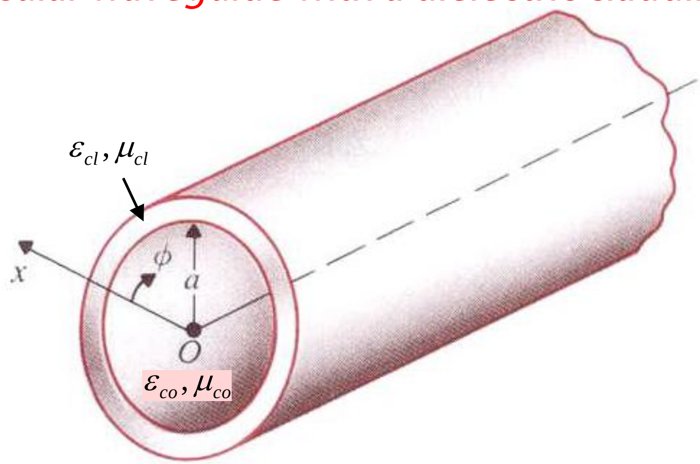
Circular Dielectric Waveguides: Optical Fibers

A circular waveguide with a dielectric cladding



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$\boxed{\mu_{co}\varepsilon_{co} > \mu_{cl}\varepsilon_{cl}}$$

Let: $\gamma = j\beta$

$$k^2 = \omega^2\mu\varepsilon = \omega^2\begin{bmatrix}\mu_{co}\varepsilon_{co}, & r \le a \\ \mu_{cl}\varepsilon_{cl}, & r > a\end{bmatrix}$$

$$\to h^2 = k_{co}^2 - \beta^2$$

$$\to q^2 = \beta^2 - k_{cl}^2$$

Let:

$$E_{co,z}(r,\phi,z) = E^0_{co,z}(r,\phi)e^{-\gamma z}$$
$$H_{co,z}(r,\phi,z) = H^0_{co,z}(r,\phi)e^{-\gamma z}$$

$\to$ *Core*

$$E_{cl,z}(r,\phi,z) = E^0_{cl,z}(r,\phi)e^{-\gamma z}$$
$$H_{cl,z}(r,\phi,z) = H^0_{cl,z}(r,\phi)e^{-\gamma z}$$

$\to$ *Cladding*

Recall:

$$\boxed{R(r) = \begin{cases} CJ_n(hr) + DN_n(hr) & \text{for } h^2 > 0 \\ CI_n(hr) + DK_n(hr) & \text{for } h^2 < 0 \end{cases}}$$

$$\to E^0_{co,z}, H^0_{co,z} \propto J_n(hr)\cos n\phi$$

$$\to E^0_{cl,z}, H^0_{cl,z} \propto K_n(hr)\cos n\phi$$

TM Modes in Circular Dielectric Waveguides

Unless n = 0, the B.C. cannot be satisfied only with E_z!

Recall:

$$\mathbf{E}_T^0 = -\frac{\gamma}{h^2}\nabla_T E_z^0 \leftarrow \nabla_T E_z^0 = \left(\mathbf{a}_r \frac{\partial}{\partial r} + \mathbf{a}_\phi \frac{\partial}{r\partial\phi}\right)E_z^0 \rightarrow \mathbf{H} = \frac{1}{Z_{TM}}\mathbf{a}_z \times \mathbf{E} \quad \leftarrow Z_{TM} = \gamma / j\omega\varepsilon$$

Core:	$E_{co,r}^0 = -\frac{j\beta}{h^2} hA_n J_n'(hr)\cos n\phi$	$H_{co,r}^0 = \frac{\omega\varepsilon_{co}}{\beta}(-1)E_{co,\phi}^0 = -\frac{j\omega\varepsilon_{co} n}{h^2 r}A_n J_n(hr)\sin n\phi$
	$E_{co,\phi}^0 = -\frac{j\beta}{h^2}\frac{(-n)}{r}A_n J_n(hr)\sin n\phi$	$H_{co,\phi}^0 = \frac{\omega\varepsilon_{co}}{\beta}E_{co,r}^0 = -\frac{j\omega\varepsilon_{co}}{h}A_n J_n'(hr)\cos n\phi$
	$E_{co,z}^0 = A_n J_n(hr)\cos n\phi$	$H_{co,z}^0 = 0$
Cladding:	$E_{cl,r}^0 = \frac{j\beta}{q^2} qC_n K_n'(qr)\cos n\phi$	$H_{cl,r}^0 = \frac{\omega\varepsilon_{cl}}{\beta}(-1)E_{cl,\phi}^0 = \frac{j\omega\varepsilon_{cl} n}{q^2 r}C_n K_n(qr)\sin n\phi$
	$E_{cl,\phi}^0 = \frac{j\beta}{q^2}\frac{(-n)}{r}C_n K_n(qr)\sin n\phi$	$H_{cl,\phi}^0 = \frac{\omega\varepsilon_{cl}}{\beta}E_{cl,r}^0 = \frac{j\omega\varepsilon_{cl}}{q}C_n K_n'(qr)\cos n\phi$
	$E_{cl,z}^0 = C_n K_n(qr)\cos n\phi$	$H_{cl,z}^0 = 0$
B.C.:	$E_{co,z}^0(a) = E_{cl,z}^0(a)$ $E_{co,\phi}^0(a) = E_{cl,\phi}^0(a)$	$H_{co,z}^0(a) = H_{cl,z}^0(a)$ $H_{co,\phi}^0(a) = H_{cl,\phi}^0(a)$

TE Modes in Circular Dielectric Waveguides

Unless n = 0, the B.C. cannot be satisfied only with H_z!

$\leftarrow Z_{TE} = j\omega\mu/\gamma$

Recall:

$$\mathbf{H}_T^0 = -\frac{\gamma}{h^2}\nabla_T H_z^0 \leftarrow \nabla_T H_z^0 = \left(\mathbf{a}_r \frac{\partial}{\partial r} + \mathbf{a}_\phi \frac{\partial}{r\partial\phi}\right) H_z^0 \rightarrow \mathbf{E} = -Z_{TE}\mathbf{a}_z \times \mathbf{H}$$

Core:	$H_{co,r}^0 = -\frac{j\beta}{h^2} h B_n J_n'(hr)\cos n\phi$	$E_{co,r}^0 = -\frac{\omega\mu_{co}}{\beta}(-1)H_{co,\phi}^0 = \frac{j\omega\mu_{co} n}{h^2 r} B_n J_n(hr)\sin n\phi$
	$H_{co,\phi}^0 = -\frac{j\beta}{h^2}\frac{(-n)}{r} B_n J_n(hr)\sin n\phi$	$E_{co,\phi}^0 = -\frac{\omega\mu_{co}}{\beta} H_{co,r}^0 = \frac{j\omega\mu_{co}}{h} B_n J_n'(hr)\cos n\phi$
	$H_{co,z}^0 = B_n J_n(hr)\cos n\phi$	$E_{co,z}^0 = 0$
Cladding:	$H_{cl,r}^0 = \frac{j\beta}{q^2} q D_n K_n'(qr)\cos n\phi$	$E_{cl,r}^0 = -\frac{\omega\mu_{cl}}{\beta}(-1)H_{cl,\phi}^0 = -\frac{j\omega\mu_{cl} n}{q^2 r} D_n K_n(qr)\sin n\phi$
	$H_{cl,\phi}^0 = \frac{j\beta}{q^2}\frac{(-n)}{r} D_n K_n(qr)\sin n\phi$	$E_{cl,\phi}^0 = -\frac{\omega\mu_{cl}}{\beta} H_{cl,r}^0 = -\frac{j\omega\mu_{cl}}{q} D_n K_n'(qr)\cos n\phi$
	$H_{cl,z}^0 = D_n K_n(qr)\cos n\phi$	$E_{cl,z}^0 = 0$
B.C.:	$H_{co,z}^0(a) = H_{cl,z}^0(a)$ $H_{co,\phi}^0(a) = H_{cl,\phi}^0(a)$	$E_{co,z}^0(a) = E_{cl,z}^0(a)$ $E_{co,\phi}^0(a) = E_{cl,\phi}^0(a)$

Hybrid Modes in Circular Dielectric Waveguides (1)

Core:

TM	TE	TM	TE
$E^0_{co,r} = -\frac{j\beta}{h^2} hA_n J_n'(hr)\cos n\phi$	$E^0_{co,r} = \frac{j\omega\mu_{co} n}{h^2 r} B_n J_n(hr)\sin(n\phi+\phi_0)$	$H^0_{co,r} = -\frac{j\omega\varepsilon_{co} n}{h^2 r} A_n J_n(hr)\sin n\phi$	$H^0_{co,r} = -\frac{j\beta}{h^2} hB_n J_n'(hr)\cos(n\phi+\phi_0)$
$E^0_{co,\phi} = -\frac{j\beta}{h^2}\frac{(-n)}{r} A_n J_n(hr)\sin n\phi$	$E^0_{co,\phi} = \frac{j\omega\mu_{co}}{h} B_n J_n'(hr)\cos(n\phi+\phi_0)$	$H^0_{co,\phi} = -\frac{j\omega\varepsilon_{co}}{h} A_n J_n'(hr)\cos n\phi$	$H^0_{co,\phi} = -\frac{j\beta}{h^2}\frac{(-n)}{r} B_n J_n(hr)\sin(n\phi+\phi_0)$
$E^0_{co,z} = A_n J_n(hr)\cos n\phi$	$E^0_{co,z} = 0$	$H^0_{co,z} = 0$	$H^0_{co,z} = B_n J_n(hr)\cos(n\phi+\phi_0)$

$$\to E^0_{co,r} = -\frac{j\beta}{h^2}\left[hA_n J_n'(hr)\cos n\phi - \frac{\omega\mu_{co} n}{\beta r} B_n J_n(hr)\sin(n\phi+\phi_0)\right]$$
$$\to E^0_{co,\phi} = -\frac{j\beta}{h^2}\left[\frac{(-n)}{r} A_n J_n(hr)\sin n\phi - \frac{\omega\mu_{co}}{\beta} B_n hJ_n'(hr)\cos(n\phi+\phi_0)\right]$$
$$\to E^0_{co,z} = A_n J_n(hr)\cos n\phi$$

$$\to H^0_{co,r} = -\frac{j\beta}{h^2}\left[\frac{\omega\varepsilon_{co} n}{\beta r} A_n J_n(hr)\sin n\phi + hB_n J_n'(hr)\cos(n\phi+\phi_0)\right]$$
$$\to H^0_{co,\phi} = -\frac{j\beta}{h^2}\left[\frac{\omega\varepsilon_{co}}{\beta} hA_n J_n'(hr)\cos n\phi - \frac{n}{r} B_n J_n(hr)\sin(n\phi+\phi_0)\right]$$
$$\to H^0_{co,z} = B_n J_n(hr)\cos(n\phi+\phi_0)$$

Cladding:

$E^0_{cl,r} = \frac{j\beta}{q^2} qC_n K_n'(qr)\cos n\phi$	$E^0_{cl,r} = -\frac{j\omega\mu_{cl} n}{q^2 r} D_n K_n(qr)\sin(n\phi+\phi_0)$	$H^0_{cl,r} = \frac{j\omega\varepsilon_{cl} n}{q^2 r} C_n K_n(qr)\sin n\phi$	$H^0_{cl,r} = \frac{j\beta}{q^2} qD_n K_n'(qr)\cos(n\phi+\phi_0)$
$E^0_{cl,\phi} = \frac{j\beta}{q^2}\frac{(-n)}{r} C_n K_n(qr)\sin n\phi$	$E^0_{cl,\phi} = -\frac{j\omega\mu_{cl}}{q} D_n K_n'(qr)\cos(n\phi+\phi_0)$	$H^0_{cl,\phi} = \frac{j\omega\varepsilon_{cl}}{q} C_n K_n'(qr)\cos n\phi$	$H^0_{cl,\phi} = \frac{j\beta}{q^2}\frac{(-n)}{r} D_n K_n(qr)\sin(n\phi+\phi_0)$
$E^0_{cl,z} = C_n K_n(qr)\cos n\phi$	$E^0_{cl,z} = 0$	$H^0_{cl,z} = 0$	$H^0_{cl,z} = D_n K_n(qr)\cos(n\phi+\phi_0)$

$$\to E^0_{cl,r} = \frac{j\beta}{q^2}\left[qC_n K_n'(qr)\cos n\phi - \frac{\omega\mu_{cl} n}{\beta r} D_n K_n(qr)\sin(n\phi+\phi_0)\right]$$
$$\to E^0_{cl,\phi} = \frac{j\beta}{q^2}\left[\frac{(-n)}{r} C_n K_n(qr)\sin n\phi - \frac{\omega\mu_{cl}}{\beta} qD_n K_n'(qr)\cos(n\phi+\phi_0)\right]$$
$$\to E^0_{cl,z} = C_n K_n(qr)\cos n\phi$$

$$\to H^0_{cl,r} = \frac{j\beta}{q^2}\left[\frac{\omega\varepsilon_{cl} n}{\beta r} C_n K_n(qr)\sin n\phi + qD_n K_n'(qr)\cos(n\phi+\phi_0)\right]$$
$$\to H^0_{cl,\phi} = \frac{j\beta}{q^2}\left[\frac{\omega\varepsilon_{cl}}{\beta} qC_n K_n'(qr)\cos n\phi + \frac{(-n)}{r} D_n K_n(qr)\sin(n\phi+\phi_0)\right]$$
$$\to H^0_{cl,z} = D_n K_n(qr)\cos(n\phi+\phi_0)$$

B.C.:

$$E^0_{co,z}(a) = E^0_{cl,z}(a)$$
$$E^0_{co,\phi}(a) = E^0_{cl,\phi}(a)$$

$$H^0_{co,z}(a) = H^0_{cl,z}(a)$$
$$H^0_{co,\phi}(a) = H^0_{cl,\phi}(a)$$

Hybrid Modes in Circular Dielectric Waveguides (2)

Boundary condition:

$$E^0_{co,z}(a)=E^0_{cl,z}(a)\quad\rightarrow A_nJ_n(ha)\cos n\phi=C_nK_n(qa)\cos n\phi\quad \boxed{\rightarrow A_n=C_n\frac{K_n(qa)}{J_n(ha)}}$$

$$E^0_{co,\phi}(a)=E^0_{cl,\phi}(a)\quad\rightarrow -\frac{j\beta}{h^2}[\frac{(-n)}{a}A_nJ_n(ha)\sin n\phi-\frac{\omega\mu_{co}}{\beta}B_nhJ'_n(ha)\cos(n\phi+\phi_0)]=\frac{j\beta}{q^2}[\frac{(-n)}{a}C_nK_n(qa)\sin n\phi-\frac{\omega\mu_{cl}}{\beta}qD_nK'_n(qa)\cos(n\phi+\phi_0)]$$

$$H^0_{co,z}(a)=H^0_{cl,z}(a)\quad\rightarrow B_nJ_n(ha)\cos(n\phi+\phi_0)=D_nK_n(qa)\cos(n\phi+\phi_0)\quad \boxed{\rightarrow D_n=B_n\frac{J_n(ha)}{K_n(qa)}}$$

$$H^0_{co,\phi}(a)=H^0_{cl,\phi}(a)\quad\rightarrow -\frac{j\beta}{h^2}[\frac{\omega\varepsilon_{co}}{\beta}hA_nJ'_n(ha)\cos n\phi-\frac{n}{a}B_nJ_n(ha)\sin(n\phi+\phi_0)]=\frac{j\beta}{q^2}[\frac{\omega\varepsilon_{cl}}{\beta}qC_nK'_n(qa)\cos n\phi+\frac{(-n)}{a}D_nK_n(qa)\sin(n\phi+\phi_0)]$$

$$\rightarrow A_n\frac{n}{h^2a}J_n(ha)\sin n\phi+B_n\frac{\omega\mu_{co}}{h\beta}J'_n(ha)\cos(n\phi+\phi_0)+\frac{n}{q^2a}C_nK_n(qa)\sin n\phi+D_n\frac{\omega\mu_{cl}}{q\beta}K'_n(qa)\cos(n\phi+\phi_0)=0$$

$$\rightarrow A_n\frac{\omega\varepsilon_{co}}{h\beta}J'_n(ha)\cos n\phi-B_n\frac{n}{h^2a}J_n(ha)\sin(n\phi+\phi_0)+C_n\frac{\omega\varepsilon_{cl}}{q\beta}K'_n(qa)\cos n\phi-D_n\frac{n}{q^2a}K_n(qa)\sin(n\phi+\phi_0)=0$$

$$\boxed{\begin{aligned}&\rightarrow B_n\left[\frac{\omega\mu_{co}}{h\beta}J'_n(ha)+\frac{\omega\mu_{cl}}{q\beta}\frac{K'_n(qa)}{K_n(qa)}J_n(ha)\right]\cos(n\phi+\phi_0)+C_n\left[\frac{n}{q^2a}+\frac{n}{h^2a}\right]K_n(qa)\sin n\phi=0\\&\rightarrow -B_n\left[\frac{n}{h^2a}+\frac{n}{q^2a}\right]J_n(ha)\sin(n\phi+\phi_0)+C_n\left[\frac{\omega\varepsilon_{co}}{h\beta}\frac{J'_n(ha)}{J_n(ha)}K_n(qa)+\frac{\omega\varepsilon_{cl}}{q\beta}K'_n(qa)\right]\cos n\phi=0\end{aligned}}$$

→ For a non-trivial solution to exist: $\rightarrow \phi_0=\pm\frac{\pi}{2}$

$$\rightarrow\left[\frac{\omega\mu_{co}}{h\beta}J'_n(ha)+\frac{\omega\mu_{cl}}{q\beta}\frac{K'_n(qa)}{K_n(qa)}J_n(ha)\right]\left[\frac{\omega\varepsilon_{co}}{h\beta}\frac{J'_n(ha)}{J_n(ha)}K_n(qa)+\frac{\omega\varepsilon_{cl}}{q\beta}K'_n(qa)\right]\cos n\phi\cos(n\phi+\phi_0)+\left[\frac{n}{h^2a}+\frac{n}{q^2a}\right]^2J_n(ha)K_n(qa)\sin n\phi\sin(n\phi+\phi_0)=0$$

Transcendental equation for modes:

$$\boxed{\rightarrow\left[\frac{\mu_{co}}{ha}\frac{J'_n(ha)}{J_n(ha)}+\frac{\mu_{cl}}{qa}\frac{K'_n(qa)}{K_n(qa)}\right]\left[\frac{\varepsilon_{co}}{ha}\frac{J'_n(ha)}{J_n(ha)}+\frac{\varepsilon_{cl}}{qa}\frac{K'_n(qa)}{K_n(qa)}\right]-n^2\left[\left(\frac{1}{ha}\right)^2+\left(\frac{1}{qa}\right)^2\right]^2\left(\frac{\beta}{\omega}\right)^2=0}$$

Characteristics of Hybrid Modes (1)

Transcendental equation for modes:

$$\left[\frac{\mu_{co}}{ha}\frac{J_n'(ha)}{J_n(ha)}+\frac{\mu_{cl}}{qa}\frac{K_n'(qa)}{K_n(qa)}\right]\left[\frac{\varepsilon_{co}}{ha}\frac{J_n'(ha)}{J_n(ha)}+\frac{\varepsilon_{cl}}{qa}\frac{K_n'(qa)}{K_n(qa)}\right]=n^2\left[\left(\frac{1}{ha}\right)^2+\left(\frac{1}{qa}\right)^2\right]^2\left(\frac{\beta}{\omega}\right)^2=0$$

For non-magnetic media: $\rightarrow \mu_{co}=\mu_{cl}=\mu_0$

$$\rightarrow\left[\frac{J_n'(ha)}{haJ_n(ha)}+\frac{K_n'(qa)}{qaK_n(qa)}\right]\left[\frac{n_{co}^2J_n'(ha)}{haJ_n(ha)}+\frac{n_{cl}^2K_n'(qa)}{qaK_n(qa)}\right]=n^2\left[\left(\frac{1}{ha}\right)^2+\left(\frac{1}{qa}\right)^2\right]^2\left(\frac{\beta}{k_0}\right)^2=0$$

Solution:

$$\rightarrow\frac{J_n'(ha)}{haJ_n(ha)}=-\frac{n_{co}^2+n_{cl}^2}{2n_{co}^2}\frac{K_n'(qa)}{qaK_n(qa)}\pm\left\{\left(\frac{n_{co}^2-n_{cl}^2}{2n_{co}^2}\right)^2\left(\frac{K_n'(qa)}{qaK_n(qa)}\right)^2+\frac{n^2}{n_{co}^2}\left(\frac{\beta}{k_0}\right)^2\left[\frac{1}{(ha)^2}+\frac{1}{(qa)^2}\right]^2\right\}^{1/2}$$

$$\leftarrow\begin{cases}J_n'(x)=-J_{n+1}'(x)+\dfrac{n}{x}J_n(x)\\ J_n'(x)=J_{n-1}'(x)-\dfrac{n}{x}J_n(x)\end{cases}$$

EH modes:

$$\rightarrow\frac{J_{n+1}(ha)}{haJ_n(ha)}=\frac{n_{co}^2+n_{cl}^2}{2n_{co}^2}\frac{K_n'(qa)}{qaK_n(qa)}+\frac{n}{(ha)^2}-\left[\left(\frac{n_{co}^2-n_{cl}^2}{2n_{co}^2}\right)^2\left(\frac{K_n'(qa)}{qaK_n(qa)}\right)^2+\frac{n^2}{n_{co}^2}\left(\frac{\beta}{k_0}\right)^2\left(\frac{1}{h^2a^2}+\frac{1}{q^2a^2}\right)^2\right]^{1/2}$$

HE modes:

$$\rightarrow\frac{J_{n-1}(ha)}{haJ_n(ha)}=-\frac{n_{co}^2+n_{cl}^2}{2n_{co}^2}\frac{K_n'(qa)}{qaK_n(qa)}+\frac{n}{(ha)^2}-\left[\left(\frac{n_{co}^2-n_{cl}^2}{2n_{co}^2}\right)^2\left(\frac{K_n'(qa)}{qaK_n(qa)}\right)^2+\frac{n^2}{n_{co}^2}\left(\frac{\beta}{k_0}\right)^2\left(\frac{1}{h^2a^2}+\frac{1}{q^2a^2}\right)^2\right]^{1/2}$$

Characteristics of Hybrid Modes (2)

EH modes: $\rightarrow \frac{J_{n+1}(ha)}{haJ_n(ha)} = \frac{n_{co}^2 + n_{cl}^2}{2n_{co}^2}\frac{K_n'(qa)}{qaK_n(qa)} + \frac{n}{(ha)^2} - \left[\left(\frac{n_{co}^2 - n_{cl}^2}{2n_{co}^2}\right)^2 \left(\frac{K_n'(qa)}{qaK_n(qa)}\right)^2 + \frac{n^2}{n_{co}^2}\left(\frac{\beta}{k_0}\right)^2 \left(\frac{1}{h^2a^2} + \frac{1}{q^2a^2}\right)^2\right]^{1/2}$

$E_z \uparrow H_z \downarrow \rightarrow$ *TM-like*

HE modes: $\rightarrow \frac{J_{n-1}(ha)}{haJ_n(ha)} = -\frac{n_{co}^2 + n_{cl}^2}{2n_{co}^2}\frac{K_n'(qa)}{qaK_n(qa)} + \frac{n}{(ha)^2} - \left[\left(\frac{n_{co}^2 - n_{cl}^2}{2n_{co}^2}\right)^2 \left(\frac{K_n'(qa)}{qaK_n(qa)}\right)^2 + \frac{n^2}{n_{co}^2}\left(\frac{\beta}{k_0}\right)^2 \left(\frac{1}{h^2a^2} + \frac{1}{q^2a^2}\right)^2\right]^{1/2}$

$E_z \downarrow H_z \uparrow \rightarrow$ *TE-like*

For n = 0:

TM modes: $\rightarrow \frac{J_1(ha)}{haJ_0(ha)} = \frac{n_{cl}^2}{n_{co}^2}\frac{K_0'(qa)}{qaK_0(qa)} = -\frac{n_{cl}^2}{n_{co}^2}\frac{K_1(qa)}{qaK_0(qa)}$

TE modes: $\rightarrow \frac{J_1(ha)}{haJ_0(ha)} = \frac{K_0'(qa)}{qaK_0(qa)} = -\frac{K_1(qa)}{qaK_0(qa)}$

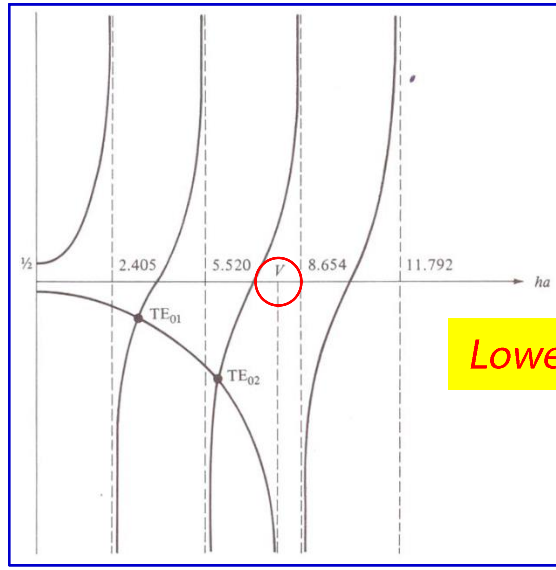
To be solved numerically

Cutoff Conditions

Graphical method

TM_{0m}: $\rightarrow \dfrac{J_1(ha)}{haJ_0(ha)} = -\dfrac{n_{cl}^2}{n_{co}^2}\dfrac{K_1(qa)}{qaK_0(qa)}$

TE_{0m}: $\rightarrow \dfrac{J_1(ha)}{haJ_0(ha)} = -\dfrac{K_1(qa)}{qaK_0(qa)}$



Lowest cutoff!

Recall:

$$\rightarrow h^2 = k_{co}^2 - \beta^2 \quad \rightarrow q^2 = \beta^2 - k_{cl}^2$$

$$\text{If } ha \rightarrow \frac{2\pi a}{\lambda_0}\sqrt{n_{co}^2 - n_{cl}^2} \equiv V,\; qa \rightarrow 0$$

$V < 2.405 \rightarrow$ Single-mode!

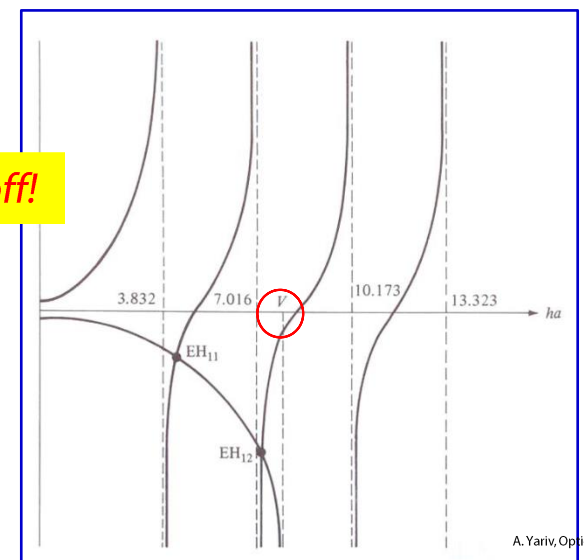
A. Yariv, Optical Electronics, 4th ed., HBJ, 1991.

EH_{1m}: $\rightarrow \dfrac{J_2(ha)}{haJ_1(ha)} = \dfrac{n_{co}^2+n_{cl}^2}{2n_{co}^2}\dfrac{K_1'(qa)}{qaK_1(qa)} + \dfrac{1}{(ha)^2} - \left[\left(\dfrac{n_{co}^2-n_{cl}^2}{2n_{co}^2}\right)^2\left(\dfrac{K_1'(qa)}{qaK_1(qa)}\right)^2 + \dfrac{1}{n_{co}^2}\left(\dfrac{\beta}{k_0}\right)^2\left(\dfrac{1}{h^2a^2}+\dfrac{1}{q^2a^2}\right)^2\right]^{1/2}$

HE_{1m}: $\rightarrow \dfrac{J_0(ha)}{haJ_1(ha)} = -\dfrac{n_{co}^2+n_{cl}^2}{2n_{co}^2}\dfrac{K_1'(qa)}{qaK_1(qa)} + \dfrac{1}{(ha)^2} - \left[\left(\dfrac{n_{co}^2-n_{cl}^2}{2n_{co}^2}\right)^2\left(\dfrac{K_1'(qa)}{qaK_1(qa)}\right)^2 + \dfrac{1}{n_{co}^2}\left(\dfrac{\beta}{k_0}\right)^2\left(\dfrac{1}{h^2a^2}+\dfrac{1}{q^2a^2}\right)^2\right]^{1/2}$

EH_{1m}:

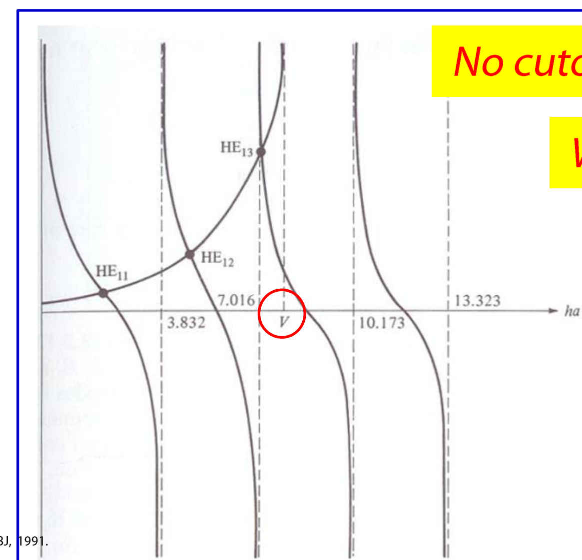
Cutoff!



HE_{1m}:

No cutoff!

Why?



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